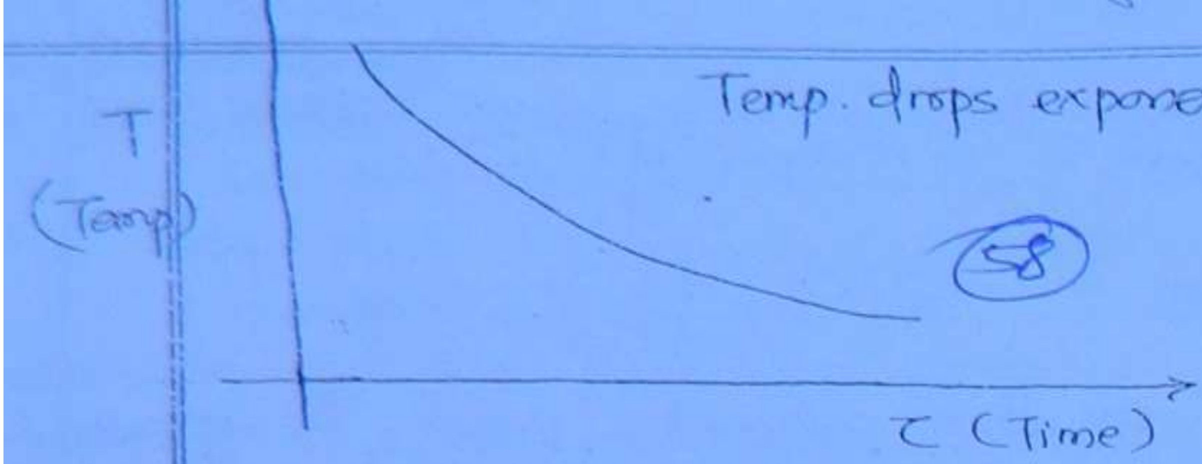


58

Since $\left(\frac{\rho V C_p}{hA}\right)$ has the unit of second. It is called time constant.

(Among all the liquids K of Hg is very high as close to steel 8 W/mK} Alcohol is also used as thermometric liquid).

Assumption ⇒
[At any instant of time τ sec body have same temp.] lumped h

In the above analysis it is assumed that internal temperature gradients within the body are neglected i.e. at any instance of time τ sec the entire body has uniform temp. Such analysis is called lumped heat capacity analysis.

Criteria for lumped heat capacity analysis
⇒ Biot Number < 0.1

Where Biot No. = $\frac{h \cdot s}{K_{solid}}$

$\therefore s = \left(\frac{V}{A}\right)$ V = Volume of body
A = Surface Area of body

For sphere $s = V/A = \dfrac{\frac{4}{3}\pi R^3}{4\pi R^2} = \dfrac{R}{3}$

Biot No. Resembles Nussell No.

$$Nu = \frac{hD}{k_{fluid}} \qquad Nu > 1$$

59

$$\text{Biot No} = \frac{(S/KA)}{(1/hA)}$$

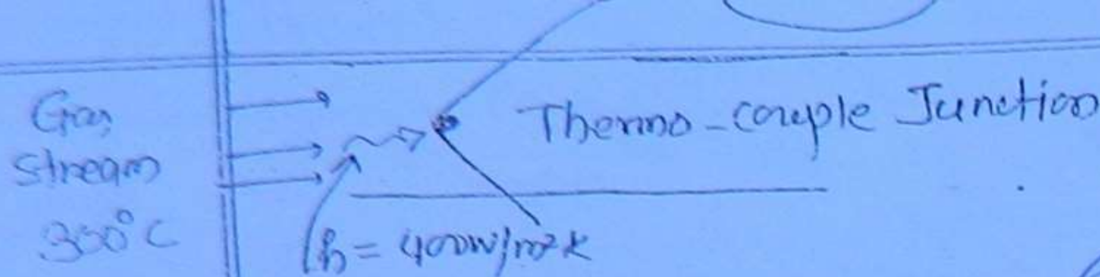
$$= \frac{\text{Internal conductive resistance}}{\text{External convective resistance}} = \frac{ICR}{ECR}$$

For Lumped heat capacity analysis is to be valid, the body is of very slender or thin shaped. And also thermal conductivity of the body is very high. e.g. Small copper sphere/ball.

$\boxed{\text{GATE 04}}$

A spherical thermal couple junction of diameter 0.706 mm is to be used for the measurement of temp of a gas stream, the convective h.t. coeff. (h) on the bead surface is 400 W/m²K. Thermo-physical properties of thermo-couple material are. K = 20 W/mK, c = 400 J/kgK & ρ = 8500 kg/m³. If thermo couple is initially at the 30°C is placed in a hot stream of 300°C, the time taken by the bead to reach 298°C is ⇒

(i) 2.35 sec (ii) 4.9 sec (iii) 14.7 sec (iv) 29.[illegible]



66

Now we have

$$\text{Biot No.} = \frac{h \cdot s}{K_{\cancel{fluid}\,solid}}$$

$$= \frac{400 \times 0.706 \times 10^{-3}}{3 \times 20 \times 2}$$

$$= 0.000235 < 0.1$$

$$\frac{T_i - T_\infty}{T - T_\infty} = e^{(hA/\rho V c_p) \cdot \tau}$$

$$\frac{30 - 300}{298 - 300} = e^{\left(\frac{400 \times 3 \times 2}{8500 \times 0.706 \times 10^{-3} \times 400}\right) \tau}$$

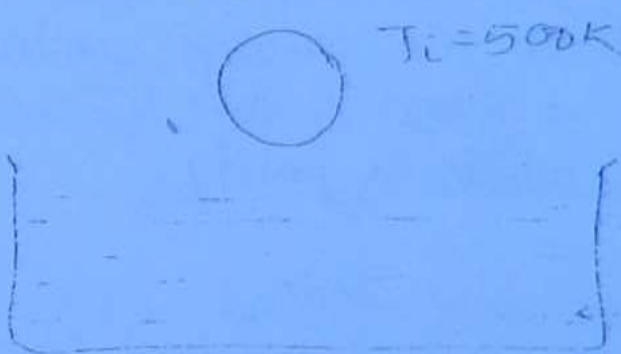
$$\boxed{\tau = 4.9 \text{ seconds.}}$$

GATE 2005

A small copper ball of 5 mm dia. at 500 K is dropped into an oil bath whose temp. is 300 K. The k of copper is 400 W/mK. Its density 9000 kg/m³ & its specific heat 385 J/kgK. If the heat transfer coeff. is 250 W/m²K. and the lumped analysis is assumed to be valid, the rate of fall of temperature of the ball at the beginning of cooling will be in K/s ⇒

(G1)

a] 8.7 b] 13.9 c] 17.3 d] 27.7



$(dT/d\tau) = ?$

Writing energy balance eqn at the very beginning of ~~the~~ dropping the ball.

$$hA(T_i - T_\infty) = -m c_p \left(\frac{dT}{d\tau}\right)_{at\ \tau = 0}$$

$$250 \times 4 \times \pi \times (2.5 \times 10^{-3})^2 (500 - 300) = -9000 \times \left(\frac{2.5}{1000}\right)^3 \times \frac{4}{3}\pi \times 385 \times \frac{dT}{d\tau}$$

$$\therefore (250 \times 200)(2.5 \times 10^{-3})^2 = -3000\left(\frac{2.5}{1000}\right)^3 \times 385 \times \frac{dT}{d\tau}$$

$$\therefore dT/d\tau = \underline{17.3}$$

As the time progresses, since the temperature diff. b/w the ball and the fluid keeps on decreasing the rate of cooling of the ball also decreasing i.e. the ball takes more time to get cool. for each °C.

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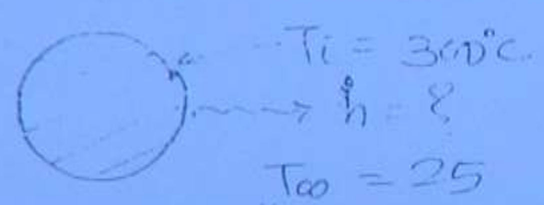
IES 2008

A copper sphere weighing 3 kg is heated in a furnace to a temp. of 300°C and is suddenly taken out & allowed to cool in ambient air at 25°C. If it takes 60 min. for the copper sphere to cool down to 35°C, what is the average surface H.T. coeff. Take $\rho_{copper} = 8950\ kg/m^3$ & $c_p = 0.383\ kJ/kg$. State the assumptions made & justify.

⟹



$$\left.\begin{aligned} T_0 &= 300°C \\ T &= 35°C \end{aligned}\right\} \text{Takes } \tau = 60 \times 60 = sec$$

$m = 3\ kg \qquad \rho = 8950\ kg/m^3$

$$\rho = \frac{m}{V}$$

$$8950 = \frac{3}{\frac{4\pi}{3} R^3}$$

$$\boxed{R = 0.043\ m}$$

Page 4

$$\frac{T_i - T_\infty}{T - T_\infty} = e^{+\left(\frac{hA}{\rho V C_p}\right)\tau}$$

Assume ⇒ (i) Lumped heat capacity analysis is valid

$$\text{Biot No} = \frac{hs}{k} < 0.1$$

Assuming highest probable value of convection heat transfer coefficient in air (forced conv.) as 400 W/m²K.

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$$\therefore \text{Biot No} = \frac{400 \times 0.043}{3 \times 385} \quad \text{— k of copper —}$$

$$= 0.0148 < 0.1$$

$$\therefore \frac{300 - 25}{35 - 25} = e^{+\left(\frac{h \, 3}{8950 \times 10 \cdot 0.043 \times 0.383}\right) 60 \times 60}$$

$$27.5 = e^{\left(\frac{h \times 3}{8950 \times 0.043 \times 0.383}\right) 3600}$$

$$\therefore \boxed{h = 45.1} \ \text{W/m}^2\text{K}$$

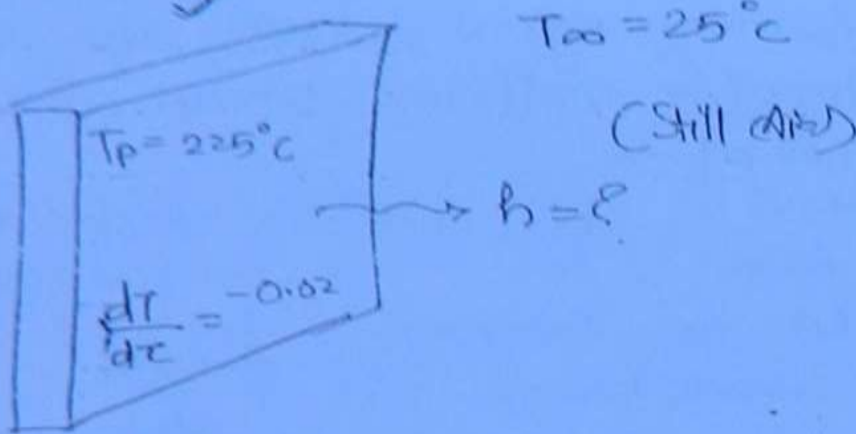
** Forced convection **

$\boxed{\text{GATE 2007}}$

The average H.T. coeff. on a thin hot vertical plate suspended in still air can be determined from observations of the change in plate temp with time as it cools. Assume the plate temp to be uniform at any instance of time & radiation heat exchange with the surrounding negligible. The ambient temp. is 25°C. The plate has a total surface area of 0.1 m² & mass of 4 kg. The specific heat of plate matl. 2.5 KJ/KgK. The h = in W/m²K at the instance when the plate temp. is 225°c & change in plate temp. with time $\frac{dT}{d\tau} = -0.02$ K/sec is

(64)

(i) 200 (ii) 20 (iii) 15 (iv) 10



At the instance of time when $\left(\frac{dT}{d\tau}\right)$ is $= -0.02$ K/sec

Energy balance eq^n

Rate of convection heat = Rate of decrease in I.E. of plate

$$h A (T_p - T_\infty) = -m c_p \left(\frac{dT}{d\tau}\right)$$

$$h \times 0.1 (225 - 25) = -4 \times 2.5 \times 10^3 \times (-0.02)$$

$$\therefore\ h = 10\ W/m^2 K$$

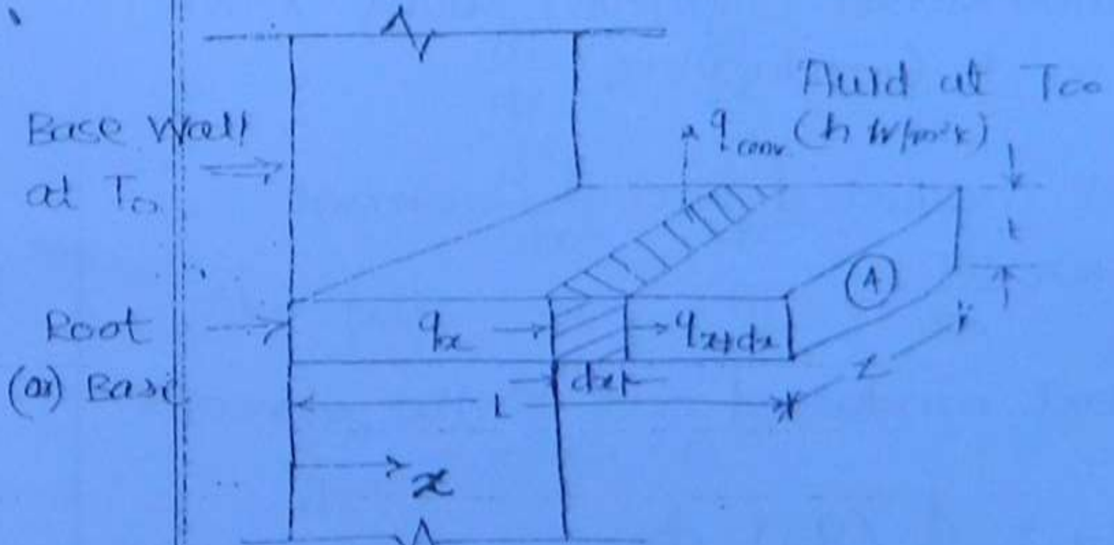
* FINS (Extended surfaces)

Fins are meant for increasing the H.T. rate from any surface to the fluid by increasing surface area of heat transfer. They are the extended surfaces projecting from the base wall. (65)

Fins are generally prefered only when the convective heat transfer coefficient (h) values are relatively low i.e. with air.

e.g. Air-cooled I.C Engine, Reciprocating air compressors, Car radiators, electric motors & transformers, Condenser of a Refrigerating Unit.

* Analysis of a Rectangular Fin.



L = Length of fin, A - Cross-sectional area of fin $= z \times t$
z = Width of fin
t = Thickness of fin

Heat is conducted into the fin at its root or base & then while conducting along the length of fin, it is simultaneously convected to the surrounding fluid at T_∞

66

Objectives of analysis

i) What is temp. of fin along x dirn.

$$T = f(x) = ?$$

ii) Heat transferred through the fin

$$q_{fin} = ?$$

Consider a differential element in the fin of length dx & let h be the convective H-T. coeff b/w the surface of fin & the fluid at T_∞

Assumptions :-

i) Steady state conditions.

ii) One dimensional conduction along x dirn

iii) Uniform K (conductivity)

Let, q_x = Heat conducted into the element

$$= -KA\frac{dT}{dx}$$

q_{x+dx} = Heat conducted out of the element

$$= q_x + \frac{d}{dx}(q_x)\cdot dx$$

$q_{convection}$ = Heat convected from the surface of element.

$$= h(P.dx)(T - T_\infty)$$

where P = Perimeter of fin

$$= (2z + 2t)$$

Writing the energy balance for steady state condi^n of element.

$$q_x = (q_{x+dx}) + q_{conv.} \qquad (67)$$

$-KA$

$$\cancel{q_x} = \cancel{q_x} + \frac{d}{dx}(q_x)\,dx + h(P.dx)(T - T_\infty)$$

$$0 = \frac{d}{dx}\left(-KA\frac{dT}{dx}\right)\cancel{dx} + h(P.\cancel{dx})(T - T_\infty)$$

$$\frac{d^2T}{dx^2} - \frac{hP}{KA}(T - T_\infty) = 0$$

Put $(T - T_\infty) = \theta$

$$\therefore \frac{dT}{dx} = \frac{d\theta}{dx}$$

$$\therefore \frac{d^2T}{dx^2} = \frac{d^2\theta}{dx^2}$$

Also put $m^2 = \left(\frac{hP}{KA}\right)$

$$\therefore \frac{d^2\theta}{dx^2} - m^2\theta = 0$$

This 2nd order D.E. in θ.

Solution to this is $\Rightarrow$

$$\theta_1 = C_1 e^{-mx} + C_2 e^{mx}$$

— where $m = \sqrt{\frac{hP}{KA}}$ (/meter)

C_1 & C_2 are const of integration to be found from boundary conditions.

One common B.C. is (68)

at $x=0$ $\quad T = T_0$ & $\theta = \theta_0 = T_0 - T_\infty$

(At the base). * The other B.C. depends upon three diff. cases.

⇒ Case (i) Fin is infinitely long (very long) then the temperature at the tip of the fin will be essentially that of the fluid.

i.e. at $x = \infty$ (Infinity)

$$T = T_\infty$$

$$\& \quad \theta \to 0$$

Then the solution will be :-

$$\boxed{\frac{\theta}{\theta_0} = \left(\frac{T - T_\infty}{T_0 - T_\infty}\right) = e^{-mx}}$$

Also heat transfer rate through the fin

$$q_{fin} = q_{\substack{\text{conducted}\\ \text{at the root}}} = -KA\left(\frac{dT}{dx}\right)_{x=0}$$

$$= \sqrt{hPKA}\ \theta_0$$

$$\boxed{q_{fin} = \sqrt{hPKA}\ (T_0 - T_\infty)} \text{ Watts.}$$

Temp. drops exponentially along the length of fin when the fin is very long.

Case (II) Fin's Tip is Insulated.

Then

$$q_{\text{conducted at } x=L} = 0$$

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$$\therefore -KA\left(\frac{dT}{dx}\right)_{x=L} = 0$$

$$\therefore \left(\frac{dT}{dx}\right)_{x=L} = 0$$

Then the solution is

$$\boxed{\frac{\theta}{\theta_0} = \frac{T - T_\infty}{T_0 - T_\infty} = \frac{\cosh[m(L-x)]}{\cosh mL}}$$

Also H.T. rate through fin $= q = \sqrt{hPKA}\,\theta_0 \tanh mL$

$$\boxed{q = \sqrt{hPKA}\;\theta_0 \tanh(mL)}$$

Case (III) Fin is finite in length & also looses heat by convection from its tip

$\Rightarrow$ Then the soln is

$$\boxed{\frac{\theta}{\theta_0} = \frac{T - T_\infty}{T_0 - T_\infty} = \frac{\cosh[m(L_c - x)]}{\cosh(mL_c)}}$$

& H.T. Rate through fin $\boxed{q = \sqrt{hPKA}\,(T_0 - T_\infty)\tanh(mL_c)}$

where L_c = Corrected Length $= (L + t/2)$

Among all the three cases, most commonly used case is " fin's tip ~~with~~ insulated."

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* Fin Efficiency *

It is defined as the ratio b/w actual H.T. rate through the fin and the max. possible H.T. rate through the fin. i.e. when the entire fin is at it's base temperature.

$$\eta_{fin} = \frac{q_{actual}}{q_{max.\ Possible}}$$

~~For fin's tip insulated case~~

Entire fin will be at its root temp. only when the mtrl of fin has infinite thermal conductivity (k)

For fin's tip insulated case :-

$$q_{fin} = \sqrt{hPKA}\ \theta_0 \tanh(mL)$$

$$q_{max.} = h(PL)(T_0 - T_\infty) \text{ Watts}$$

$$\eta_{fin} = \frac{\sqrt{hPKA}\ \cancel{\theta_0} \tanh(mL)}{hPL\ \cancel{(T_0 - T_\infty)}}$$

$$\boxed{\eta_{fin} = \frac{\tanh(mL)}{mL}}$$

$$\boxed{\eta_e \propto \sqrt{K}}$$

* Fin Effectiveness

It is ratio b/w heat transfer rate with fin and the heat transfer rate without fin.

$$\epsilon_{fin} = \frac{q_{with\ fin}}{q_{without\ fin}}$$

7

This parameter highlights the usefulness of fin i.e. whether the fins are really worth keeping or not.

$$\therefore \epsilon_{fin} = \frac{\sqrt{hPKA}\ \theta_0 \tanh(mL)}{hA(T_0 - T_\infty)}$$

$$\boxed{\epsilon_{fin} = \frac{\tanh(mL)}{\sqrt{hA/KP}}}$$

$$\boxed{\epsilon_{fin} \propto \sqrt{k}}$$

Hence the material of the fin should be of very high thermal conductivity. e.g. aluminium

$$\boxed{\epsilon_{fin} \propto 1/\sqrt{h}}$$

Hence fins do not really help when h value is very high. e.g. with water. So fins are kept when air only.

[IES 2004]

A electronic semiconductor device generates heat equal to 480×10^{-3} Watts. In order to keep the surface temp. at the upper safe limit of $70^\circ c$. The generated heat has to be dissipated to the surrounding which is at $30^\circ c$. To accomplish this task, aluminium fins of 0.7 mm square & 12 mm long are attached to the surface. The $K_{Aluminium} = 170$ W/mk. If the H.T. coeff. is 12 W/m^2K. Calculate the no. of fins required. Assume no heat loss from the tip of fins.

$\Longrightarrow$

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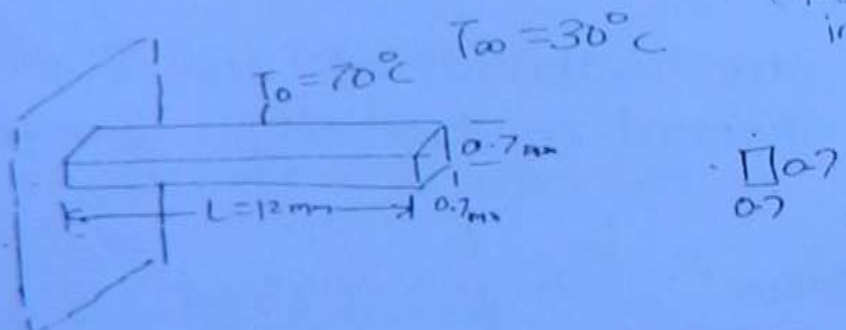
$$q = 480 \times 10^{-3} \text{ watts.}$$

$$T = 70^\circ c \qquad T_\infty = 30^\circ c$$

$$A_{fin} = 0.7 \text{ mm}^2 \qquad L = 12 \text{ mm}$$

$$K_{Aluminium} = 170 \text{ W/mk} \qquad h = 12 \text{ W/m}^2\text{K}$$

(Free convection in air)



$$q = \sqrt{hPKA}\ \theta_0 \tanh(mL)$$

$$m = \sqrt{hP/KA} = 12 \times$$

$$P = 2 \times z + 2 \times t$$

$$= 2 \times 0.7 + 2 \times 0.7$$

$$=$$

$$A = \left(\frac{0.7}{1000} * \frac{0.7}{1000}\right) m^2$$

$=$

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$\therefore\ m = 20\ /m$

Now Heat transfer rate through each fin (Insulated Tip)

$$q = \sqrt{hPKA}\ \theta_0 \tanh(mL)$$

$$q = \sqrt{12 \times P \times 170 \times A}\ (70 - 30) \tanh(20L)$$

$$\therefore q = 16 \times 10^{-3}\ \text{Watts}$$

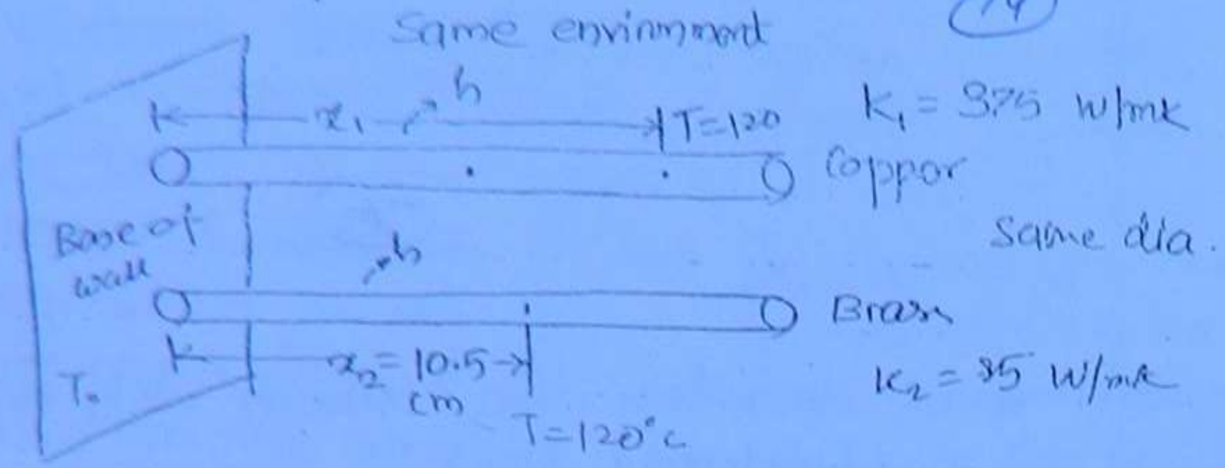
$$\text{No. of finss} = \frac{480 \times 10^{-3}}{16 \times 10^{-3}}$$

$= 30$ Fins are required.

IES 1993

Two long rods of same diameter one made of brass $k = 85 W/mK$ & other made of copper have one of their ends inserted in furnace. Both rods are exposed to the same environment at a section 10.5 cm away from the furnace end, the temp of the brass rod is 120°C at what distance from the furnace end, the same temp would be reached $\Rightarrow$ in the copper rod.

(74)



$$x_1 = ?$$

For very long rod

$$\frac{\theta}{\theta_0} = \frac{T - T_\infty}{\underbrace{T_0 - T_\infty}_{\text{This factor remains same for both rods}}} = e^{-mx}$$

For copper rod

$$\frac{\theta}{\theta_0} = e^{-mx_1}$$

For brass rod

$$\frac{\theta}{\theta_0} = e^{-mx_2}$$

$$\Rightarrow 1 = \frac{e^{-mx_1}}{e^{-mx_2}} = e^{-mx_1 + m_2x_2}$$

$$\log_e 1 = \log\left(e^{-mx_1 + m_2x_2}\right)$$

$$0 = -m_1x_1 + m_2x_2$$

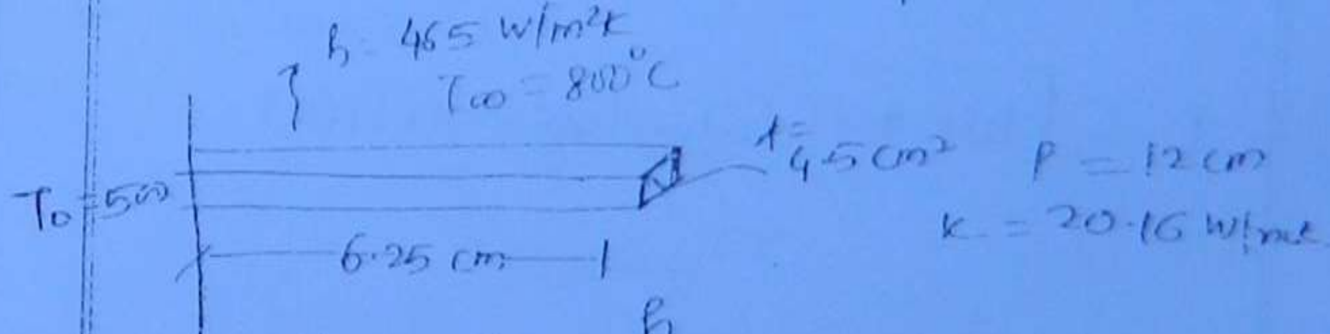
$m_1 x_1 = m_2 x_2$

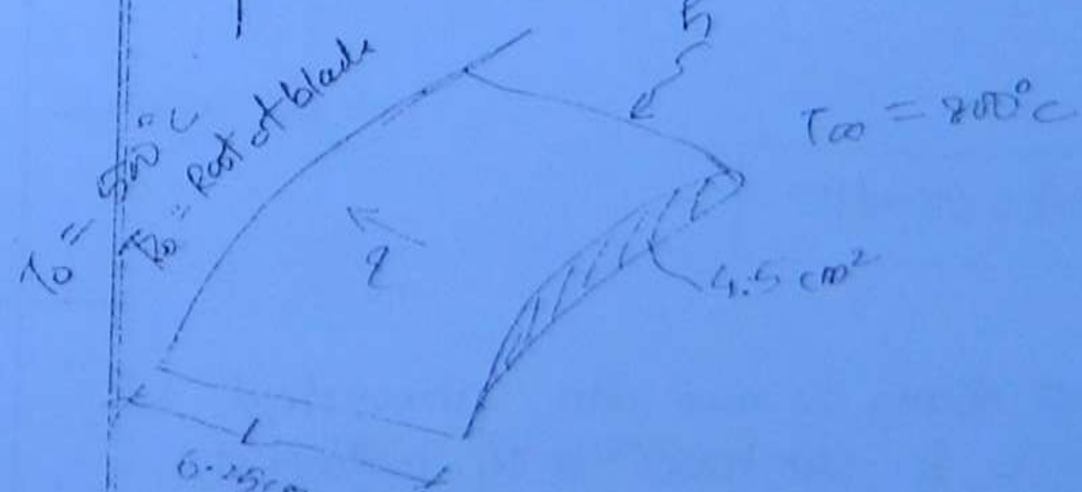
$$\frac{m_1}{m_2} = \frac{x_2}{x_1} = \sqrt{\frac{k_2}{k_1}}$$

75

$$\therefore \boxed{x_2 = 22\ cm}$$

A turbine blade ~~is~~ 6.25 cms long, c/s area 4.5 cm², Perimeter 12 cm is made of stainless steel k = 26.16 W/mK. The temp. of root is 500°C. The blade is exposed to a hot gas at 800°C & the average heat transfer coeff. is 0.465 kW/m²K. Determine the temp. at the tip & the rate of heat flow ~~be~~ at the root of the plate. Assume that tip is insulated.





Heat Transfer is from gases to the plate

) To get temp. at tip of fin. (76)

$$\frac{\theta}{\theta_0} = \frac{T-T_\infty}{T_0-T_\infty} = \frac{\cosh[m(L-x)]}{\cosh(mL)}$$

Put $x = L$ $\qquad m = \sqrt{\frac{hP}{kA}} = 68.84\,/m$

$$\frac{T_{x=L}-T_\infty}{T_0-T_\infty} = \frac{\cosh[m(L-L)]}{\cosh(mL)}$$

$$\frac{T_{x=L}-800}{500-800} = \frac{\cosh(0)}{\cosh(mL)}$$

$$T_{x=L} = 791.7^\circ C$$

$q = 243$ Watts

at $L=0$

$$q = \sqrt{hPKA}\,(T_0-T_\infty)\tanh(0)$$

$$\boxed{q = 243 \text{ Watts}}$$

IES 2000

Steel ball ~~dia~~ 12 mm dia. annealed by heated to 800°c & ~~st~~ then slowly cooling to 127°c in air at 50°c. The $h_{air} = 20\ W/m^2K$

Calculate the time required for cooling process. The properties of steel are taken as $k = 45 W/mK$

$\rho = 7830 \ kg/m^3$ & $c_p = 600 \ J/kgK$.

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→

$$hA(T_0 - T_\infty) = -m c_p \left(\frac{dT}{d\tau}\right)$$

$$\text{Biot No} = \frac{hs}{k} = \frac{hR}{3K} = 0.0008 < 0.1$$

Lumped heat capacity analysis can be done.

To get time of cooling

$$\frac{T_0 - T_\infty}{T - T_\infty} = e^{\left(\frac{hA}{\rho V c_p}\right)\tau}$$

$$\frac{800 - 50}{127 - 50} = e^{\left(\frac{20 \times 3}{7830 \times 6 \times 10^{-3} \times 600}\right)\tau}$$

$$\frac{750}{77} = e^{(60/783 \times 6 \times 6)\tau}$$

$$\frac{750}{77} = e^{(10/783 \times 6)\tau}$$

Taking $\log_e$

$$\left(\frac{10}{783 \times 6}\right)\tau =$$

$$\therefore \tau = 17.8 \ min.$$

IES 2000

A steel pipe having 10 cm bore & 12 cm outside diameter carries hot water at 80 °C, when the surrounding temp. is 15 °C. The thermal conductivity of pipe material is 54 W/mK & the inner & outer H.T. coeff. are 1 kW/m²K & 9 W/m²K. Calculate the heat loss per meter length of the coil & surface temp. Also calculate the heat loss & the surface temp. when the pipe is covered with a 4 cm thick insulation having thermal conductivity of 0.048 W/mK. Outer h reduced to 7 W/m²K.

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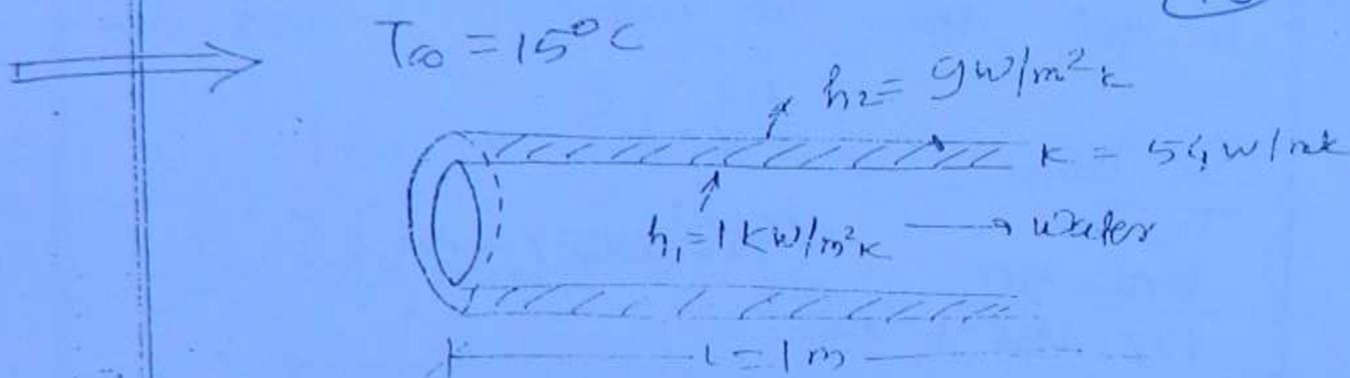

q/L 79

$T_w = 80°C$ $\quad \frac{1}{h_1 2\pi r_1 L} \quad \frac{\ln(r_2/r_1)}{2\pi K_1 L} \quad \frac{1}{h_2 2\pi r_2 L} \quad T_0 = 15°C$

$r_1 = 5$ cm

$r_2 = 6$ cm

$$\frac{q}{L} = \frac{80-15}{\frac{1}{h_1 2\pi r_1} + \frac{\ln(r_2/r_1)}{2\pi K_1} + \frac{1}{h_2 2\pi r_2}}$$

$$\therefore q/L = \frac{80-15}{\frac{1}{1000\times 2\times\pi\times 5/100} + \frac{\ln(6/5)}{2\pi\times 54} + \frac{1}{9\times 2\times\pi\times 6/100}}$$

$$\therefore \frac{q}{L} =$$

$$\therefore \frac{q}{L} = 217.79 \text{ W/m}$$

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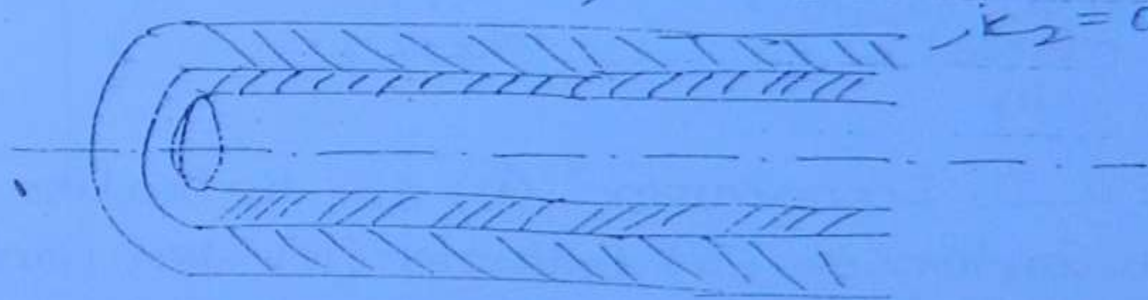
Now with insulation , $h = 7\ \text{W/m}^2\text{K}$

, $k_2 = 0.048$ W/mK



80 $\xrightarrow{q}$ $\frac{1}{h_1 2\pi r_1 L}$ — $\frac{\ln(r_2/r_1)}{2\pi K_1 L}$ — $\frac{\ln(r_3/r_2)}{2\pi K_2 L}$ — $T = ?$ — $\frac{1}{h_2 2\pi r_3 L}$ — $T_\infty = 15°C$

$$\therefore \frac{q}{L} = \frac{80-15}{\frac{1}{1000\times 2\pi\times\frac{5}{100}} + \frac{\ln(6/5)}{2\pi\times 54} + \frac{\ln(10/6)}{2\pi\times 0.048} + \frac{1}{7\times 2\pi\times\frac{10}{100}}}$$

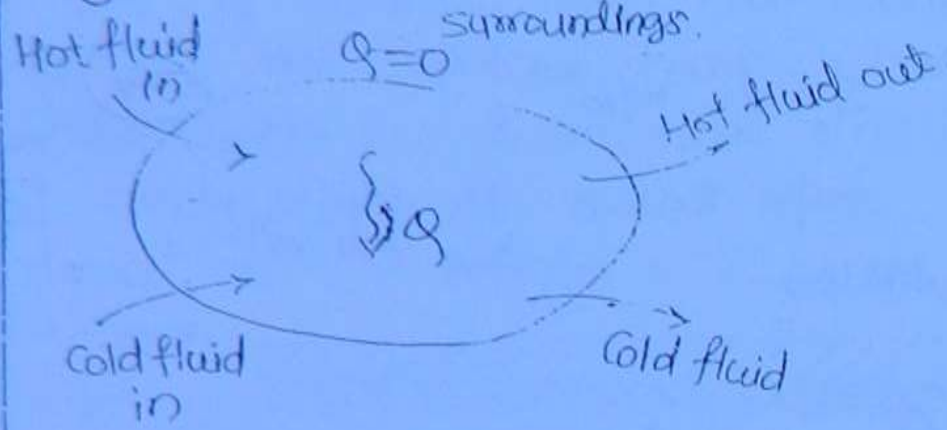
$$\therefore \frac{q}{L} =$$

$$\frac{q}{L} =$$

Heat Exchangers.

* **Heat Exchanger** –

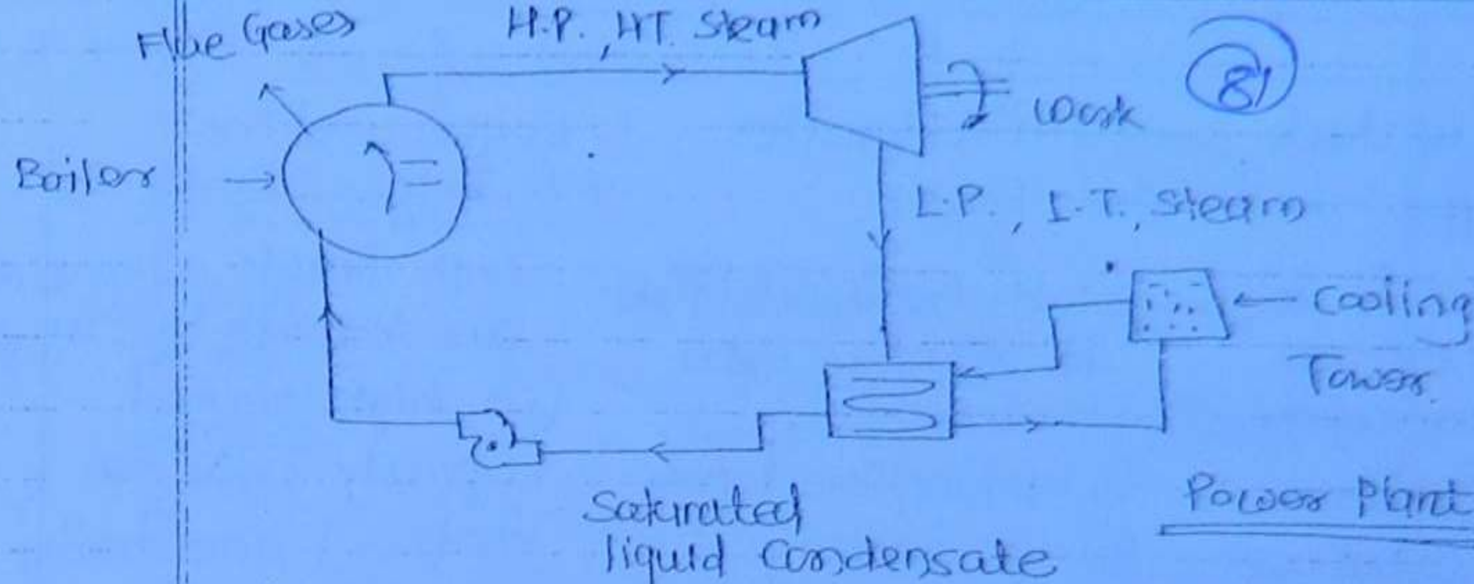
Heat exchanger is an adiabatic device in which two flowing streams of fluids exchange heat between themselves due to temperature difference without loosing or gaining any heat from the surroundings.



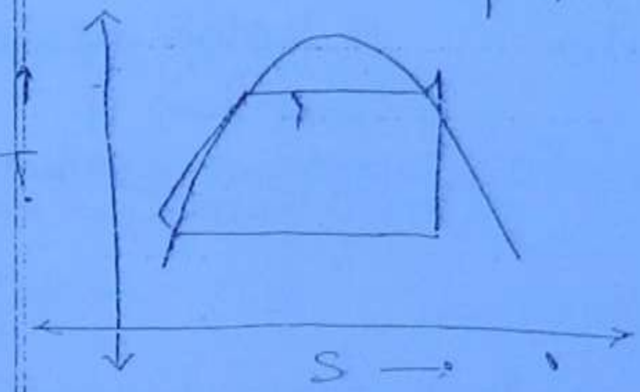
E.g. (i) Economiser (ii) Air Preheater (iii) Steam Condenser (iv) Radiator (iv) Steam Generators (v) Cooling Towers (vi) Evaporator.

(i) Economiser	Hot flue gases →	Feed water.
(ii) Air Preheater	—"— →	Air going for combustion
(iii) Steam Condenser	Steam →	Cooling water
(iv) Radiator	Hot water →	Atm. air
(v) Steam Generator	Hot flue gases →	Steam or sat. water
(vi) Cooling Tower	Hot water →	Atm. air.
(vii) Evaporator	Atm. air →	Refrigerant.

16 Jan '10

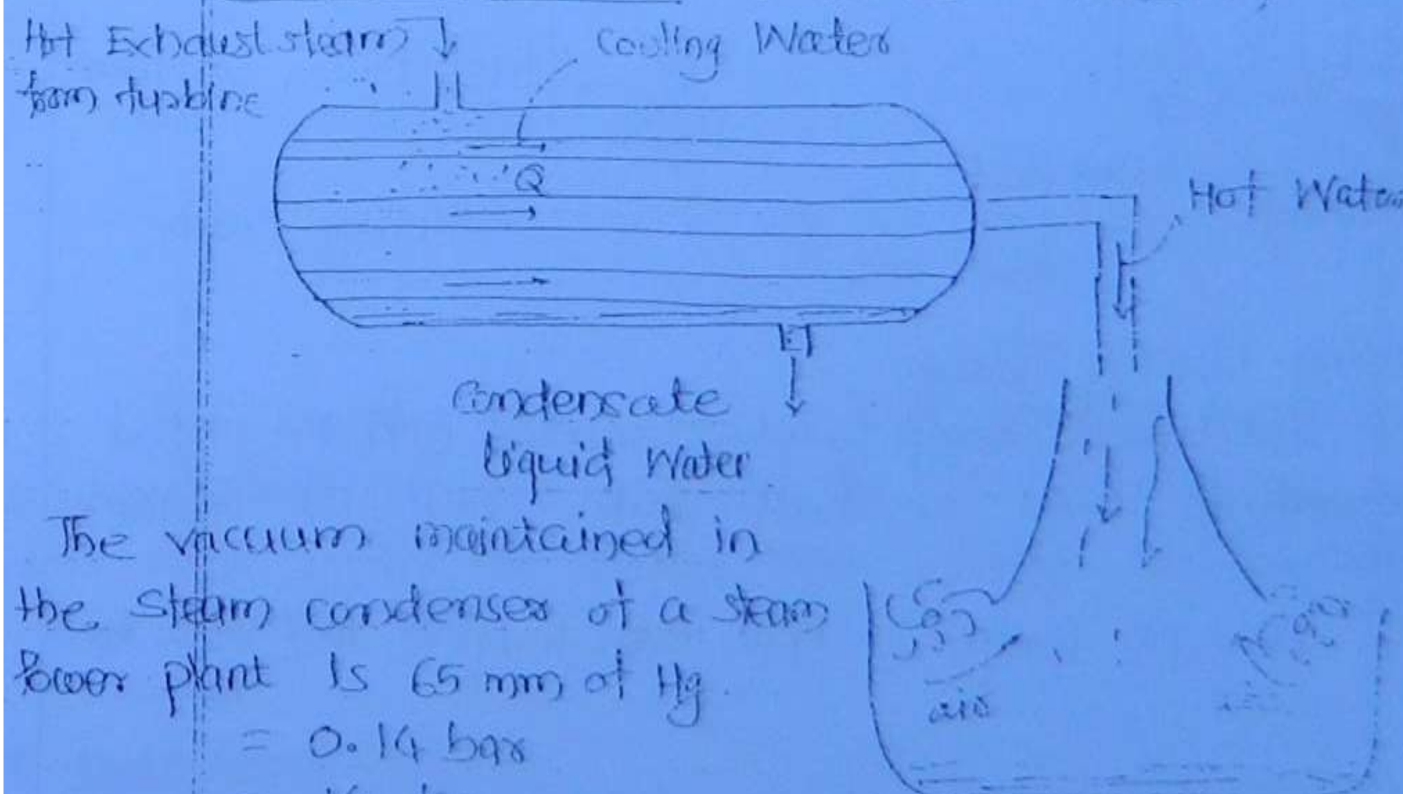


Power Plant



Ideal
Rankine Cycle

* Steam Condenser (Surface Condenser)



The vacuum maintained in the steam condenser of a steam power plant is 65 mm of Hg

= 0.14 bar

= 14 kPa

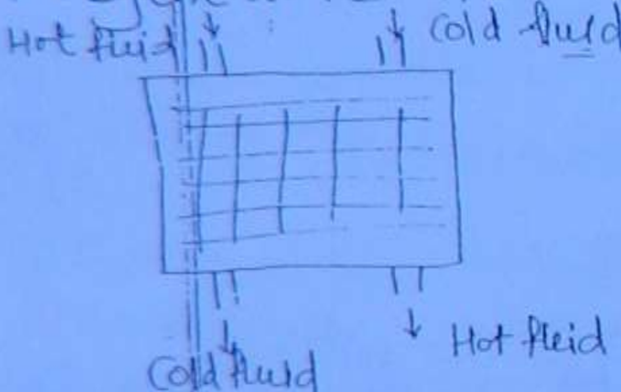
Classification of Heat Exchangers (82)

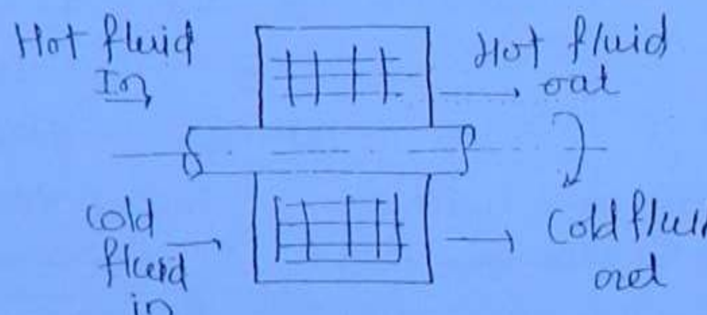
Direct contact Type
Both hot & cold fluids mix-up with each-other
i) Cooling Tower
ii) Jet condenser

Direct Transfer Type
Both hot & Cold fluids do not come into contact.
(i) Surface Condenser
(ii) Economiser
(iii) Air Preheater

Regenerative Type.
Both fluids alternatively pass through the HE (i.e. high thermal capacity cellulose matrix) one heating it & other picking up heat from it.
e.g. Jungstorm air preheater used in gas turbine power plant.

Regenerative H.E.





Rotating Matrix.

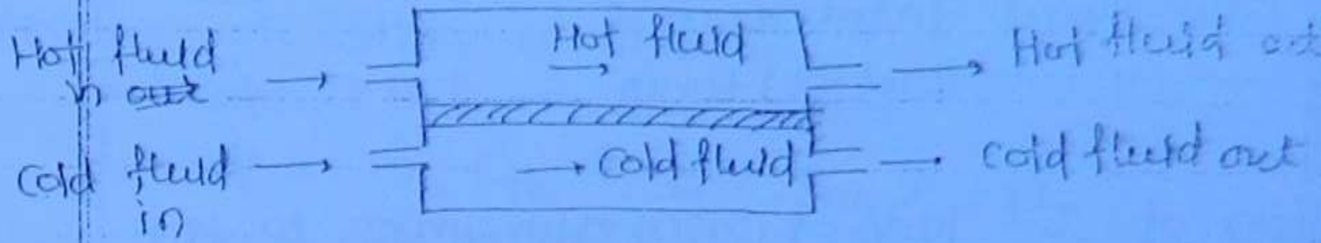
Regarding Cooling Tower.

The lowest temp. to which water can be cooled in any cooling tower is wet bulb temp. of ingoing air.

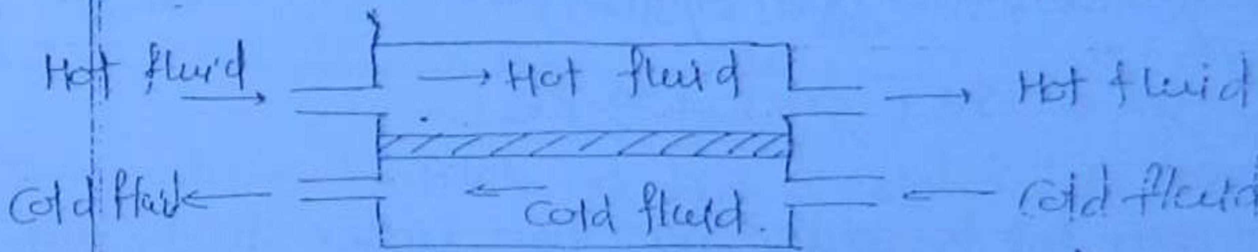
Dew pt. Temp < Wet Bulb temp < Dry Bulb Temp.

* Direct Transfer type H.E.

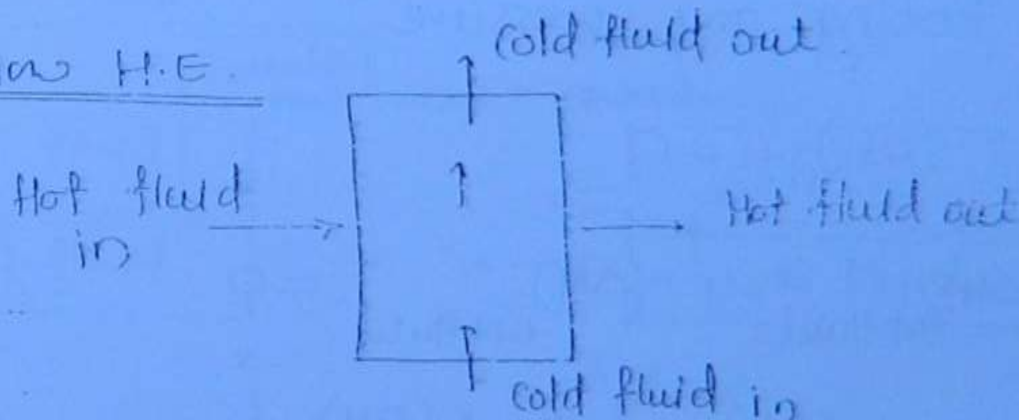
(i) Parallel Flow H.E.



(ii) Counter flow H.E.



(iii) Cross flow H.E.



The counter-flow H.E. could transfer more heat as compared to the parallel flow H.E. for a given H.E. area (Generally among three counter flow has more heat transfer rate).

* Notations :-

$\dot{m}_h$ = Mass flow rate of hot fluid (Kg/min)

$\dot{m}_c$ = ——— " ——— cold —" ———

C_{p_h} = Sp. heat of Hot fluid in kJ/kg·K
C_{p_c} = —— " —— cold —— " ——
T_{hi} = Hot fluid inlet temp.
T_{he} = —— " —— outlet (exit) —— " ——

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T_{ci} = Cold fluid inlet temp
T_{ce} = —— " —— (outlet) (exit) temp.

Application of Ist law of thermodynamics to H.E.

* SFEE (Steady flow Energy Equation)

(Steady flow open system)

$$\cancel{Q}^{0} - \cancel{W}^{0} = \Delta H + \cancel{\Delta KE}^{0} + \cancel{\Delta PE}^{0}$$

Assumptions

(i) Both hot & cold fluids flow under steady state without loosing any pressure.

$$(\Delta H)_{H.E.} = 0$$

$$(\Delta H)_{\text{Hot fluid}} + (\Delta H)_{\text{cold fluid}} = 0$$

$$\Longrightarrow \quad -(\Delta H)_{\text{Hot fluid}} = +(\Delta H)_{\text{cold fluid}}$$

The rate of enthalpy decrease of hot fluid = Rate of enthalpy increase of cold fluid

$$\dot{m}_h C_{p_h} (T_{hi} - T_{he}) = \dot{m}_c C_{p_c} (T_{ce} - T_{ci})$$

** Also known as energy balance eq^n or heat balance eq^n.

From thermodynamic rule of

$$Q_{P=C} = \Delta H$$
(Isobaric)

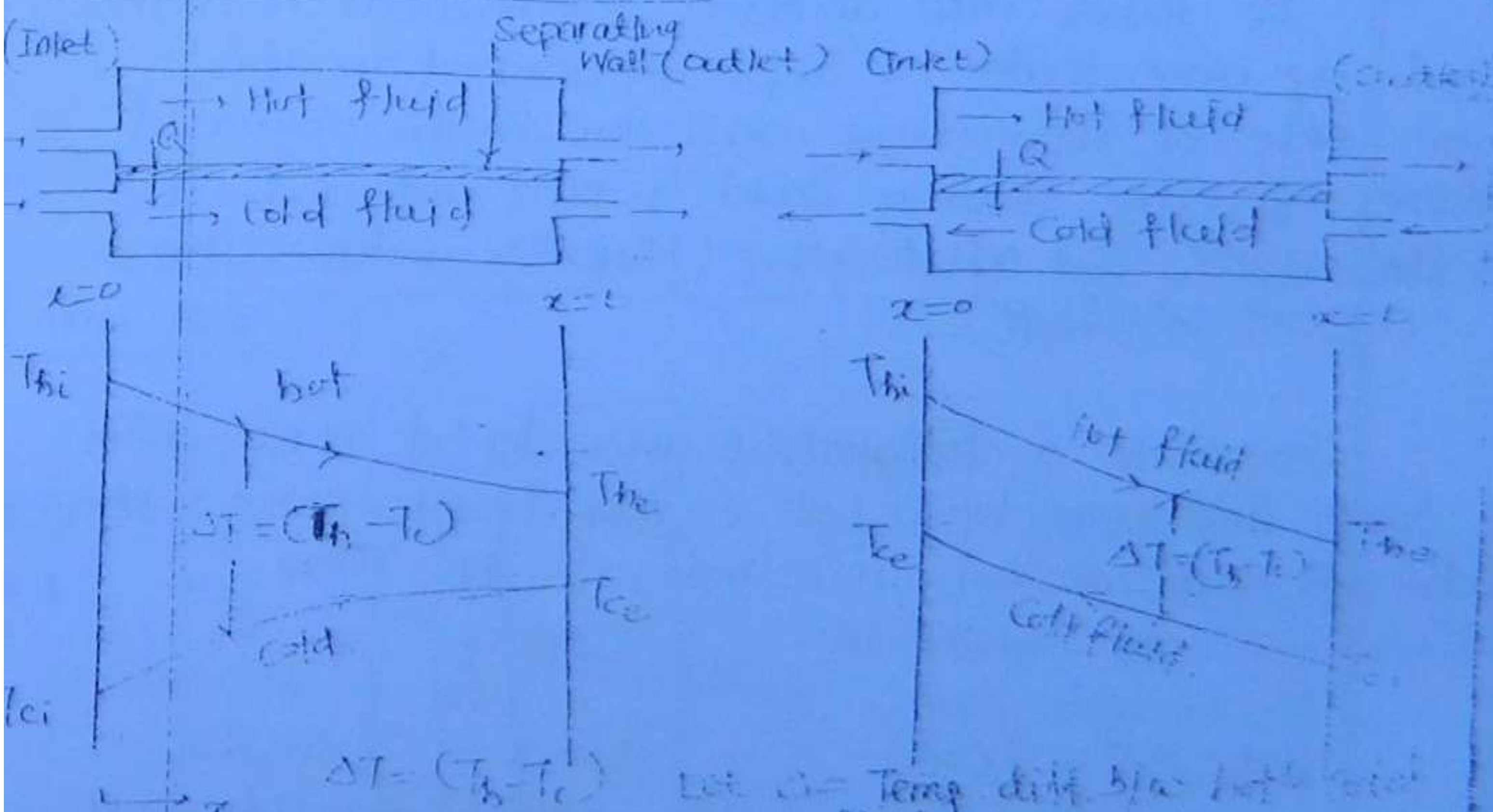
We can say that the rate of H.T. b/w hot & cold fluids is equal to the rate of change of enthalpy of either of fluids.

Rate of H.T.

$$\therefore Q = \dot{m}_h C_{P_h} (T_{hi} - T_{he}) \quad \text{or}$$
$$Q = \dot{m}_c C_{P_c} (T_{ce} - T_{ci})$$

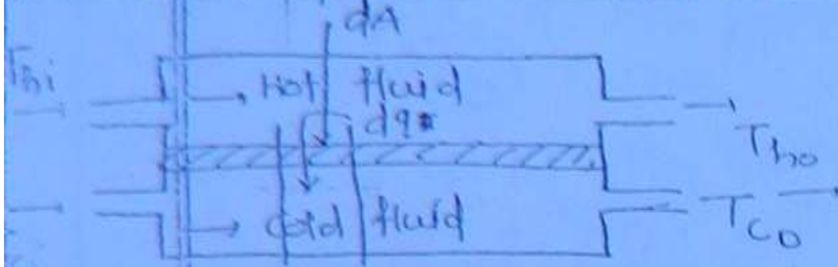
$$\boxed{C_{P_{water}} = 4.187 \ \text{kJ/kg°C} \qquad C_{P_{air}} = 1.005 \ \text{kJ/kg°C}}$$

* Temperature Profiles of Hot & Cold Fluids in Parallel & Counter Flow H.E

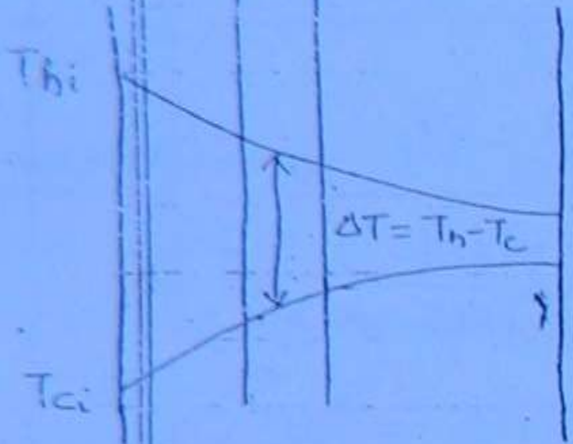


Let ΔT = Temp diff b/w hot & cold fluid at any section at a dist. x from inlet. $\Delta T = f(x)$

Mean Temperature Difference (MTD) $[\Delta T_m]$



(86)



Let h_1 = Convection H.T. coeff. on hot side.
h_2 = convection H.T. coeff on cold side.
Overall H.T. coeff

$$\frac{1}{U} = \frac{1}{h_1} + \frac{1}{h_2} + F_1 + F_2$$

(Neglecting conducting resistance of pipe or wall)

where F_1 & F_2 are Fouling Factors.

Fouling Factor (F) ~~test~~

It takes into account the thermal resistance offered by any scale or deposite formed on the either side of separating wall due to the chemical reaction b/w the flowing fluid & pipe material. It has the units of $m^2K/Watt$. (Usually values are $F = 0.0002\ m^2K/watt$)

Consider a differential area dA of H.E, where the temp. difference b/w hot & cold fluids is ΔT through which the differential H.T. rate is dq. Then,

$$dq = U \Delta T\, dA$$

But $\Delta T = f(dA)$

$$dA = B.dx$$ (B = width of separating wall $\perp^{r}$ to plane of fig.)

$dq = U B\,dx\,\Delta T$

By integrating,

$$\int_{\text{Inlet}}^{\text{Exit}} dq = Q = \text{Total H.T. rate in the H.E.} = \int_{\text{Inlet}}^{\text{Exit}} U \Delta T\, dA$$

$$= U \int_{\text{Inlet}}^{\text{Exit}} \Delta T\, dA \quad \text{—— (i)}$$

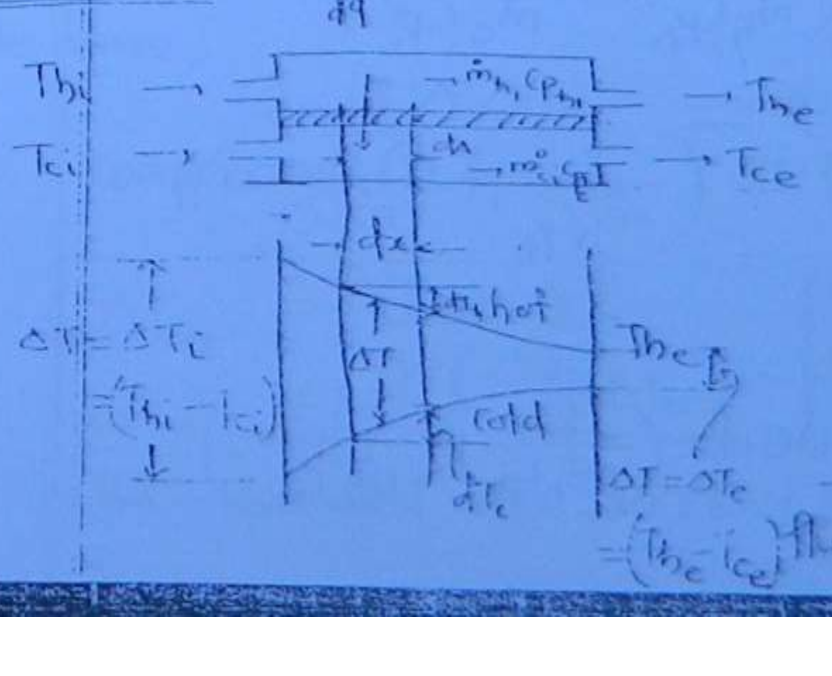
Now defining ΔT_m from the eqn

$Q = U A \Delta T_m$ —— (ii), where A = Total H.T. area of H.E.

Comparing (i) & (ii)

$$\Delta T_m = \frac{1}{A} \int_{\text{Inlet}}^{\text{Exit}} \Delta T\, dA$$

** To derive Mean Temp. Diff. for a parallel flow H.E. *



Consider a diff. area of ~~length~~ dx & length dx through which H.T. rate is dq

Let, $\Delta T = T_h - T_c$

Temp diff b/w hot & cold fluids at dx

$dq = U \Delta T \, dA$

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$= U \Delta T \, B . dx$

Also

$dq = -\dot{m}_h C_{p_h} \, dT_h$

$dq = +\dot{m}_c C_{p_c} \, dT_c$

At $x=0$ $\Delta T = \Delta T_i$
$= T_{hi} - T_{ci}$

At $x=L$ $\Delta T = \Delta T_e$
$= T_{he} - T_{ce}$

let $\Delta T = T_h - T_c$

$d(\Delta T) = dT_h - dT_c$

$$= -\frac{dq}{\dot{m}_h C_{p_h}} - \frac{dq}{\dot{m}_c P_c}$$

$$= -dq \left(\frac{1}{\dot{m}_h C_{p_h}} + \frac{1}{\dot{m}_c C_{p_c}} \right)$$

$$d(\Delta T) = -U \Delta T \, B \, dx \left(\frac{1}{\dot{m}_h C_{p_h}} + \frac{1}{\dot{m}_c C_{p_c}} \right)$$

$$\frac{d(\Delta T)}{\Delta T} = -U B \, dx \left(\frac{1}{\dot{m}_h C_{p_h}} + \frac{1}{\dot{m}_c C_{p_c}} \right)$$

$$\int_{\Delta T_i}^{\Delta T_e} \frac{d(\Delta T)}{\Delta T} = \int_0^L -U B \, dx \left(\frac{1}{\dot{m}_h C_{p_h}} + \frac{1}{\dot{m}_c C_{p_c}} \right)$$

$$\therefore -\int_{\Delta T_i}^{\Delta T_e} \frac{d(\Delta T)}{\Delta T} = \int_0^L U B \, dx \left(\frac{1}{\dot{m}_h C_{p_h}} + \frac{1}{\dot{m}_c C_{p_c}} \right)$$

$$\ln\left(\frac{\Delta T_i}{\Delta T_e}\right) = UB\left(\frac{1}{\dot{m}_h C_{p_h}} + \frac{1}{\dot{m}_c C_{p_c}}\right)L$$

But $(B \times L)$ = Total H.T. area of H.E.

$$\therefore \ln\left(\frac{\Delta T_i}{\Delta T_e}\right) = UA\left(\frac{1}{\dot{m}_h C_{p_h}} + \frac{1}{\dot{m}_c C_{p_c}}\right)$$

Energy Balance Eq^n

$$Q = \dot{m}_h C_{p_h}(T_{hi} - T_{he}) = \dot{m}_c C_{p_c}(T_{ce} - T_{ci})$$

$$\frac{1}{\dot{m}_h C_{p_h}} = \left(\frac{T_{hi} - T_{he}}{Q}\right), \quad \frac{1}{\dot{m}_c C_{p_c}} = \left(\frac{T_{ce} - T_{ci}}{Q}\right)$$

$$\ln\left(\frac{\Delta T_i}{\Delta T_e}\right) = UA\left(\frac{T_{hi} - T_{he}}{Q} + \frac{T_{ce} - T_{ci}}{Q}\right)$$

$$= \frac{UA}{Q}\left[(T_{hi} - T_{ci}) - (T_{he} - T_{ce})\right]$$

$$\therefore Q = \frac{UA\left[\Delta T_i - \Delta T_e\right]}{\ln(\Delta T_i / \Delta T_e)} \quad \text{——(i)}$$

We have,

$$Q = UA\,\Delta T_m \quad \text{——(ii)}$$

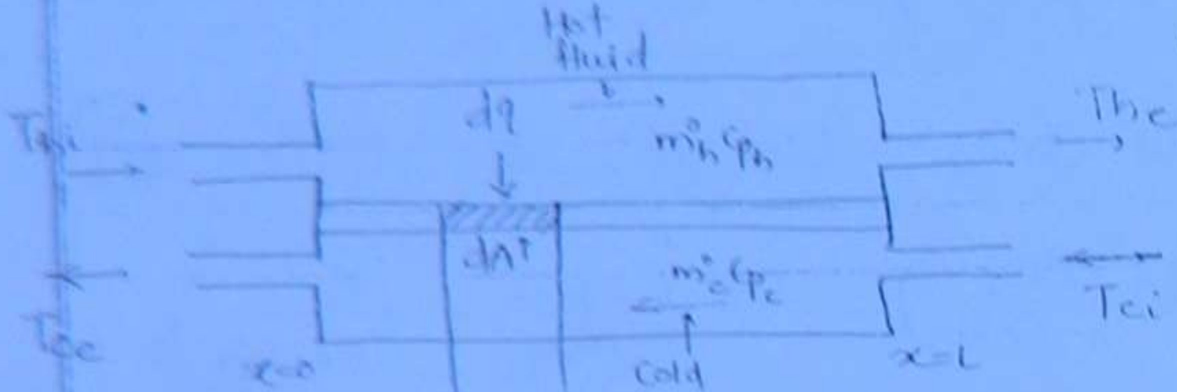
Comparing (i) & (ii)

$$\boxed{\Delta T_m = \frac{\Delta T_i - \Delta T_e}{\ln(\Delta T_i / \Delta T_e)}}$$

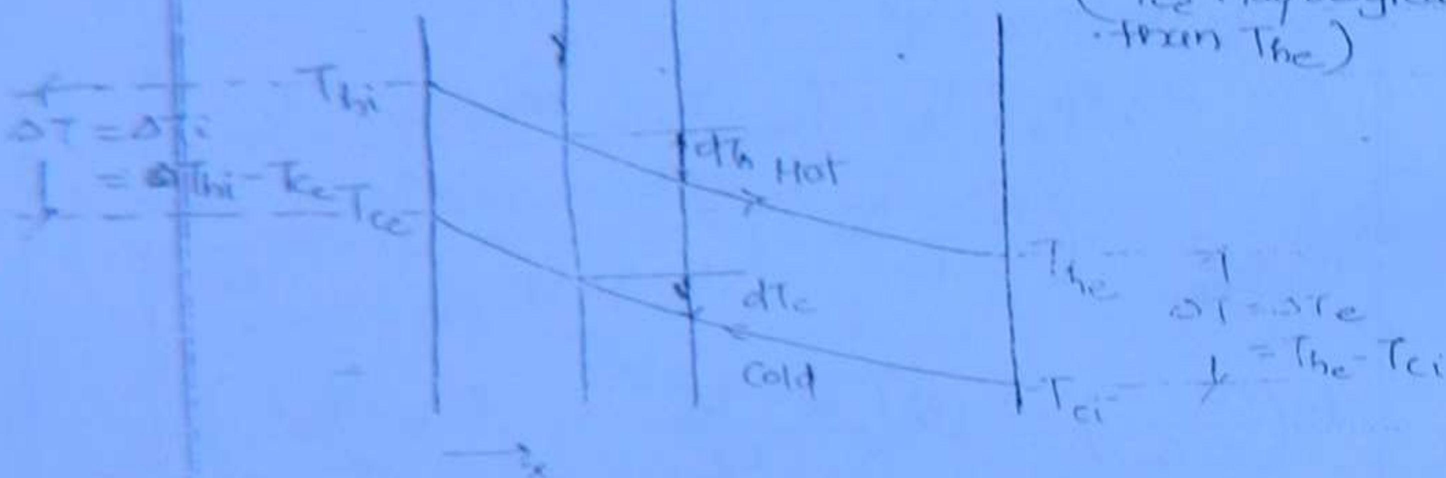
This is called as LMTD (Log mean Temp. diff.)

LMTD For Counter Flow Heat Exchangers

(90)



(T_{ce} may be greater than T_{he})



Consider a differential area dA of length dx through which the differential H.T. rate b/w hot & cold fluid is dq.

Let $\Delta T = T_h - T_c = f(x)$

At $x = 0$ $\Delta T = \Delta T_i = T_{hi} - T_{ce}$

At $x = L$ $\Delta T = \Delta T_e = T_{he} - T_{ci}$

Then $dq = U \Delta T \, dA$

$= U \Delta T \, B \, dx$

Also $dq = -\dot{m}_h c_{p_h} dT_h$

$= -\dot{m}_c c_{p_c} dT_c$

Let $\Delta T = T_h - T_c$

$$d(\Delta T) = dT_h - dT_c$$

$$= -\frac{dq}{\dot{m}_h C_{p_h}} + \frac{dq}{\dot{m}_c C_{p_c}} \quad (9i)$$

$$= -dq \left(\frac{1}{\dot{m}_h C_{p_h}} - \frac{1}{\dot{m}_c C_{p_c}} \right)$$

$$d(\Delta T) = -U \Delta T \, B.dx \left(\frac{1}{\dot{m}_h C_{p_h}} - \frac{1}{\dot{m}_c C_{p_c}} \right)$$

$$-\int_{\Delta T_i}^{\Delta T_e} \frac{d(\Delta T)}{\Delta T} = \int_0^L UB\,dx \left(\frac{1}{\dot{m}_h C_{p_h}} - \frac{1}{\dot{m}_c C_{p_c}} \right)$$

$$\ln\left(\frac{\Delta T_i}{\Delta T_e}\right) = UB \left(\frac{1}{\dot{m}_h C_{p_h}} - \frac{1}{\dot{m}_c C_{p_c}} \right) L$$

$\therefore B \times L = A$ (Area of H.T)

$$\ln\left(\frac{\Delta T_i}{\Delta T_e}\right) = UA \left(\frac{1}{\dot{m}_h C_{p_h}} - \frac{1}{\dot{m}_c C_{p_c}} \right) \quad \text{--- (i)}$$

We have,

From Energy Balance eqn.

$$Q = \dot{m}_h C_{p_h} (T_{hi} - T_{he}) = \dot{m}_c C_{p_c} (T_{ce} - T_{ci})$$

$$\frac{1}{\dot{m}_h C_{p_h}} = \left(\frac{T_{hi} - T_{he}}{Q} \right) ; \quad \frac{1}{\dot{m}_c C_{p_c}} = \left(\frac{T_{ce} - T_{ci}}{Q} \right)$$

$$\ln\left(\frac{\Delta T_i}{\Delta T_e}\right) = UA\left[\left(\frac{T_{hi}-T_{he}}{Q}\right) - \left(\frac{T_{ce}-T_{ci}}{Q}\right)\right]$$

$$\ln\left(\frac{\Delta T_i}{\Delta T_e}\right) = \frac{1}{Q}(UA)\left[(T_{hi}-T_{ce}) - (T_{he}-T_{ci})\right]$$

$$Q = \frac{UA\left[\Delta T_i - \Delta T_e\right]}{\ln(\Delta T_i/\Delta T_e)} \quad \text{--- (ii)}$$

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We have $Q = UA\Delta T_m$ (iii) Comparing (ii) & (iii)

$$\boxed{\Delta T_m = \frac{\Delta T_i - \Delta T_e}{\ln(\Delta T_i/\Delta T_e)}}$$

~~ΔT_e~~ This is eqn of LMTD

Eventhough the formula for LMTD is same in both parallel flow & counter flow H.E. The defn of ΔT_i & ΔT_e are different b/w them

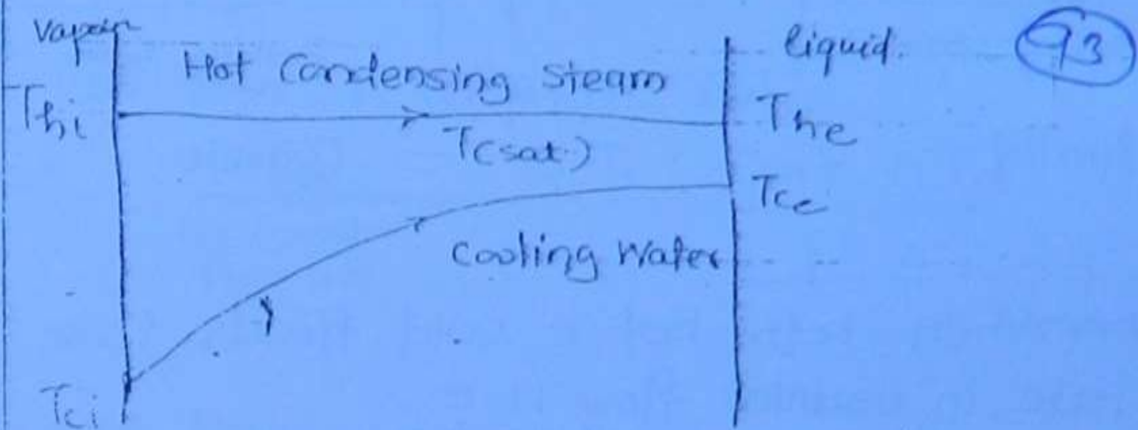
For the same inlet & exit temp of hot & cold fluids and for the same mass flow rate of the same fluid, the counter flow H.E. would occupy lesser Heat Transfer Area. (It is more compact as compared to parallel flow H.E.)

For the same H.T. area provided in both parallel flow & counter flow modes, the counter flow H.E. can transfer higher H.T. rate than parallel flow H.E. i.e. hot fluid exit temp lesser in counter flow

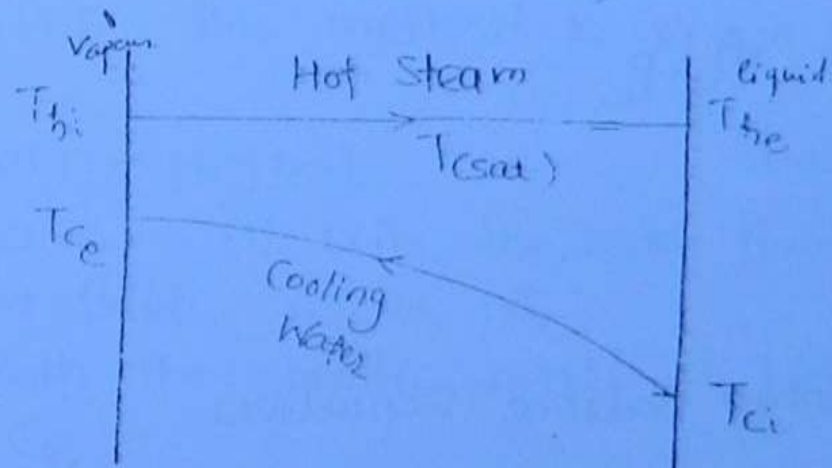
Two Special Cases.

(Case - I) ⇒ When one of the fluids is undergoing change of phase like in steam condenser or evaporator ; then temperature profile will like ⇒

Example ⇒ Steam Condenser.



(Parallel flow H.E.)

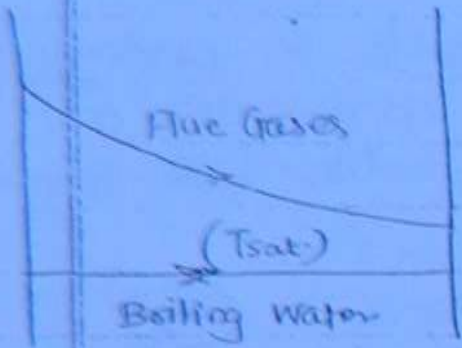


(Counter flow H.E.)

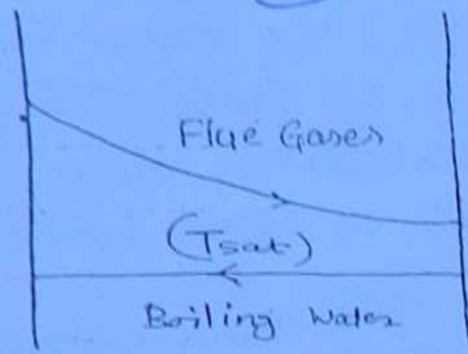
In this case $(\Delta T_m)_{\text{||el H.E}} = (\Delta T_m)_{\text{Counter flow H.E.}}$

Hence it is immaterial, what type of H.E. is to be designed.

In case of (evaporator)/Steam Generator.



Parallel



Counter

(Case II) ⇒ When both hot & cold fluids have equal capacity rate in counter flow H.E.

Capacity rate $= (\dot{m} \times C_p)$

i.e.

$$\dot{m}_h C_{p_h} = \dot{m}_c C_{p_c}$$

(kg/s) (J/kg K)

⇓

(Watt/K)

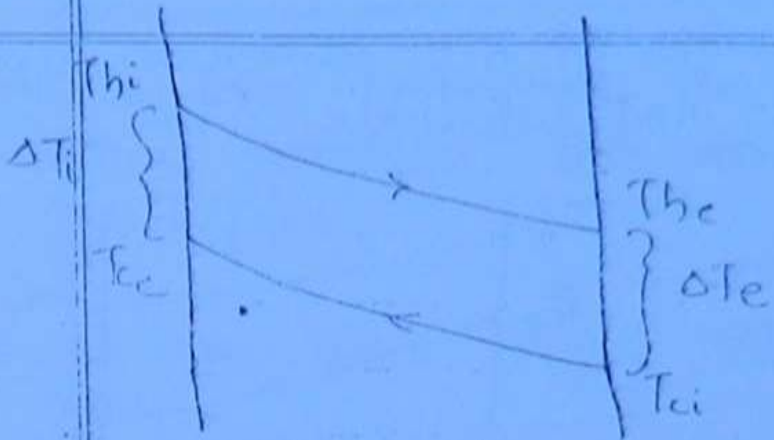
Then from Energy Balance Equation

$$\dot{m}_h C_{p_h}(T_{hi} - T_{he}) = \dot{m}_c C_{p_c}(T_{ce} - T_{ci})$$

$$T_{hi} - T_{he} = T_{ce} - T_{ci}$$

$$T_{hi} - T_{ce} = T_{he} - T_{ci}$$

$$\boxed{\Delta T_i = \Delta T_e}$$



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But from L' Hospital's Rule

$\underline{\Delta T_m = \Delta T_i \text{ (or) } \Delta T_e}$

For counter flow H.E.

Design of Heat Exchangers

Here the objective is to calculate the H.T area of H.E. This method is known as LMTD method

LMTD Method

Given data ⇒ (i) Both the mass flow rates of hot and cold fluids $\dot{m}_h$ & $\dot{m}_c$

(ii) Both the specific heats of hot & cold fluids c_{p_h} & c_{p_c}

(iii) Overall Heat Transfer coeff. (U) in W/m²K

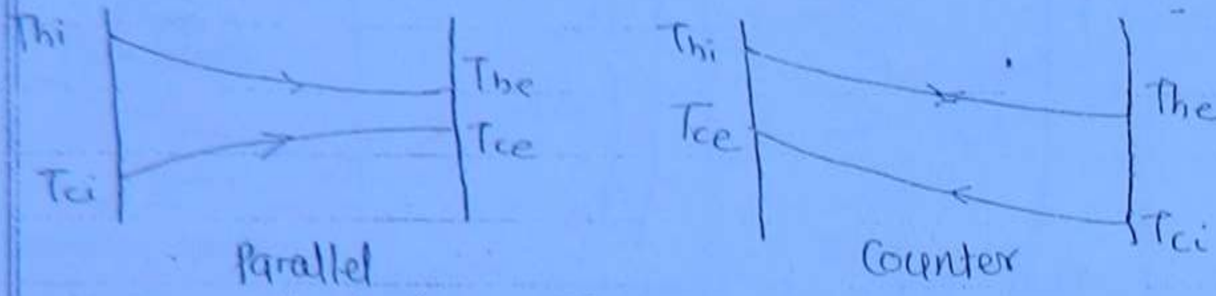
(iv) Only 3 Temps among four temperatures (T_{hi}, T_{ci}, T_{he}, T_{ce})

To find ⇒ Area (A) of H.T. in H.E. = ?

Step (i) Calculate the 4th unknown temperature from energy balance equation

i.e. $\dot{m}_h c_{p_h} (T_{hi} - T_{he}) = \dot{m}_c c_{p_c} (T_{ce} - T_{ci})$

Step (ii) Draw the temp. profiles of H.E. based on type of H.E to be designed. And calculate ΔT_m



Hence calculate ΔT_m

$$\Delta T_m = \frac{\Delta T_i - \Delta T_e}{\ln(\Delta T_i / \Delta T_e)}$$

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Step (iii) obtain H.T. rate Q b/w hot & cold fluid

$$Q = \dot{m}_h C_{p_h}(T_{hi} - T_{he}) = \dot{m}_c C_{p_c}(T_{ce} - T_{ci})$$

J/kgK. Watts

Step (iv) Then obtain the area of H.t from

$$Q = UA\Delta T_m$$

$$\therefore \left(A = \frac{Q}{U\Delta T_m}\right) m^2$$

IES 1999

Light lubricant oil $c_p = 2090$ J/kgK. is cooled by allowing it to exchange energy to with water in a small H.E The oil enters & leaves the H.E at 375 K & 350 K respectively. And flows at rate of 0.5 kg/s. Water at 280 K is available in sufficient quantity to allow 0.201 kg/s. to be used for cooling surface. Determine the require H.T. area for counterflow operation. The overall H.T. coeff. is taken as 250 W/m²K.

(97)

$\Longrightarrow$ We have energy balance eqⁿ.

$$\dot{m}_h C_{p_h} (T_{hi} - T_{he}) = \dot{m}_c C_{p_c} (T_{ce} - T_{ci})$$

$$(0.5)(2090)(375-350) = (0.201)(4.187\times10^3)(T_{ce}-280)$$

$$26125 = 841.587\,(T_{ce}-280)$$

$$\therefore \quad T_{ce} = 311.043\ ^\circ K$$

Now

$$\Delta T_m = \frac{63.957 - 70}{\ln(63.957/70)}$$

$$\Delta T_m = 67 \text{ K}$$

$$Q = (0.5)(2090)(375-350)$$

$$= 26125 \text{ Watts}$$

$$A = \frac{26125}{250 \times 67} = 1.56 \ \text{m}^2$$

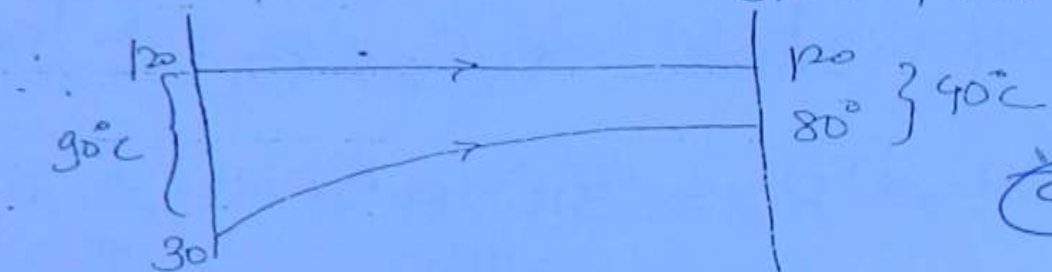
In a condenser water enters at 30°C & flows at the rate of 1500 kg/hr. The condensing steam at temp. of 120°C & cooling water leaves the condenser at 80°C $C_{p_{water}} = 4.187$ KJ/kgK. If $U = 2000$ W/m²K. The H.T. area is

(i) $0.707\ m^2$ (ii) $7.07\ m^2$ (iii) $70.7\ m^2$ (iv) $141.0\ m^2$

$\Rightarrow$

$$\frac{1500}{3600} \times 4.187 \times 10^3 \times (80 - 30) = \theta$$

87.229 KW



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$$\Delta T_m = \frac{\Delta T_i - \Delta T_e}{\ln(\Delta T_i / \Delta T_e)} = \frac{90 - 40}{\log_e (90/40)}$$

$\Delta T_m = 61.657\ °C$

$\therefore\ 87.229 = U A \Delta T_m$

$\therefore\ 87.229 \times 10^3 = 2000 \times A \times 61.657$

$\therefore\ \boxed{A = 0.707\ m^2}$

GATE 2008

Counterflow

The LMTD of H.E. is 20°c. The cold fluid enters at 20°C and hot fluid enters at 100°C Mass flow rate of cold fluid is twice that of hot fluid, sp. heat at const. pressure of hot fluid is twice that of cold fluid. The exit temp. of cold fluid is ⟹

a] 40°C b] 60°C c] 80°C d] Can't be determined

(99)

$\Delta T_m = 20°$

$T_{ci} = 20°C$

$T_{hi} = 100°C$

$\dot{m}_c = 2\,\dot{m}_h$

$C_{p_h} = 2\,C_{p_c}$

$\dot{m}_c C_{p_c} = \dot{m}_h C_{p_h}$

Equal capacity rates on hot side & cold side

$\therefore \Delta T_m = \Delta T_i$ or ΔT_e

We have,

$$\dot{m}_h C_{p_h}(T_{hi} - T_{he}) = \dot{m}_c C_{p_c}(T_{ce} - T_{ci})$$

$\therefore$ ~~$100 - T_{he} = T_{ce} - 20$~~

~~Mass flow rate~~

By applying L' Hospital's Rule

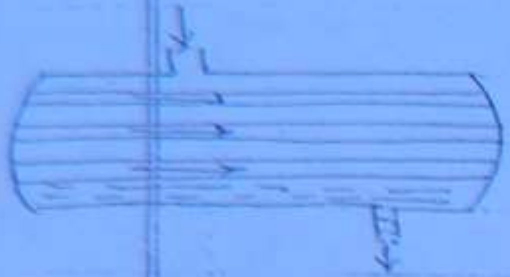
$$\Delta T_m = \Delta T_i \text{ or } \Delta T_e$$

$$\therefore 20 = T_{hi} - T_{ce}$$

$$\therefore 20 = 100 - T_{ce}$$

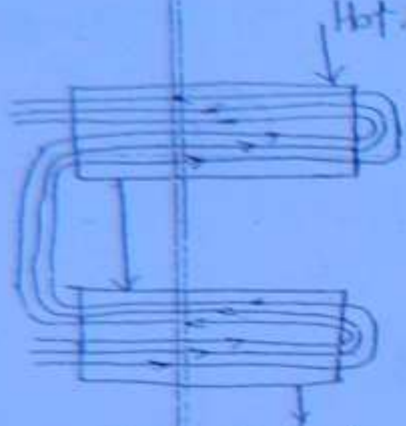
$$\therefore \boxed{T_{ce} = 80°C}$$

* Shell & Tube Heat Exchanger



Single Shell Pass & Single Tube Pass

2 Tube Pass & single shell Pass H.E.



2 Shell Pass & 4 Tube Pass H.E.

In this cases

$$\underset{(LMTD)}{\Delta T_m} = \underset{\text{Counterflow}}{\Delta T_m} \times F$$

F = Correction Factor.

IES 2009

Find the surface area required, for a surface condenser dealing with a 25000 kg of saturated steam per hour at a pressure of 0.5 bar. Temperature of condensate water is 25°c. Cooling water is heated from 15°C to 25°C while passing through the condenser. Assume H.T. coeff of 10 KW/m²K. The condenser has two water passes with a tubes of 19 mm od. & 1.2 mm thickness. Find the length & no. of tubes per pass. Assume velocity of water is 1 m/s. Assume correction factor for 2 tube pass

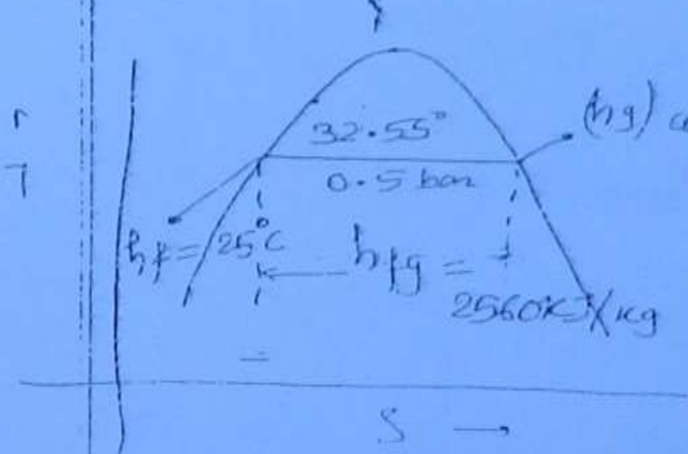
exchanger as 0.86. At 0.5 bar, saturation temp is 32.55°C and latent heat is 2560 kJ/kg.
$C_{p_{water}} = 4.187$ kJ/kgK $\rho = 1000$ kg/m³

$\Longrightarrow$ $\dot{m}_h = 25000/3600 = 6.94$ kg/s

$T_{c_i} = 15°C$ $T_{c_e} = 25°C$

$U = 10 \times 10^3$ W/m²K

101



For 1 kg of steam, the heat rejected by steam

$$= (\Delta h) \text{ kJ/kg} = (h_g)_{at\ 32.55°C} - (h_f)_{at\ 25°C}$$

T(sat)	h_f (kJ/kg)
32.55°C	4.187 × 32.55
25°C	4.187 × 25

$\Delta h = \cancel{136.29} - \cancel{104.68}$

$\therefore \Delta h = \cancel{31.325}$ kJ/kg

$(h_g)_{32.55} = (h_f + h_{fg})$

$= 2696$

When there is change of phase, then

$$\boxed{Q = \dot{m}_{steam} \times \text{Heat Rejected per kg}}$$

(102)

$\therefore$ Heat Rejected per kg

$$(\Delta h) = (2696 - 104.6)$$

$$= 2591.4 \text{ KJ/kg}$$

$$\therefore Q = 2591.4 \times 6.94$$

$$= (17997) \text{ KJ/sec}$$

$$\Delta T_m = \frac{\Delta T_i - \Delta T_e}{\ln(\Delta T_i / \Delta T_e)}$$

$$=$$

$$= 8.75°c$$

$17997 \times 10^3 = 10 \times 10^3 (A)(8.75) \times 0.86$

$\therefore A = 240 \ m^2$

$\therefore A = \pi d L \times n \times p$ (103)

P = No. of Pass
n = no. of tubes per pass
d = dia. of tube
L = length of tube

$\dot{m}_w = \rho A V n$ A = c/s area

$= \rho \frac{\pi}{4} di^2 \times \bar{V} \times n$

$\dot{m}_w = 1000 \times \frac{\pi}{4} \times (16.6)^2 \times 1 \times n$

We have $Q = \dot{m}_w \times 4.187 \times 10^3 \times (25-15)$

$17997 = \dot{m}_w \times$

$\dot{m}_w = 429.830$ kg/sec

$429.83 = 1000 \times \frac{\pi}{4} (16.6)^2 \times 1 \times n$

$n = 1986$

$240 = \pi \times 19 \times 10^{-3} \times L \times 1986 \times 2$

$L = 1.01$ m.

Effectiveness of H.E. (ϵ)

It is defined as the actual H.T. rate b/w hot & cold fluids & the maximum possible H.T. rate b/w them.

(04)

$$\epsilon_{HE} = \frac{Q_{actual}}{Q_{max}}$$

$$Q_{actual} = \dot{m}_h c_{p_h} (T_{hi} - T_{he})$$
$$= \dot{m}_c c_{p_c} (T_{ce} - T_{ci})$$

Q_{max} = Maximum possible H.T. rate
$$= (\dot{m} c_p)_{small} \times (T_{hi} - T_{ci})$$

where $(\dot{m} c_p)_{small}$ is smaller capacity rate b/w $\dot{m}_h c_{p_h}$ & $\dot{m}_c c_{p_c}$.

If $\dot{m}_h c_{p_h} < \dot{m}_c c_{p_c}$

Then,
$$\epsilon_{H.E.} = \frac{\dot{m}_h c_{p_h} (T_{hi} - T_{he})}{\dot{m}_h c_{p_h} (T_{hi} - T_{ci})}$$

$$\boxed{\epsilon_{H.E.} = \frac{T_{hi} - T_{he}}{T_{hi} - T_{ci}}}$$

& If $\dot{m}_c c_{p_c} < \dot{m}_h c_{p_h}$

Then,
$$\epsilon_{H.E.} = \frac{\dot{m}_c c_{p_c} (T_{ce} - T_{ci})}{\dot{m}_c c_{p_c} (T_{hi} - T_{ci})}$$

$$\boxed{\epsilon_{H.E.} = \frac{T_{ce} - T_{ci}}{T_{hi} - T_{ci}}}$$

Number of Transfer Units (NTU)

It is defined as the ratio b/w product of UA and the smaller capacity rate b/w hot & cold fluid.

$$NTU = \frac{UA}{(\dot{m}C_p)_{small}} \qquad U \Rightarrow W/m^2K \qquad C_p \Rightarrow J/kg.K$$

105

= No units

Since NTU being proportional to the area of H.E, it signifies the overall size of H.E.

* Capacity Rate Ratio (C)

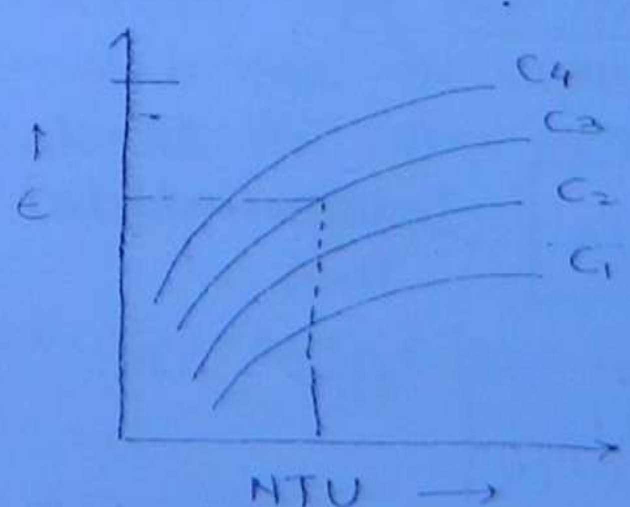
$$C = \frac{(\dot{m}C_p)_{small}}{(\dot{m}C_p)_{Big}}$$

$\in < 1$ $\qquad 0 \le C \le 1 \qquad NTU > 1$ or $NTU < 1$

$C = 0$ if one of the fluids change its phase like steam condenser or evaporator.

For any heat exchanger

$$\in = f(NTU, C)$$



Various capacity rate ratio

For parallel flow heat exchanger

$$\epsilon_{Parallel} = \frac{1-e^{-(1+C)NTU}}{1+C}$$ (106)

For counter flow heat exchanger

$$\epsilon = \frac{1-e^{-(1-C)NTU}}{1-Ce^{-(1-C)NTU}}$$

When $C=0$ (When one of the fluids undergoes phase change)

$$\boxed{\epsilon_{Parallel} = 1-e^{-NTU}}$$

$$\boxed{\epsilon_{counter} = 1-e^{-NTU}}$$

When $C=1$ (Equal capacity rate)
i.e. $\dot{m}_c c_{p_c} = \dot{m}_h c_{p_h}$

$$\boxed{\epsilon_{Parallel} = \frac{1-e^{-2NTU}}{2}}$$

$$\boxed{\epsilon_{counter} = 0/0}$$ Indeterminate

Then

$$\epsilon_{counter} = \frac{NTU}{1+NTU}$$

Effectiveness – NTU Method.

This method is adopted whenever both the exit temperatures of hot & cold fluids T_{he} & T_{ce} are unknown for a given designed heat exchanger area A.

Given data (i) Both the mass flow rates of hot & cold fluids ($\dot{m}_h$ & $\dot{m}_c$)

(ii) Both the specific heats c_{p_h} & c_{p_c}

(iii) Overall H.T. coefficient (U) in W/m^2K

(iv) Only two inlet temperatures (T_{hi} & T_{ci})

(v) Area of heat exchanger (A)

To find T_{he} & T_{c_e} = ?

Step(i) Calculate both the capacity rates of hot & cold fluid i.e. $\dot{m}_h c_{p_h}$ & $\dot{m}_c c_{p_c}$. Hence get capacity rate ratio (C).

Step(ii) Calculate NTU from UA & $(\dot{m}c_p)_{small}$. Keep c_p in J/kg.K

Step(iii) Calculate the effectiveness of heat exchanger since it is a function of NTU & C

$$\epsilon = f^n(NTU, C)$$

Step(iv) Calculate only one exit temp. of the H.E. based on which side the capacity rate is smaller.

Step(v) Calculate the other exit temp. based upon energy balance eqn.

i.e. $Q = \dot{m}_h c_{p_h}(T_{hi} - T_{he}) = \dot{m}_c c_{p_c}(T_{c_e} - T_{ci})$

** Effectiveness -NTU method can be used for evaluating the area of heat exchanger with the same data given in the LMTD method. But LMTD method can't be used for evaluating both the exit temperatures with the data given in E-NTU method. **

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IES 2005

A process energy industry employs a counter flow heat exchanger to cool 0.8 kg/sec of oil ($C_p = 2.5$ kJ/kg.K) from 140°C to 40°C by the use of water entering at 20°C. The overall H.T coeff. is estimated to be 1600 W/m²K. It is assumed that the exit temp. of water will not exceed 80°C. Using NTU method & NTU = 4 in this case. Calculate the following
(i) Mass flow rate of water.
(ii) Surface area required.
(iii) Effectiveness of Heat Exchanger.

→

$$C_{p_h} = 2.5 \times 10^3 \text{ J/kg.K}$$
$$\dot{m}_h = 0.8 \text{ kg/sec}$$
$$T_{h_i} = 140^\circ C \qquad T_{h_e} = 40^\circ C$$
$$T_{c_i} = 20^\circ C \qquad U = 1600 \text{ W/m}^2\text{K}$$
$$T_{c_e} = 80^\circ C \qquad NTU = 4 \qquad C_{p_c} = 4.187 \text{ kJ/kgK}$$

capacity heat ratio
$$\dot{m}_h C_{p_h} = \dot{m}_c C_{p_c}$$

Energy balance eq^n

$$\dot{m}_h c_{p_h} (T_{hi} - T_{he}) = \dot{m}_c c_{p_c} (T_{ce} - T_{ci})$$

$$0.8 \times 2.5 \times 10^3 (120 - 40) = \dot{m}_c \, 4.187 \times 10^3 (80 - 20)$$

$$\therefore \dot{m}_c = \cancel{0.78 \text{ kg/s}} \quad 0.64 \text{ kg/s}$$

& $$\dot{m}_h c_{p_h} (140 - 40) = \dot{m}_c c_{p_c} (80 - 20)$$

$$\therefore \frac{\dot{m}_h c_{p_h}}{\dot{m}_c c_{p_c}} = \frac{60}{100} = \frac{3}{4} < 1$$

$$\dot{m}_h c_{p_h} < \dot{m}_c c_{p_c}$$

capacity rate is smaller on hot side

$$\therefore \epsilon = \frac{T_{hi} - T_{he}}{T_{hi} - T_{ci}}$$

$$= \frac{120 - 40}{120 - 20}$$

$$= 0.8$$

$$NTU = 4 = \frac{UA}{(\dot{m} c_p)_{small}} = \frac{UA}{c_{p_h} \dot{m}_h}$$

$$4 = \frac{1600 \times A}{2.5 \times 10^3 \times 0.8} \quad \therefore A = 5 \text{ m}^2$$

Date
Page

GATE 08

In a parallel flow H.E operating under steady state. The heat capacity rates of hot & cold fluids are equal. The hot fluid flowing at 1 kg/s with $C_p = 4\ kJ/kg\cdot K$ enters the exchanger at the 102°C. While the cold fluid has inlet temp. of 15°C. The overall H.T. coeff. for the H.E. is estimated to be $1\ kW/m^2K$ & the corresponding heat transfer surface area is $5\ m^2$. Neglect H.T. b/w H.E & the ambient. The exit temp in °C for the cold fluid, is

a] 45°C b] 55 c] 65 d] 75

1/0

$C = 1$ Capacity rate ratio.

For parallel flow H.E.

$$\epsilon = \frac{1 - e^{-2(NTU)}}{2}$$

$$NTU = \frac{UA}{(\dot{m}C_p)_{small}}$$

$$= \frac{1000 \times 5}{1 \times 4 \times 10^3}$$

$$= 1.25$$

$$\epsilon = \frac{1 - e^{-2(1.25)}}{2}$$

$$\therefore \epsilon = 0.458$$

$$0.458 = \frac{T_{ce} - T_{ci}}{T_{hi} - T_{ci}} = \frac{T_{ce} - 15}{102 - 15} \quad \boxed{T_{ce} = 54.84}$$

IES 1998 (25 Marks)

In a counterflow double pipe H.E. water flows through a copper tube (19 mm outer dia. & 16 mm inner dia.). At a flow rate of 1.48 m/s the oil flows through the annulus formed by inner copper tube & outer steel tube (30 mm O.D. & 26 mm I.D.). The steel tube is insulated from outside. The oil enters at the 0.4 kg/sec & is cooled from 65°C to 50°C whereas water enters at 32°C. Neglecting the resistance of copper tube, calculate the length of the tube required.

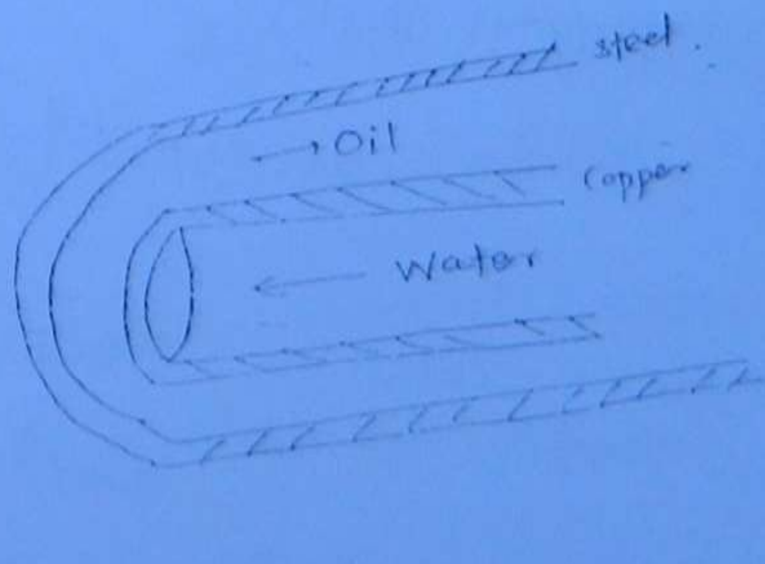
Data given ⇒ Fouling Factor on water side = $0.0005\ m^2K/W$

Fouling Factor on oil side = $0.0008\ m^2K/W$

$Nu = 0.023\ Re^{0.8}\ Pr^{0.3}$ ⇒ Mc Adam's Eq^n For turbulent flow through pipe

Property	Oil	Water
ρ (kg/m³)	850	995
C_p (kJ/kgK)	1.89	4.187
K (W/mK)	0.138	0.615
ν (m²/s)	7.44×10^{-6}	4.18×10^{-7}

1/1



$Nu = 0.023\, Re^{0.8}\, Pr^{0.3}$

$$\frac{hD}{K} = (0.023)\left(\frac{VD}{\nu}\right)^{0.8}\left(\frac{\mu C_p}{K}\right)^{0.3}$$

* To calculate ($h_{waterside}$) (1/2)

$$Re_{water} = \frac{1.48 \times 16\times10^{-3}}{4.18\times10^{-7}}$$

$$= 56650.717 \text{ (Flow is turbulent)}$$

$$Pr_{water} = \frac{\mu C_p}{K}$$

$$= \frac{\nu \times \rho \times C_p}{K} \qquad \left(\frac{\frac{kg}{m\,sec} \times J/kg\,K}{W/mK}\right)$$

$$= \frac{995 \times 4.18\times10^{-7} \times 4.187\times10^{3}}{0.615}$$

$$= 2.83$$

(Pr. no. is property of fluid)

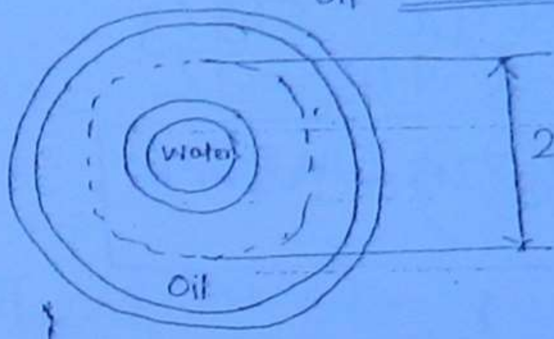
$$\frac{h \times 16\times10^{-3}}{0.615} = (0.023)(56650.717)^{0.8}(2.83)^{0.3}$$

$$h_{waterside} = 7666.176 \ W/m^2K$$

* To calculate ($h_{oilside}$)

$$Re_{oil} = \frac{VD}{\nu} \qquad \vec{V}_{oil} = \frac{\dot{m}_{oil}}{\frac{\pi}{4}\left[\left(\frac{26}{1000}\right)^2 - \left(\frac{19}{1000}\right)^2\right] \times 850}$$

$$\vec{V}_{oil} = \underline{1.9 \text{ m/s}}$$



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$$\therefore Re_{oil} = \frac{\vec{V} D}{\nu} = \frac{1.9 \times 22 \times 10^{-3}}{7.44 \times 10^{-6}}$$

$$= 5618$$ (~~Laminar~~ flow) Turbulent

$$\boxed{Re = \frac{\text{Inertia force}}{\text{Viscous force}}}$$ ($Re < 2000$) laminar

$$Pr_{oil} = \frac{\mu C_P}{K}$$

$$= \frac{\nu \times \rho \times C_P}{K} = \frac{7.44 \times 10^{-6} \times 850 \times 1.89 \times 10^{3}}{0.138}$$

$$= 86.611$$

$$\frac{h \times 22 \times 10^{-3}}{0.138} = (0.023)(5618)^{0.8}(86.611)^{0.3}$$

$$\therefore h_{oilside} = 549.69 \ W/m^2K$$

Now

$$\frac{1}{U} = \frac{1}{h_{waterside}} + \frac{1}{h_{oilside}} + F_{waterside} + F_{oilside}$$

$$= \frac{1}{7666.176} + \frac{1}{549.69} + 0.0005 + 0.0008$$

$$\boxed{U = 307.725 \ W/m^2K}$$

(114)

Energy balance eq^n

$$\dot{m}_h C_{p_h} (T_{hi} - T_{he}) = \dot{m}_c C_{p_c} (T_{ce} - T_{ci})$$

$$0.4 \times 1.89 \times (65 - 50) = \left[\rho \frac{\pi}{4} (d_i)^2_{copper} \times V \right] \times 4.187 \times 10^3 (T_{ce} - 32)$$

$$11.34 \times 10^3 = 1239.702 (T_{ce} - 32)$$

$$\boxed{T_{ce} = 41.15^\circ C}$$

$$\Delta T_m = \frac{\Delta T_i - \Delta T_e}{\ln(\Delta T_i / \Delta T_e)}$$

$$= \frac{(T_{hi} - T_{ce}) - (T_{he} - T_{ci})}{\ln\left(\frac{T_{hi} - T_{ce}}{T_{he} - T_{ci}}\right)}$$

$$= \frac{(65 - 41.15) - (50 - 32)}{\ln\left(\frac{65 - 41.15}{50 - 32}\right)}$$

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$\Delta T_m = 20.7°C$

$Q = UA\,\Delta T_m$

$\cancel{Q = \dot{m}}$ $\quad Q = \dot{m}_h c_{p_h} (T_{hi} - T_{he})$

$= 11.34 \times 10^3$

$\therefore 11.34 \times 10^3 = 307.725 \times A \times 20.7$

$\therefore A = 1.78 \ m^2$

$\pi D_o L = 1.78 \ m^2$

$\uparrow$ copper

$$L = \frac{1.78}{\pi \times 19 \times 10^{-3}} = 29.820 \ m$$

$$\frac{1}{U_i} = \frac{1}{h_i} + \frac{r_i}{k_1}\left(\ln r_o / r_i\right) + \frac{r_i}{r_o}\frac{1}{h_o}$$

Radiation

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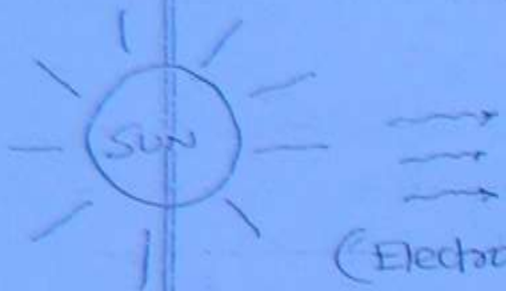
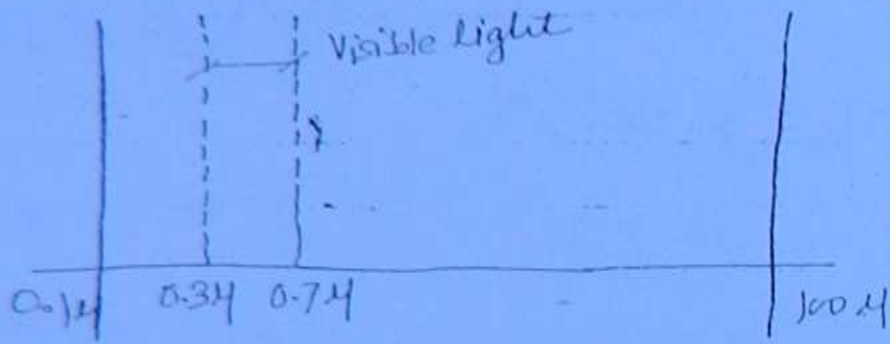


(Electromagnetic Waves)



spectrum of Thermal Radiation

All bodies at all temperatures emit thermal radiation, the rate of emission or the radiation energy coming from the surface of a body per unit time & per unit area being directly proportional to the fourth power of the absolute temp. of body. Hence radiation completely pre-dominates over conduction & convection particularly when, the temp. diff. is quite high. e.g. Heat transfer from hot flue gases to the refractory wall in a large furnace is predominantly by radiation.

All high temperature bodies like Sun emit radiation at the short wavelengths, whereas any earth bound body at long wavelengths.

Basic Def^n

(1) Total hemispherical Emissive Power (E)

It is defined as the radiation energy emitted from a surface of the body per unit time per unit area in all hemispherical directions at all wavelength

$$E \Rightarrow \frac{J}{sec \cdot m^2} = \frac{W}{m^2}$$

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(2) Total emissivity (ϵ)

It is defined as the ratio b/w total hemispherical emissive power of a non-black surface and total hemispherical emissive power of black surface both being at the same temperature.

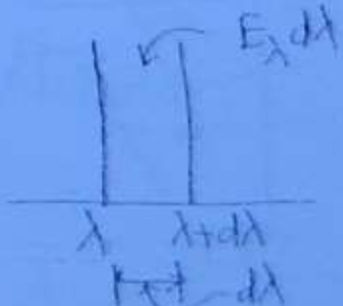
$$\epsilon \Rightarrow \frac{E}{E_b}$$

Black body is the body which absorbs all the incident radiation falling upon it.

A thermally black body need not appear black in colour to the human eye e.g snow, ice.

A thermally black body is not only good absorber & also an ideal emitter.

(3) Monochromatic (Spectral) Emissive Power (E_λ)



Unit $\frac{J}{sec \cdot m^2 \cdot \mu m}$

It is defined as the quantity which when multiplied by $d\lambda$, shall give the radiation energy