KINEMATICS OF MACHINERY

Unit – I-Basics of Mechanism

1.Terminologies
- Machine
- Mechanism
- Kinematic Pair
- Links
- Kinematic Chain

2.DOF
- Kutzhback Equation
- Grubler Equation

3.Grashoff Law

4.Mechanism
- Four Bar
- Single-Slider Crank
- Double-Slider

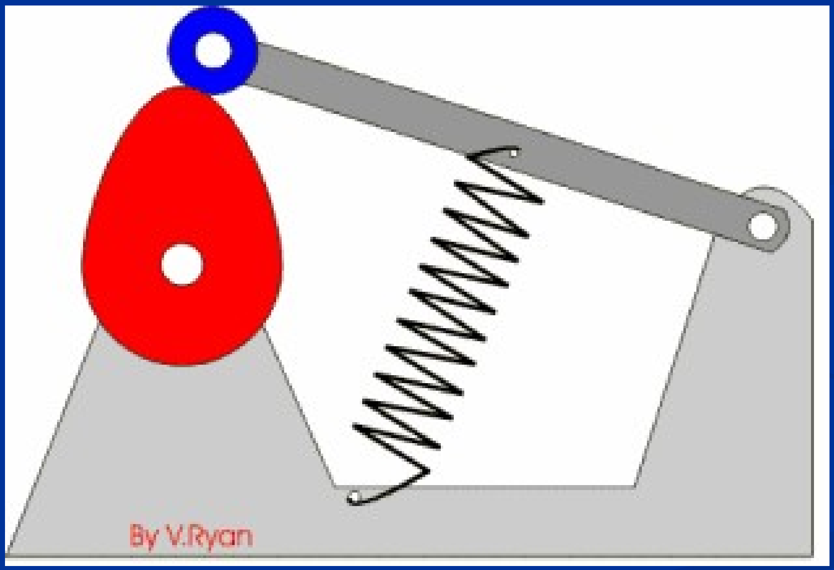
5.Inversion of Mechanism

6.Mechanical Advantage

MECHANISM

Mechanism – Part of a machine, which transmit motion and power from input point to output point

Example for Mechanism



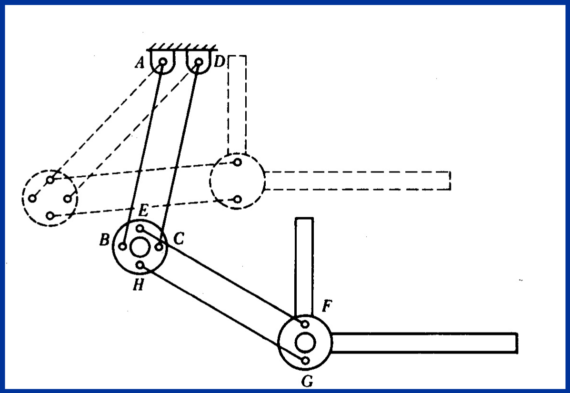
PLANAR MECHANISMS

When all the links of a mechanism have plane motion, it is called as a planar mechanism. All the links in a planar mechanism move in planes parallel to the reference plane.

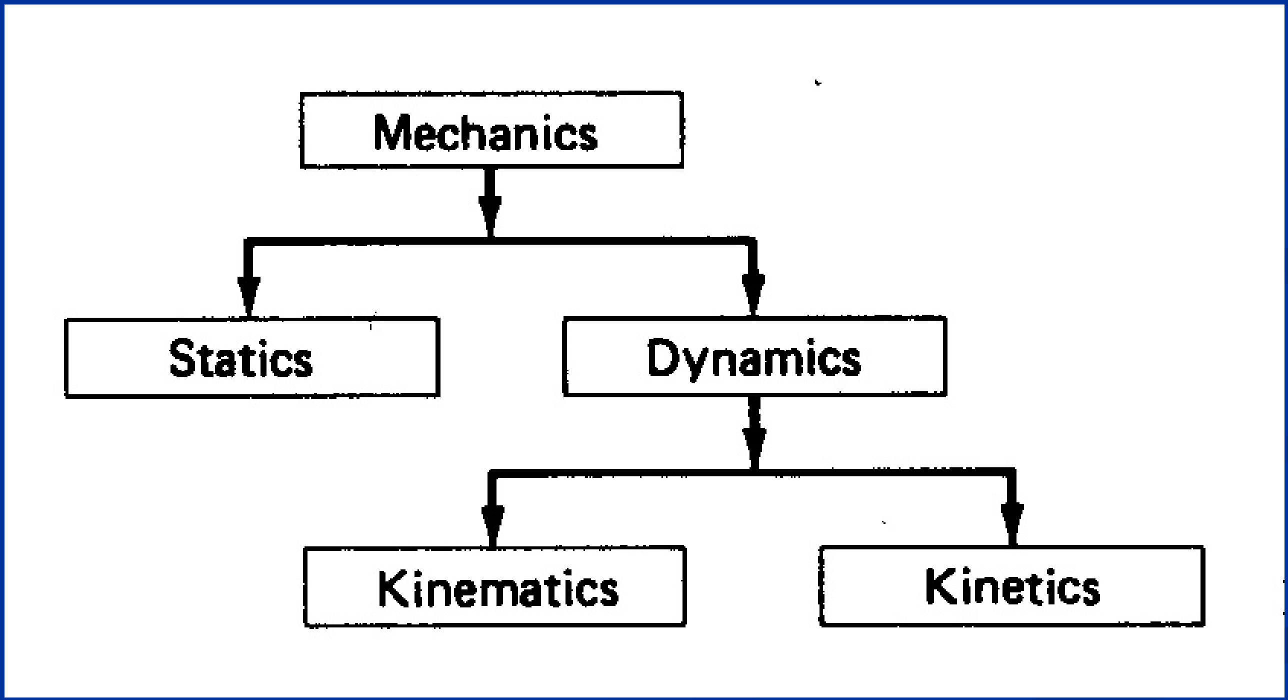
MACHINE

A machine is a mechanism or collection of mechanisms, which transmit force from the source of power to the resistance to be overcome.

Though all machines are mechanisms, all mechanisms are not machines



KINEMATICS



RELEVANCE OF KINEMATIC STUDY

- Motion requirements
- Design requirements

MOTION STUDY

Study of position, displacement, velocity and acceleration of different elements of mechanism

Given input → Desired output

Motion requirement

The 4 Axis Packing Robot

DESIGN REQUIREMENTS

Design: determination of shape and size

1. Requires knowledge of material
2. Requires knowledge of stress

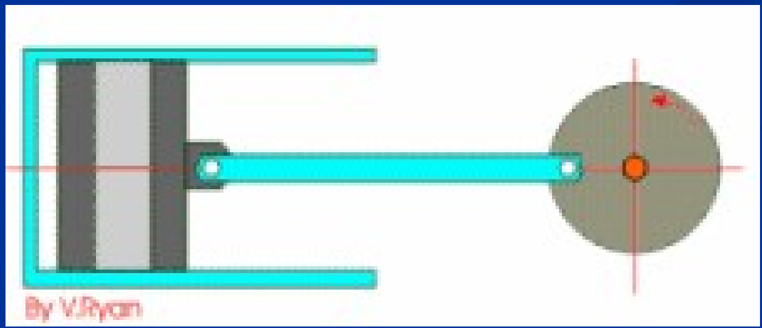
Requires knowledge of load acting

(i) static load

(ii) dynamic/inertia load

DYNAMIC/INERTIA LOAD

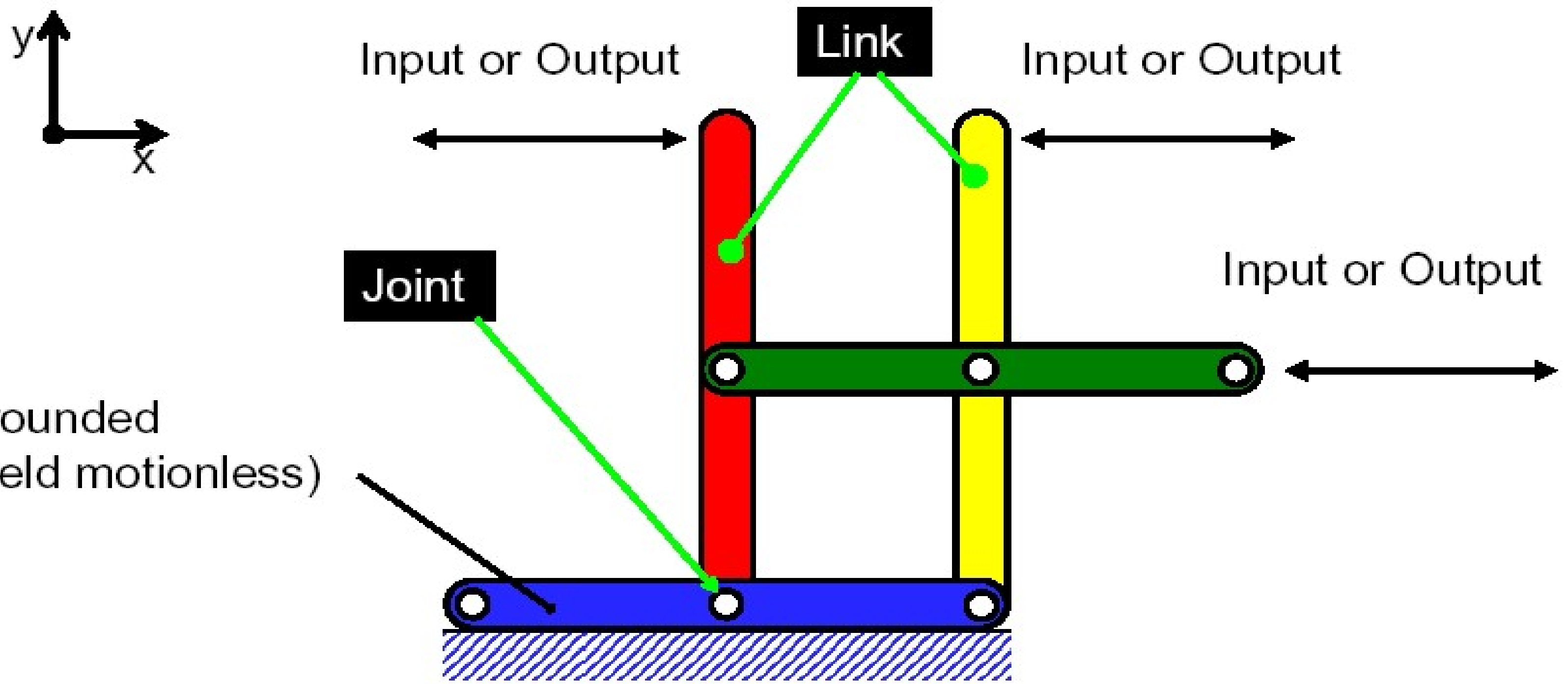
Inertia load require acceleration



What is a linkage?

What is a linkage?

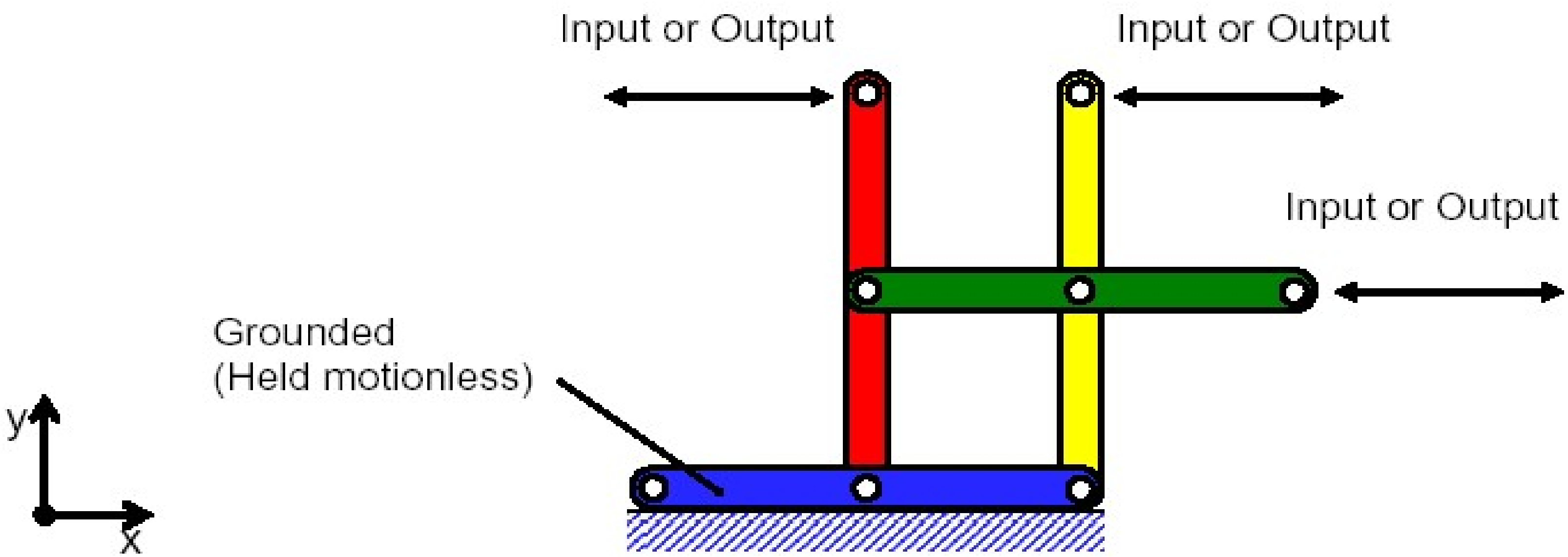
A mechanism used to define motion (kinematics) &/or transfer energy using links & joints



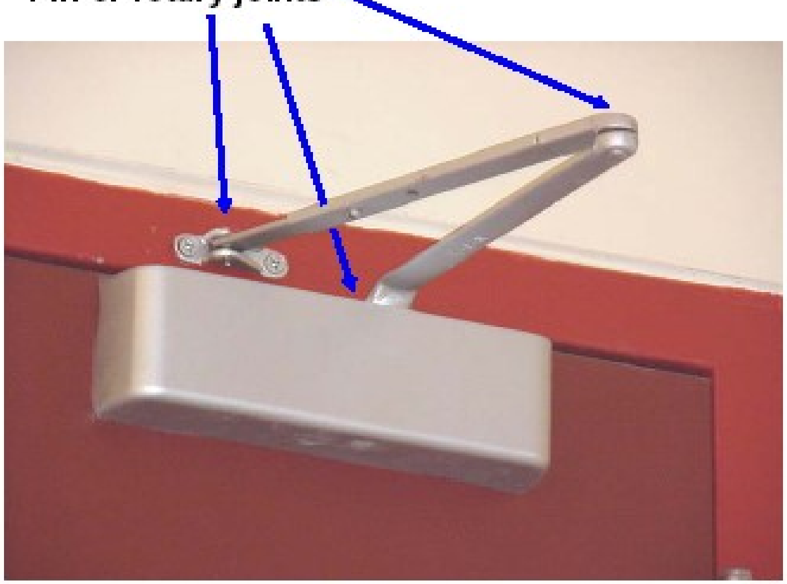
Linkage classification

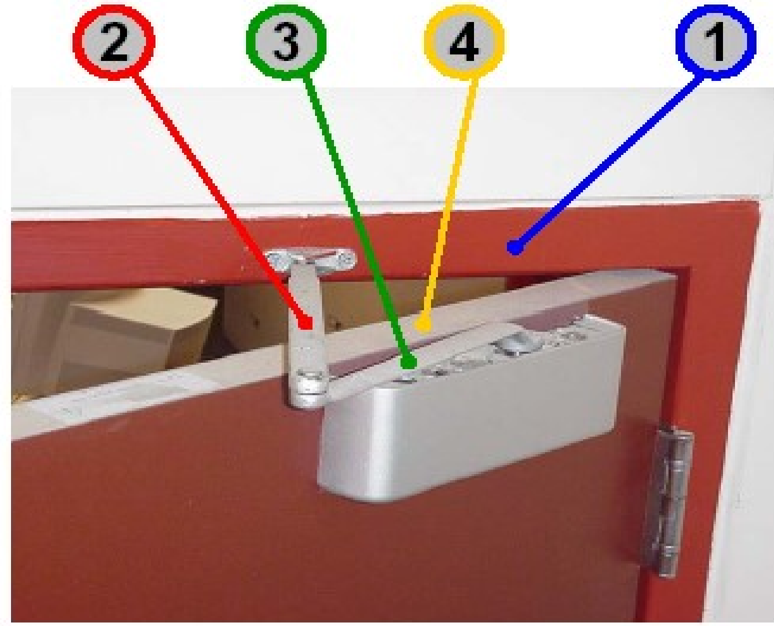
Linkages are classified by:

- The number of links (we will deal with 4-bar linkages)
- Type of links (we will deal with bars and sliders)
- Connection between links (we will deal with pinned, spherical and sliding joints)



Example: 4 bar door damper linkage



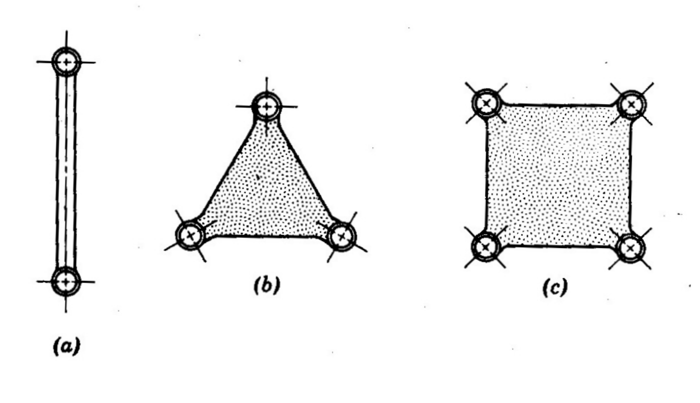


1	= Wall	or	Link 1	This is the grounded (held still)
2	= Bar 2	or	Link 2	
3	= Bar 3	or	Link 3	
4	= Door	or	Link 4	

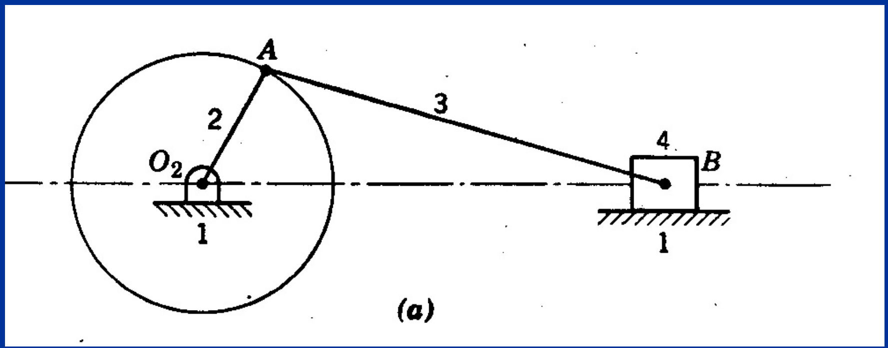
LINK OR ELEMENT

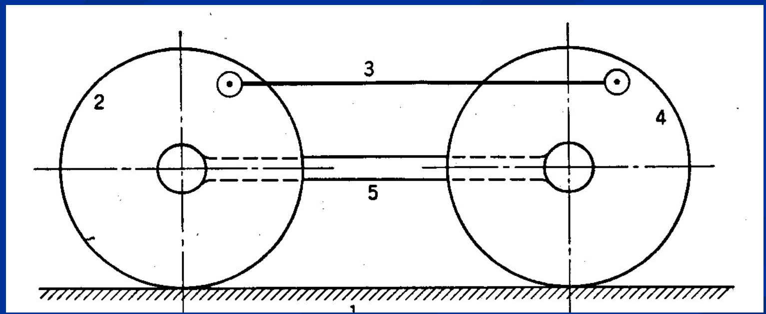
Any body (normally rigid) which has motion relative to another

- **Binary link**
- **Ternary link**
- **Quaternary link**



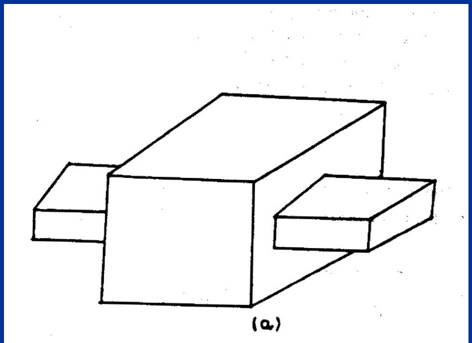
Examples of rigid links

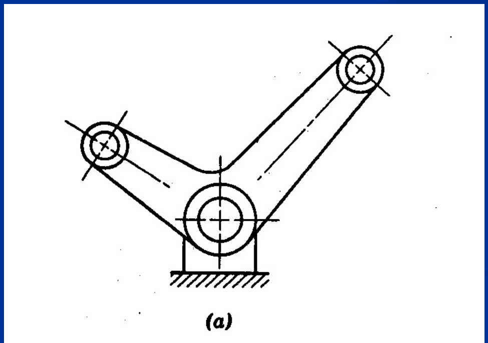




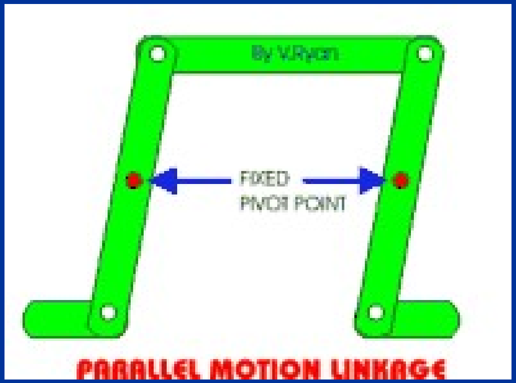
PAIRING ELEMENTS

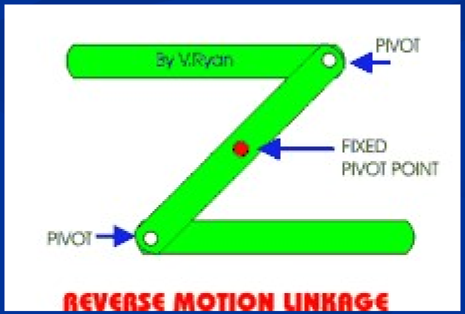
Pairing elements: the geometrical forms by which two members of a mechanism are joined together, so that the relative motion between these two is consistent. Such a pair of links is called **Kinematic Pair**.



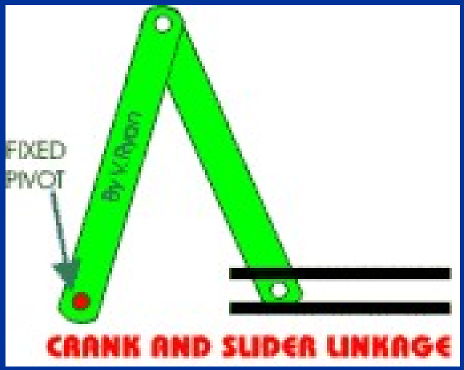


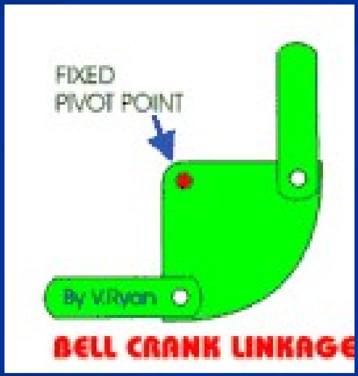
PAIRING ELEMENTS



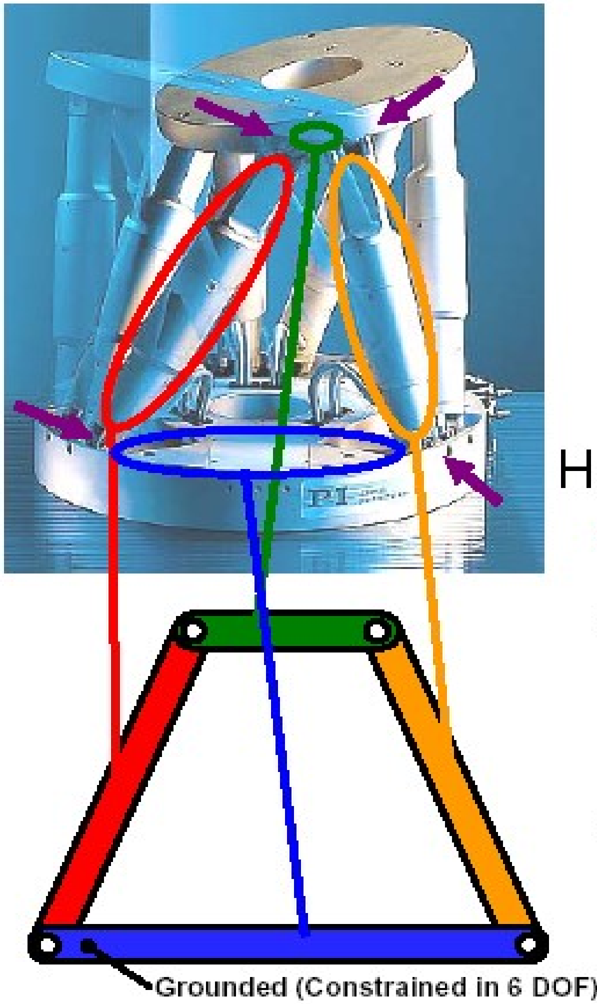


PAIRING ELEMENTS





Example: Stewart platform



How to identify links and joints

Step 1: Find the link that is not moving (ground)

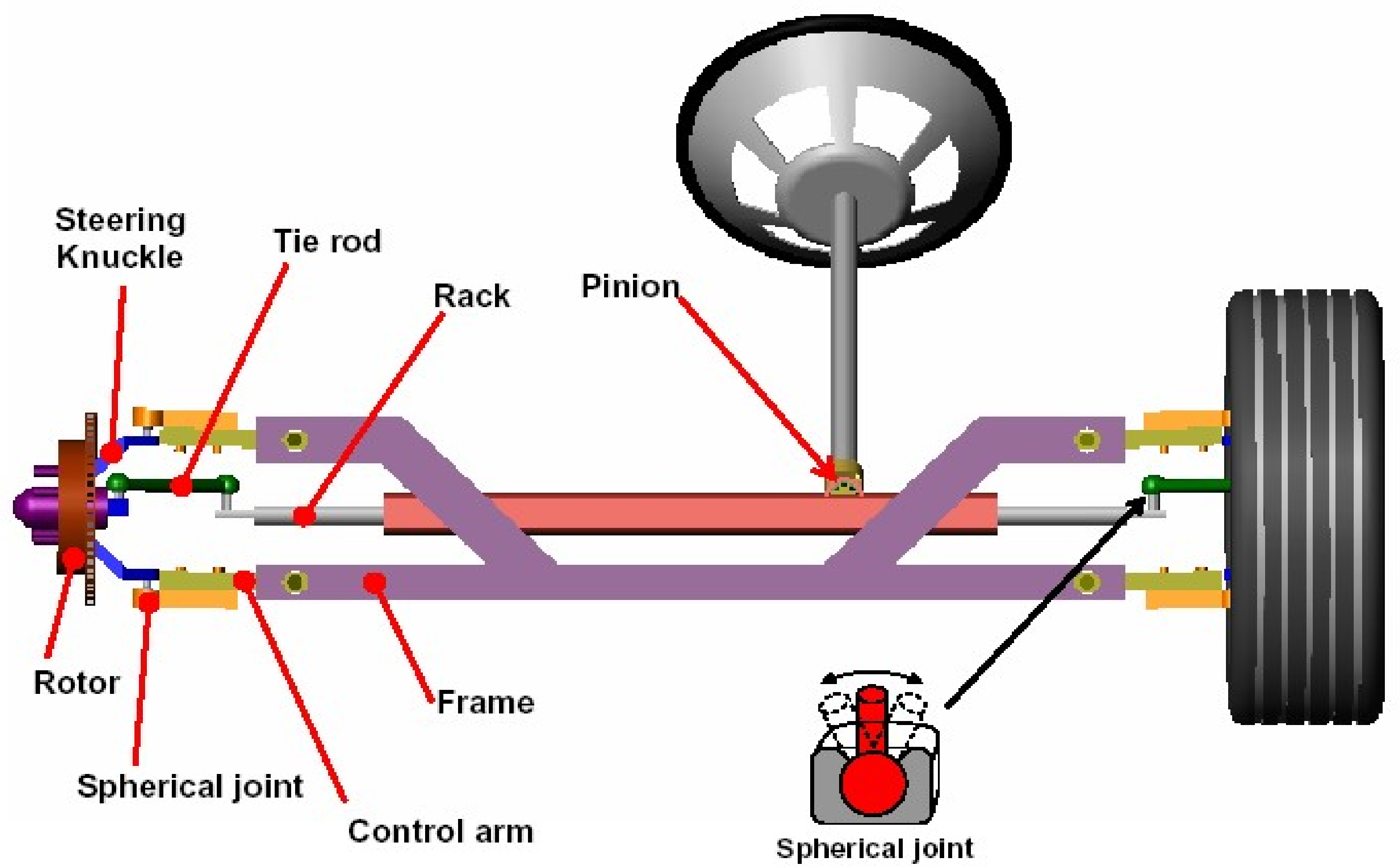
Step 2: Find a feature that experiences relative motion (joint)

Step 3: The next link lies between this joint and the next pt of relative motion

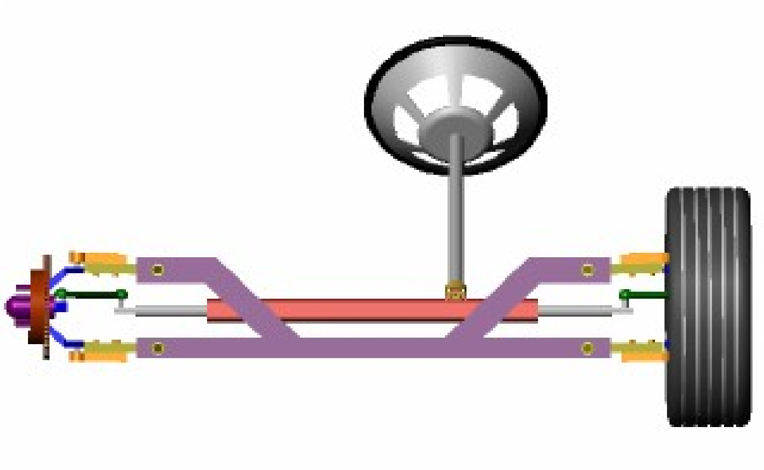
Step x: Continue through links/joints to shortest path to ground

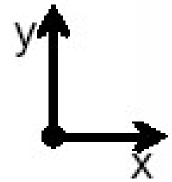
This sounds easy, but…

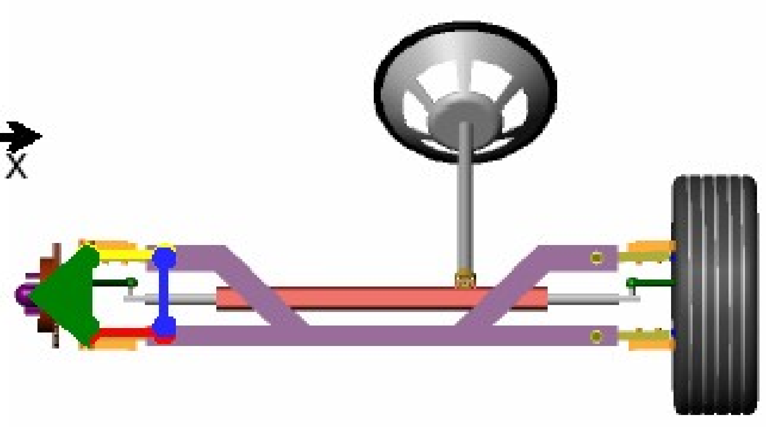
Example: Automotive suspension



Example: Automotive suspension







KINEMATIC PAIRS

- A mechanism has been defined as a combination so connected that each moves with respect to each other. A clue to the behavior lies in in the nature of connections, known as kinetic pairs. The degree of freedom of a kinetic pair is given by the number independent coordinates required to completely specify the relative movement.

TYPES OF KINEMATIC PAIRS

Based on nature of contact between elements

- (i) **Lower pair** : The joint by which two members are connected has surface contact. A pair is said to be a lower pair when the connection between two elements are through the area of contact. Its 6 types are

Revolute(Or)TurningPair

Prismatic(Or)SlidingPair

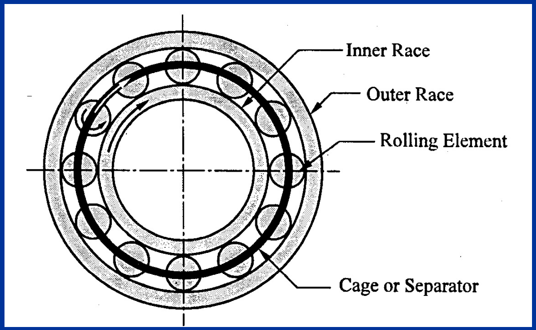
Screw(Or)HelicalPair

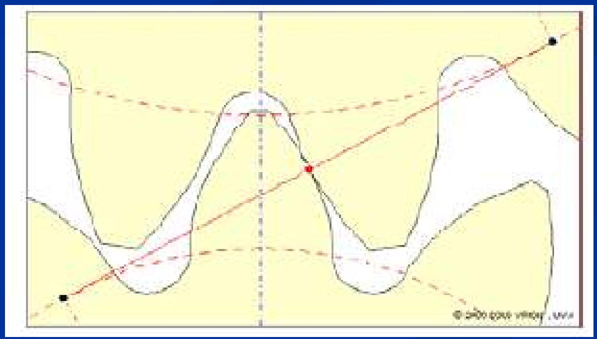
CylindricalPair

Spherical(Or)GlobularPair

Flat(or)PlanarPair

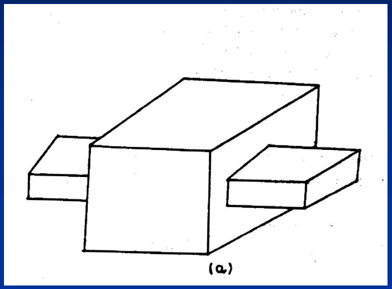
(ii) Higher pair: The contact between the pairing elements takes place at a point or along a line.



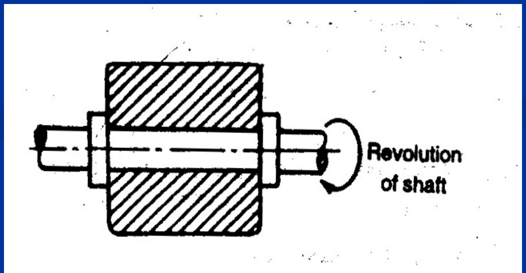


Based on relative motion between pairing elements

(a) Siding pair [DOF = 1]

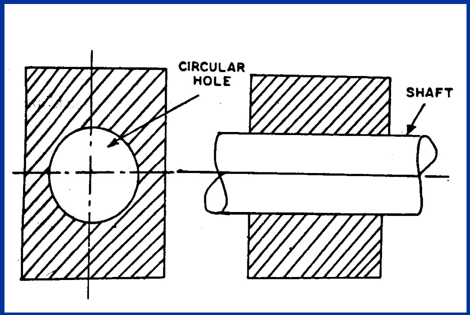


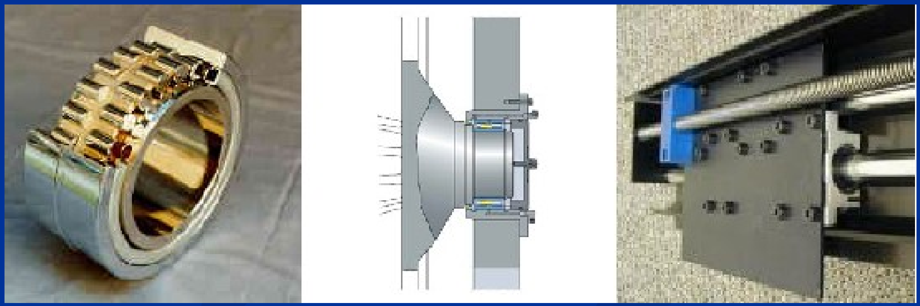
(b) Turning pair (revolute pair) [DOF = 1]



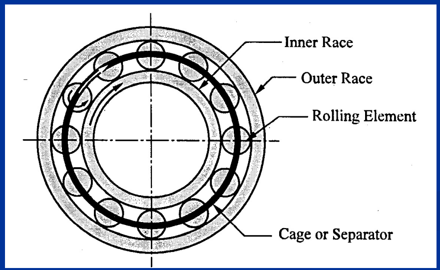
Based on relative motion between pairing elements

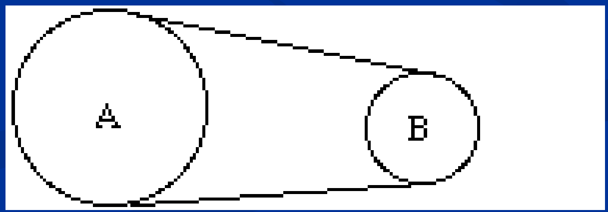
(c) Cylindrical pair [DOF = 2]





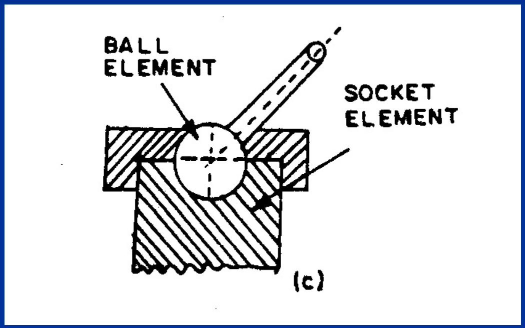
(d) Rolling pair [DOF = 1]

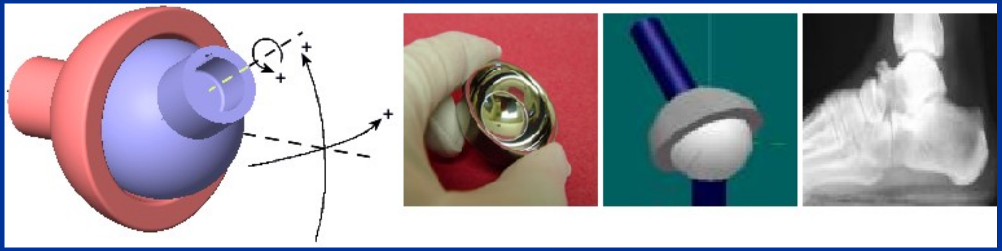




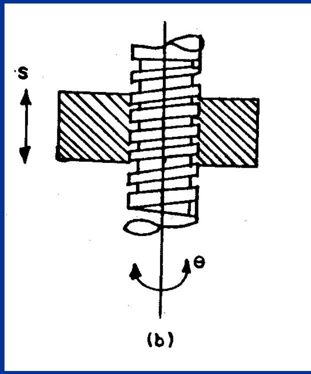
Based on relative motion between pairing elements

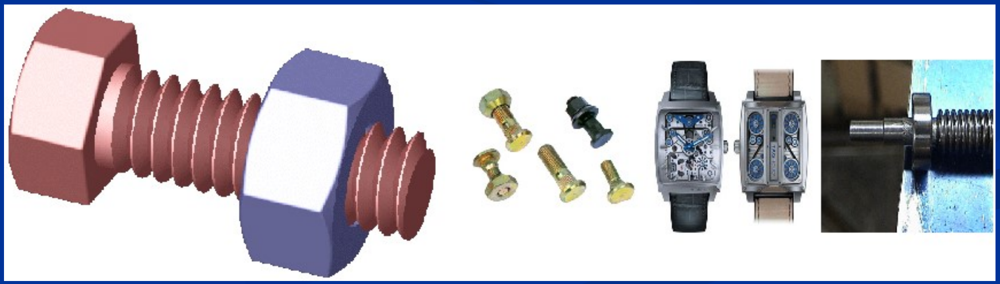
(e) Spherical pair [DOF = 3]





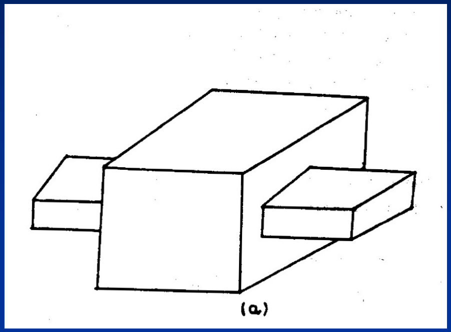
(f) Helical pair or screw pair [DOF = 1]



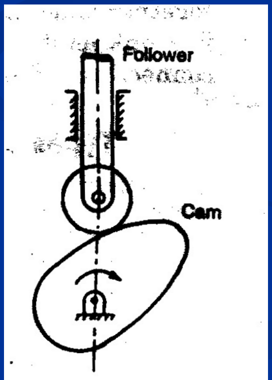


Based on the nature of mechanical constraint

(a) Closed pair



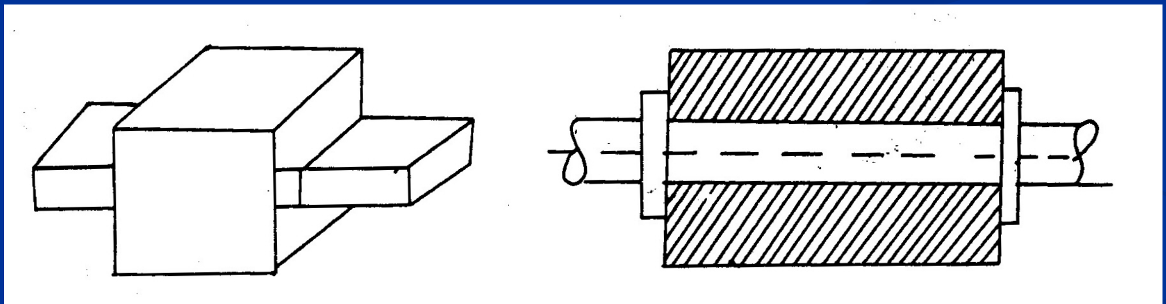
(b) Unclosed or force closed pair



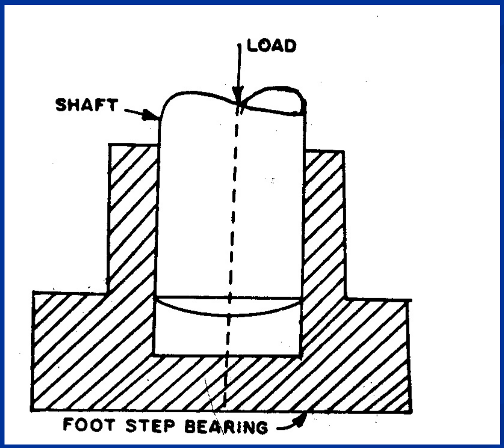
CONSTRAINED MOTION

one element has got only one definite motion relative to the other

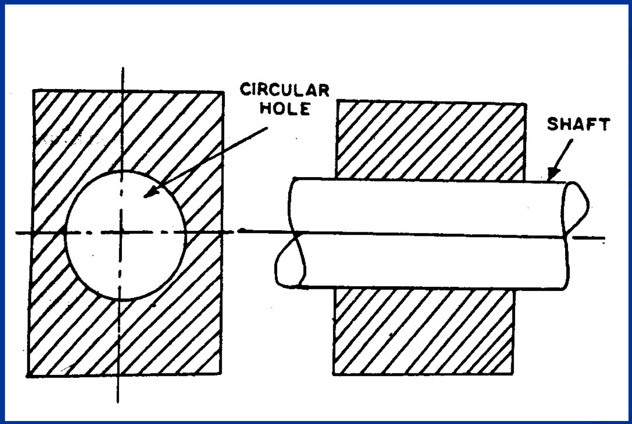
(a) Completely constrained motion



(b) Successfully constrained motion



(c) Incompletely constrained motion



KINEMATIC CHAIN

Group of links either joined together or arranged in a manner that permits them to move relative to one another.

Kinematic Chain

Relation between Links, Pairs and Joints

$L = 2P - 4$

$J = (3/2)\,L - 2$

L => No of Links

P => No of Pairs

J => No of Joints

L.H.S > R.H.S => Locked chain

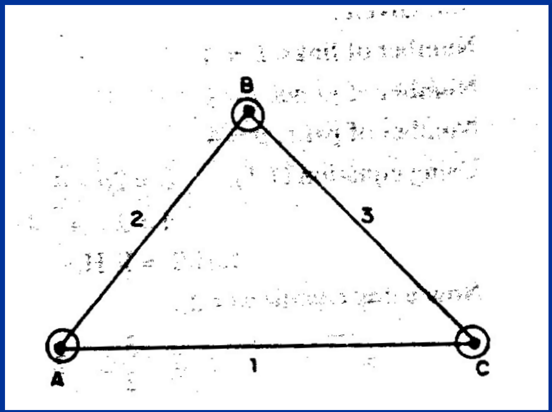
L.H.S = R.H.S => Constrained Kinematic Chain

L.H.S < R.H.S => Unconstrained Kinematic Chain

LOCKED CHAIN (Or) STRUCTURE

Links connected in such a way that no relative motion is possible.

L=3, J=3, P=3 L.H.S>R.H.S

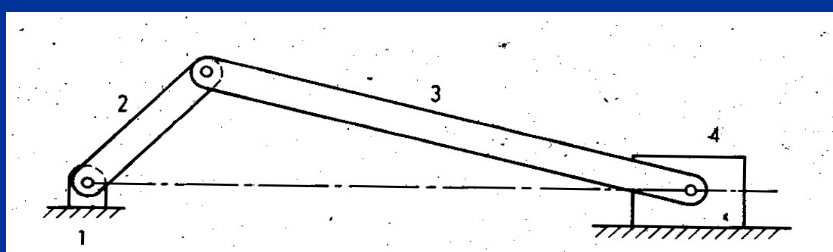


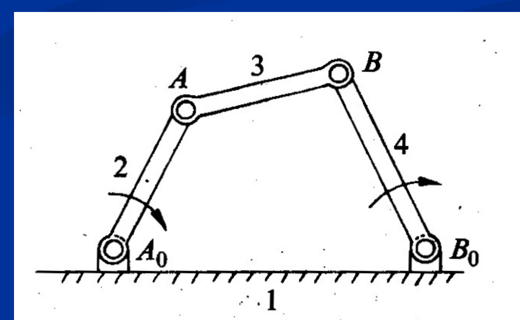
Kinematic Chain Mechanism

Slider crank and four bar mechanisms

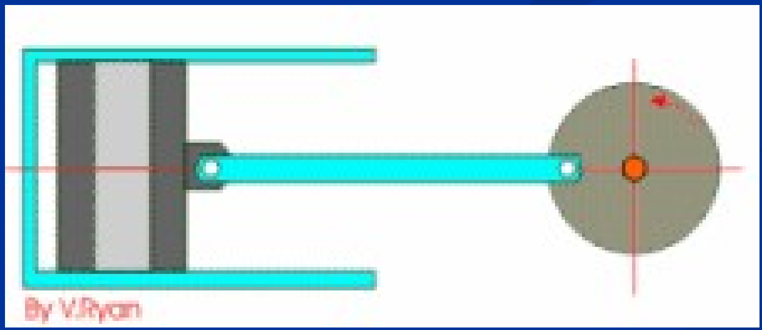
L=4, J=4, P=4

L.H.S=R.H.S





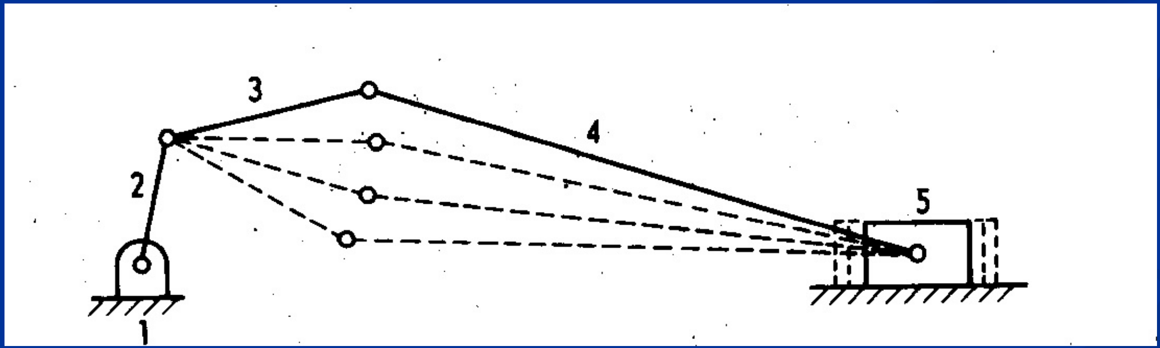
Working of slider crank mechanism



Unconstrained kinematic chain

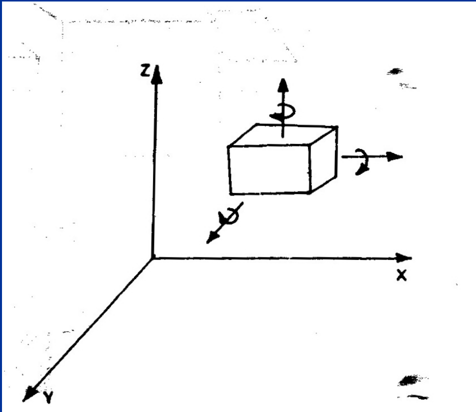
L=5,P=5,J=5

L.H.S < R.H.S



DEGREES OF FREEDOM (DOF):

It is the number of independent coordinates required to describe the position of a body.



Degrees of freedom/mobility of a mechanism

It is the number of inputs (number of independent coordinates) required to describe the configuration or position of all the links of the mechanism, with respect to the fixed link at any given instant.

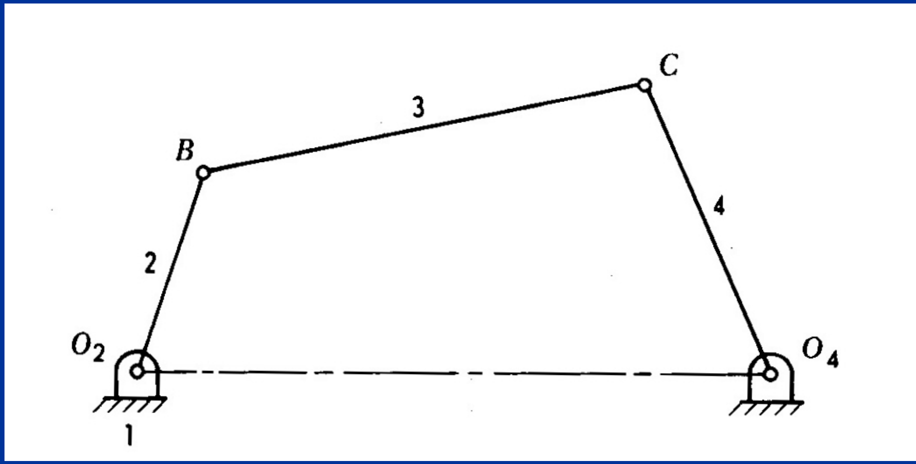
GRUBLER'S CRITERION

Number of degrees of freedom of a mechanism is given by

$$F = 3(n-1)-2l-h.$$ Where,

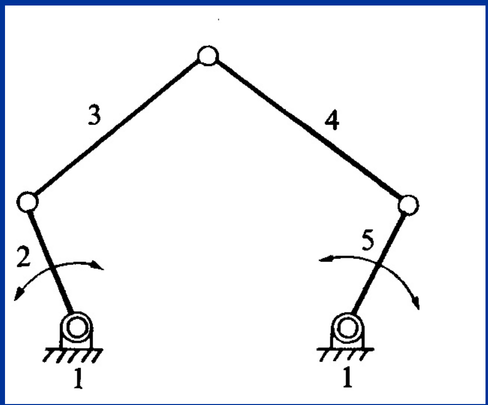
- F = Degrees of freedom
- n = Number of links in the mechanism.
- l = Number of lower pairs, which is obtained by counting the number of joints. If more than two links are joined together at any point, then, one additional lower pair is to be considered for every additional link.
- h = Number of higher pairs

Examples - DOF



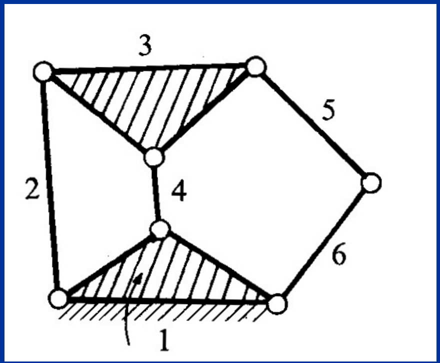
- $F = 3(n-1)-2l-h$
- Here, $n = 4, l = 4$ & $h = 0$.
- $F = 3(4-1)-2(4) = 1$
- I.e., one input to any one link will result in definite motion of all the links.

Examples - DOF



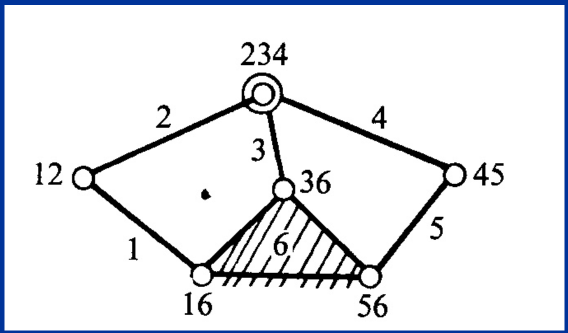
- $F = 3(n-1)-2l-h$
- Here, $n = 5$, $l = 5$ and $h = 0$.
- $F = 3(5-1)-2(5) = 2$
- I.e., two inputs to any two links are required to yield definite motions in all the links.

Examples - DOF



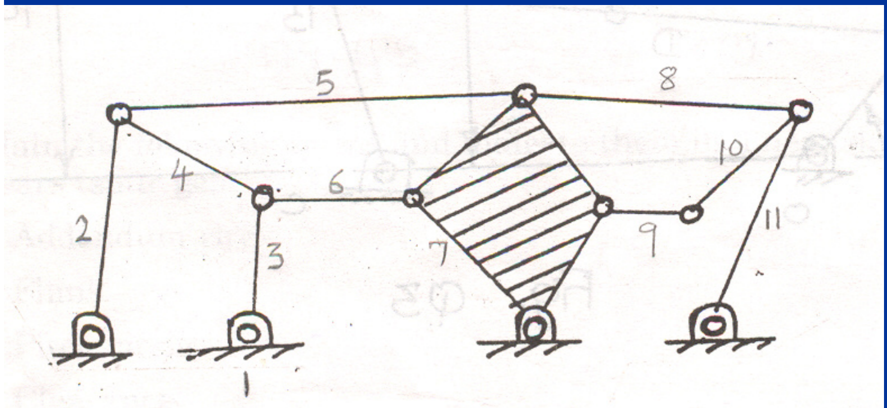
- F = 3(n-1)-2l-h
- Here, n = 6, l = 7 and h = 0.
- F = 3(6-1)-2(7) = 1
- I.e., one input to any one link will result in definite motion of all the links.

Examples - DOF



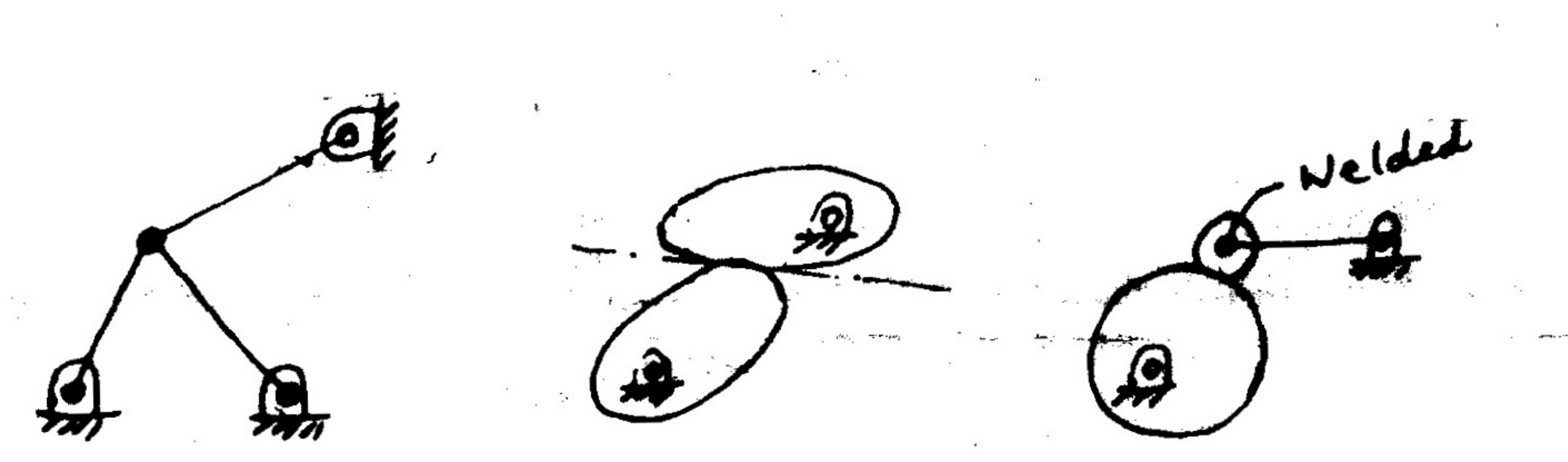
- F = 3(n-1)-2l-h
- Here, n = 6, l = 7 (at the intersection of 2, 3 and 4, two lower pairs are to be considered) and h = 0.
- F = 3(6-1)-2(7) = 1

Examples - DOF



- F = 3(n-1)-2l-h
- Here, n = 11, l = 15 (two lower pairs at the intersection of 3, 4, 6; 2, 4, 5; 5, 7, 8; 8, 10, 11) and h = 0.
- F = 3(11-1)-2(15) = 0

Examples - DOF



(a)
F = 3(n-1)-2l-h
Here, n = 4, l = 5 and h = 0.
F = 3(4-1)-2(5) = -1
I.e., it is a structure

(b)
F = 3(n-1)-2l-h
Here, n = 3, l = 2 and h = 1.
F = 3(3-1)-2(2)-1 = 1

(c)
F = 3(n-1)-2l-h
Here, n = 3, l = 2 and h = 1.
F = 3(3-1)-2(2)-1 = 1

Determining DOF and Pairs

- N_b=No of Binary Links
- N_t=No of Ternary Links
- N_o=No of Other Links
- N=Total No of Links
- L=No of Loops
- P=No of Pairs
- M=Mobility or DOF

P=N+L-1

M=N-(2L+1)

Determining DOF and Pairs

$P=N+L-1$

$M=N-(2L+1)$

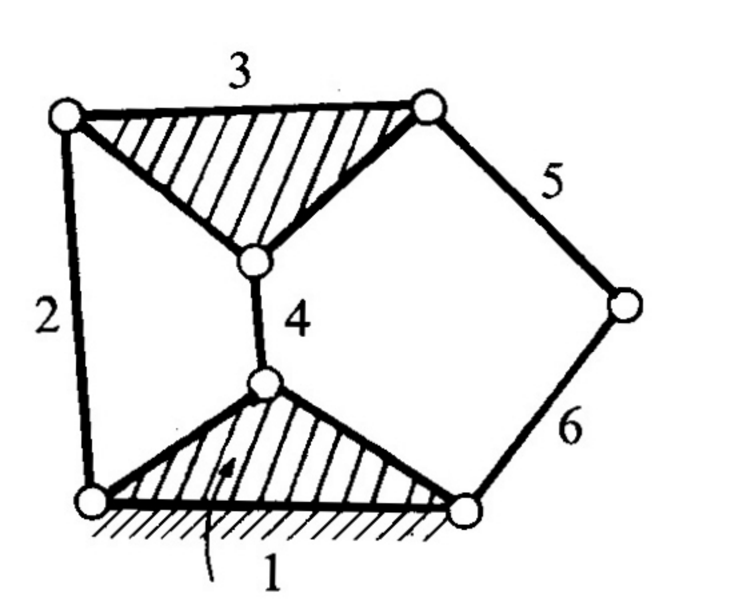
$N_b = 4, N_t=2, N_0=0$

$N=6, L=2$

Sol:

$P=6+2-1=7$

$M=6-(2x2 +1)=1$



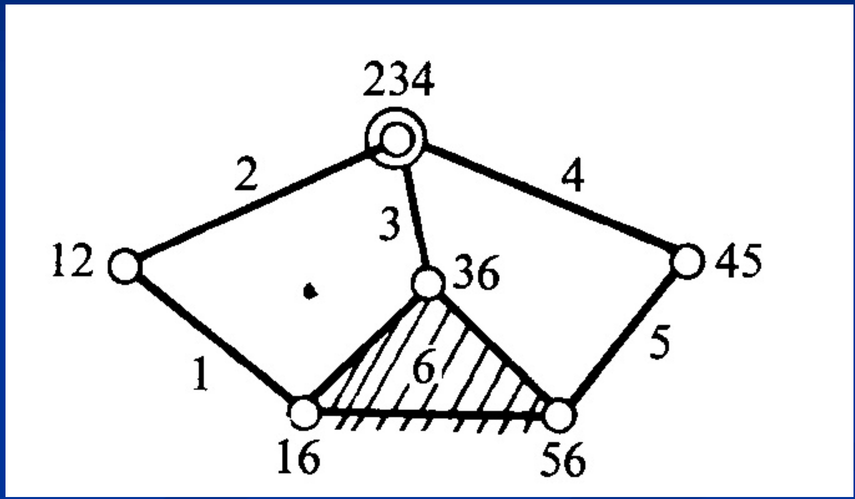
Determining DOF and Pairs

$P=N+L-1$

$M=N-(2L+1)$

$N_b = 5, N_t=1, N_0=0$

$N=6, L=2$



Sol:

$P=6+2-1=7$

$M=6-(2x2 +1)=1$

Determining DOF and Pairs

$P=N+L-1$

$M=N-(2L+1)$

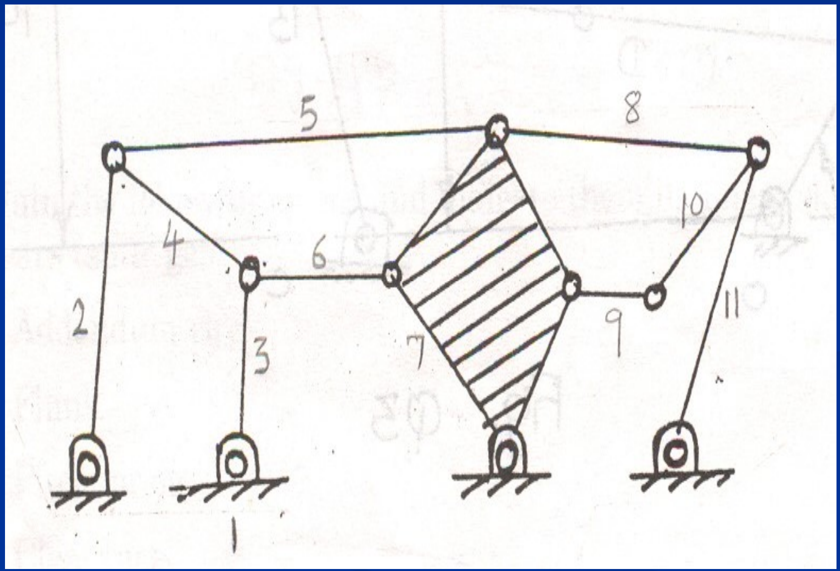
$N_b = 9, N_t=0, N_0 = 2$

$N=11, L=5$

Sol:

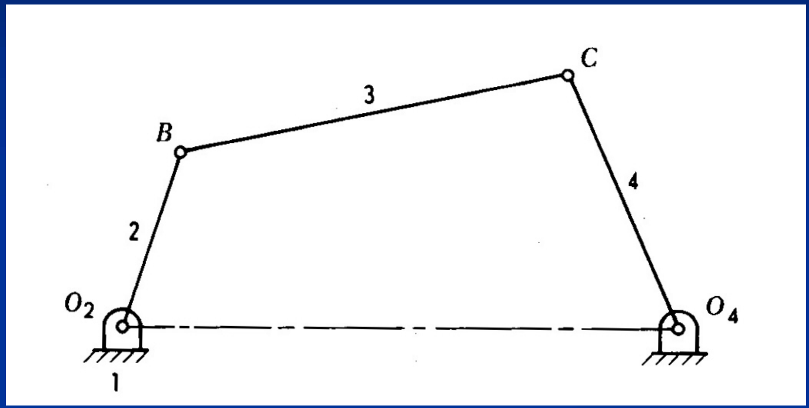
$P=11+5-1=15$

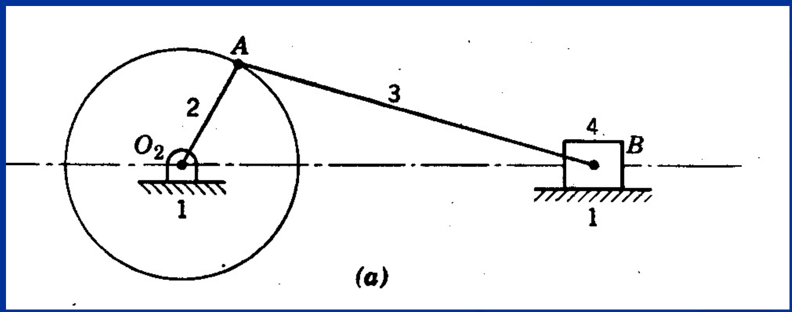
$M=11-(2x5 +1)=0$



Grashoff Law

- The sum of the shortest and longest link length should not exceed the sum of the other two link lengths.





$s+l < p+q$

(e.x) $(1+2) < (3+4)$

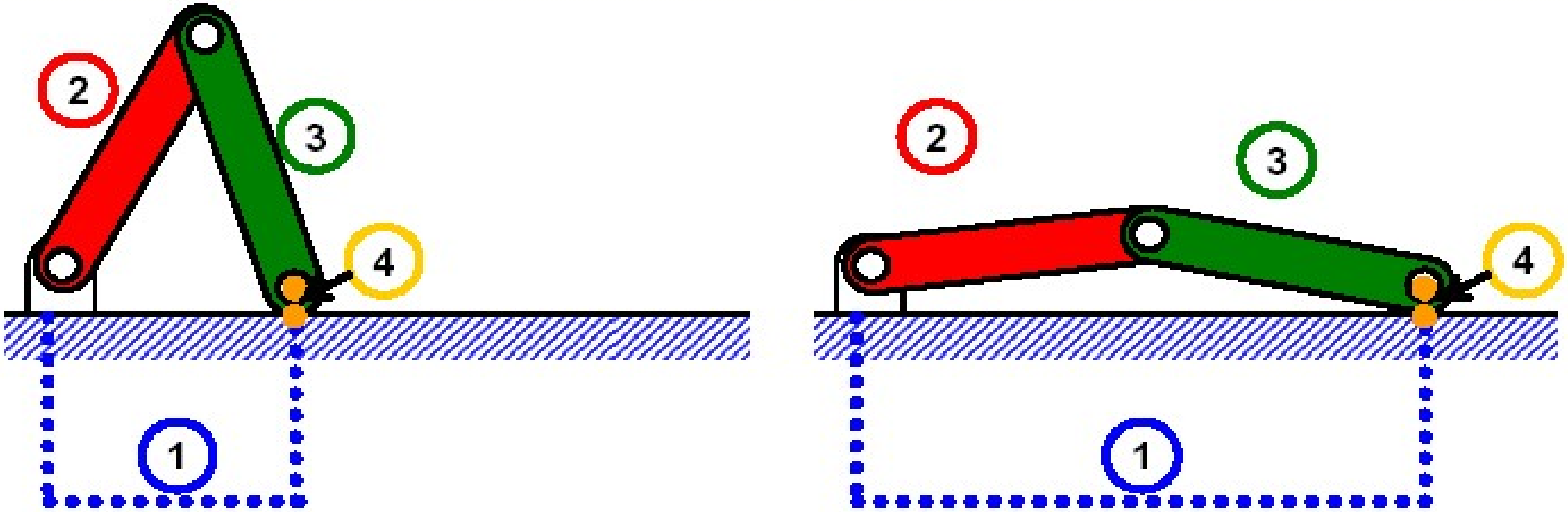
INVERSIONS OF MECHANISM

A mechanism is one in which one of the links of a kinematic chain is fixed. Different mechanisms can be obtained by fixing different links of the same kinematic chain. These are called as inversions of the mechanism.

INVERSIONS OF MECHANISM

- 1.Four Bar Chain
- 2.Single Slider Crank
- 3.Double Slider Crank

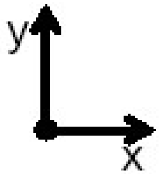
Effective 4-bar linkage



Appears that there are only 3 links

Link 4 is an effective link

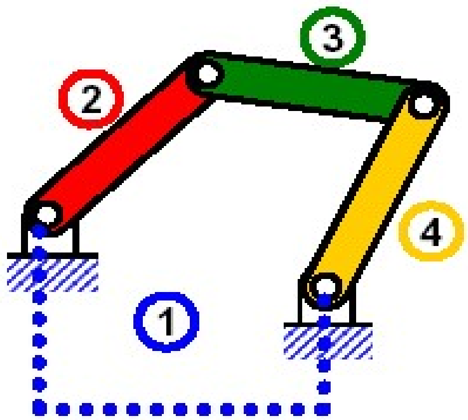
This linkage can be modeled as a 4 bar linkage

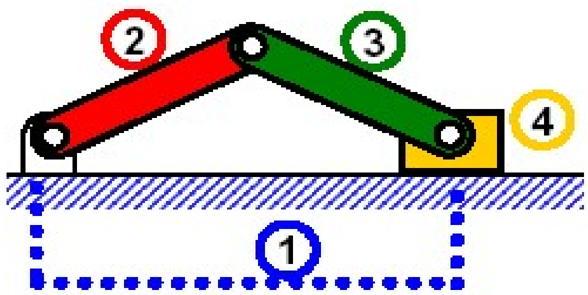


Length of link 1 varies with linkage position

An effective link is characterized by a "rigid" non-changing length that is needed to define the motion of the mechanism

Common 4-bar linkage types



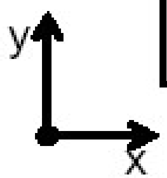


4 BAR COUPLER:

Consists of 4 links:
- 1 = Ground
- 2 = Crank or follower
- 3 = Coupler
- 4 = Crank or follower

Crank is the link driven by force/torque

Links can cross each other



4 BAR SLIDER-CRANK:

Consists of 3 links and slider:
- 1 = Ground
- 2 = Crank
- 3 = Coupler
- 4 = Slider

Crank is link driven by Force/torque

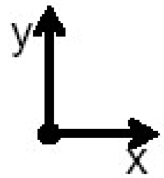
Link # 4 can be:
- A pin, block, or tube

Link # 4 slides in/on one of these:
- Another part, rod or tube

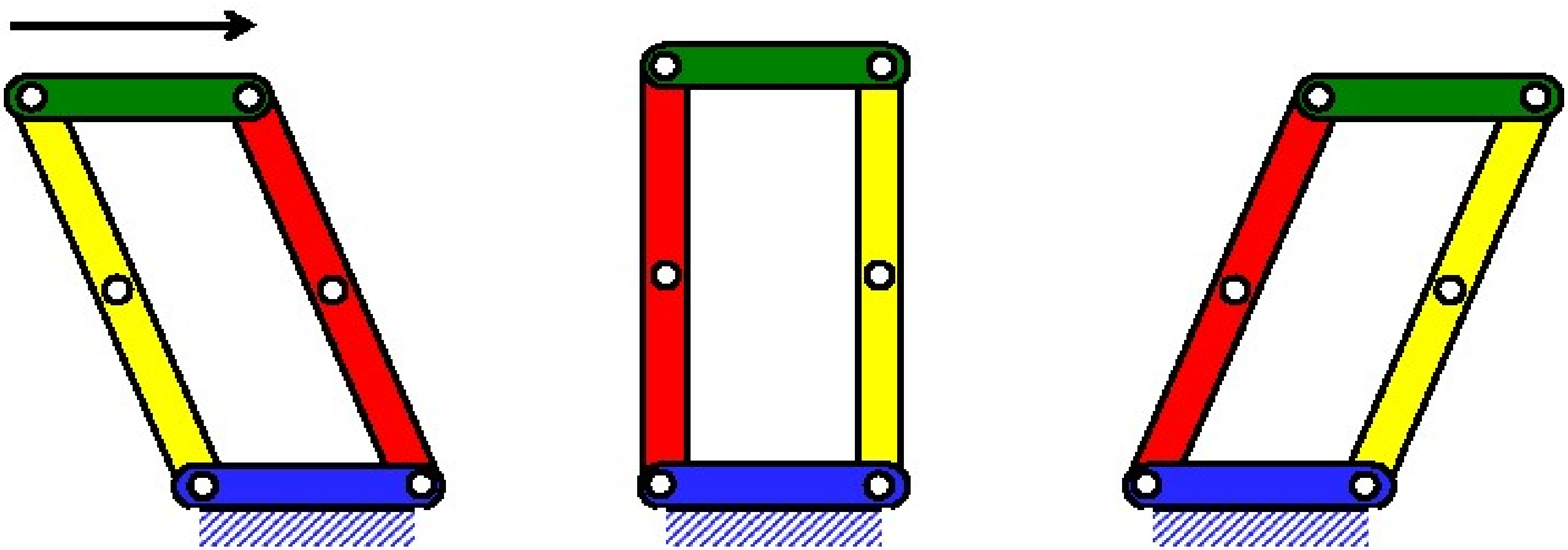
Links can cross each other

Parallel link 4 bar mechanism

Parallel 4 bar linkage:

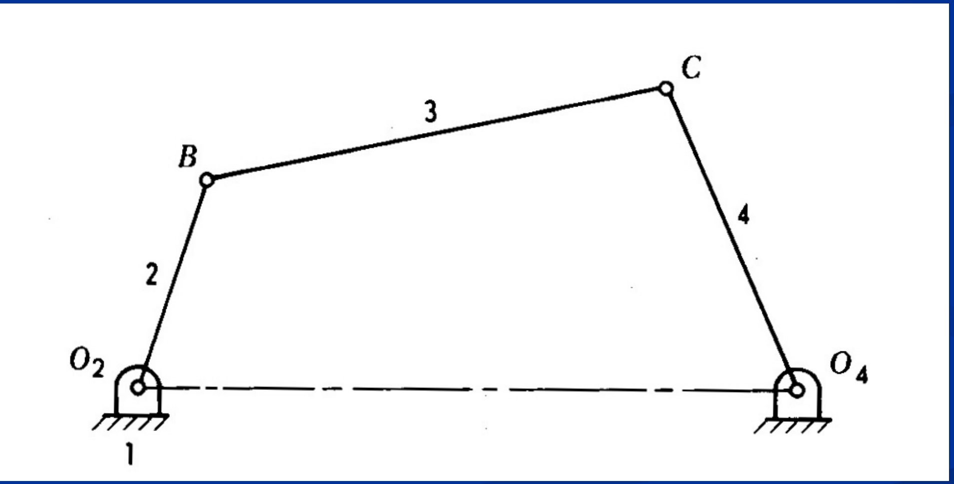


- Opposing links have equal lengths
- $\theta_1 = 180^\circ$ $|R_1| = |R_3|$ $|R_2| = |R_4|$
- Opposing links remain parallel



1. FOUR BAR CHAIN

- (link 1) frame
- (link 2) crank
- (link 3) coupler
- (link 4) rocker

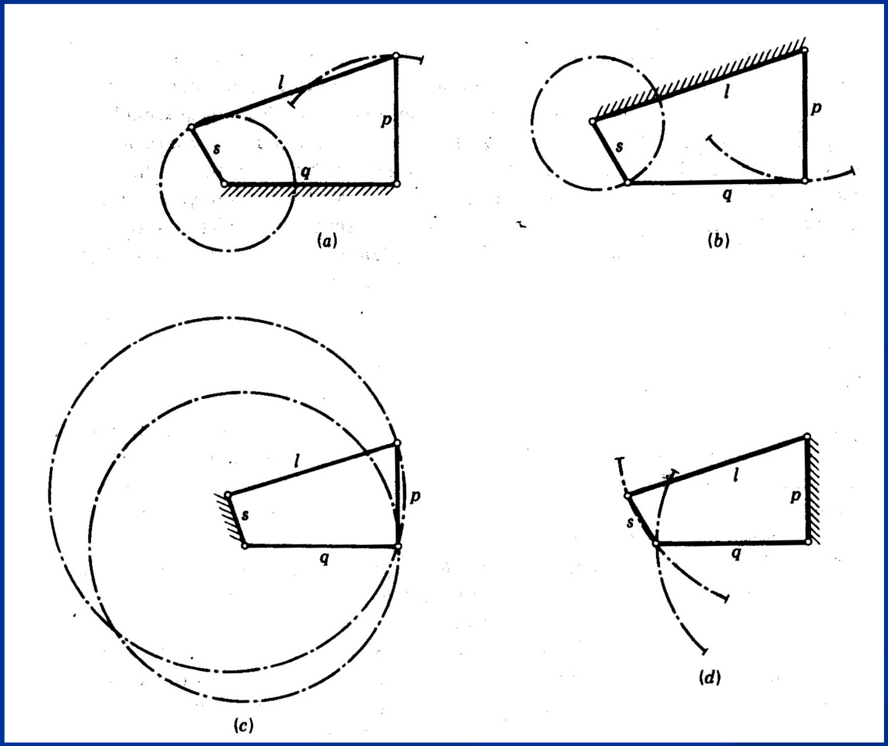


INVERSIONS OF FOUR BAR CHAIN

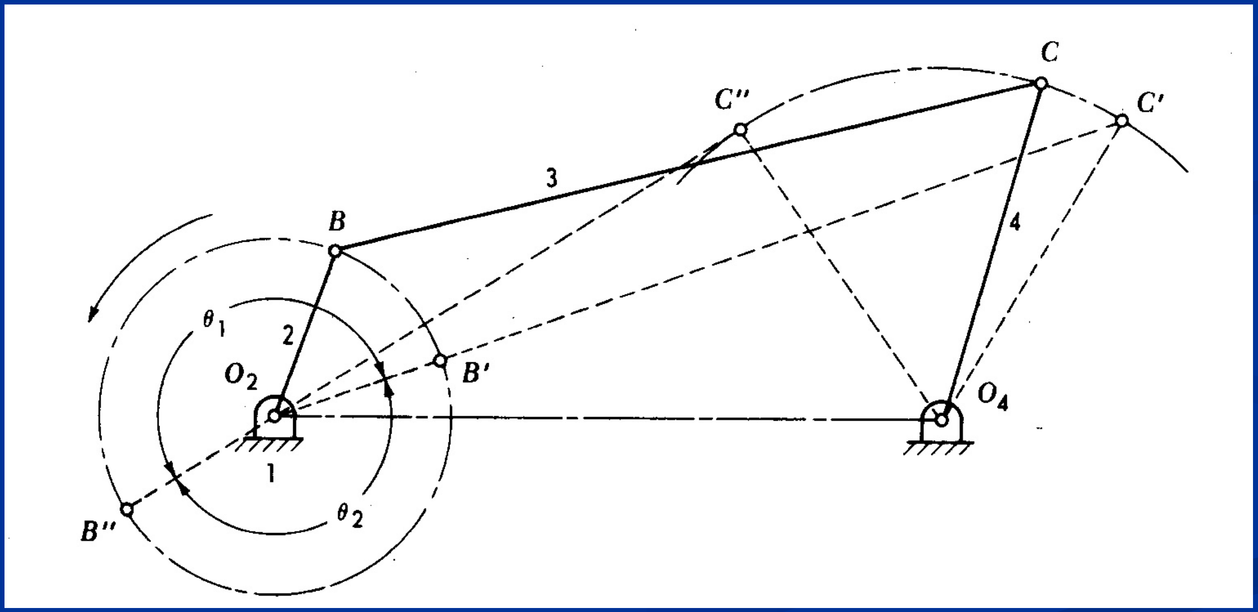
Fix link 1& 3. Crank-rocker
or Crank-Lever
mechanism

Fix link 2. Drag link
or Double Crank
mechanism

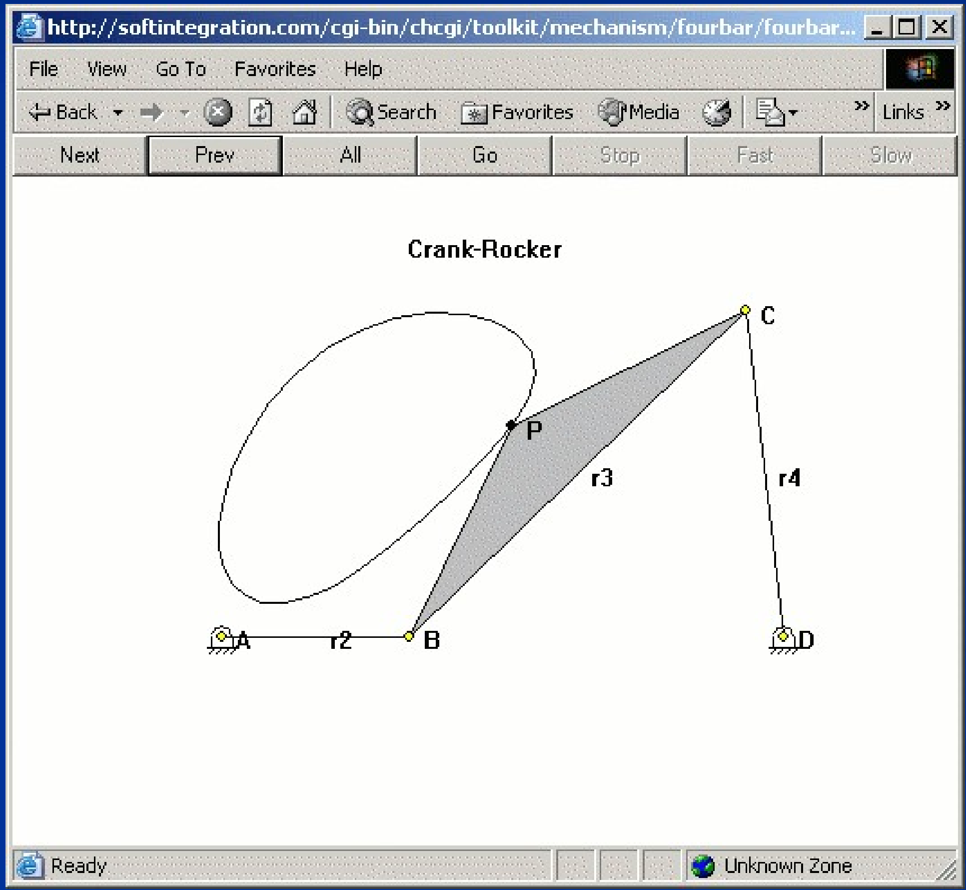
Fix link 4. Double rocker
mechanism
Pantograph



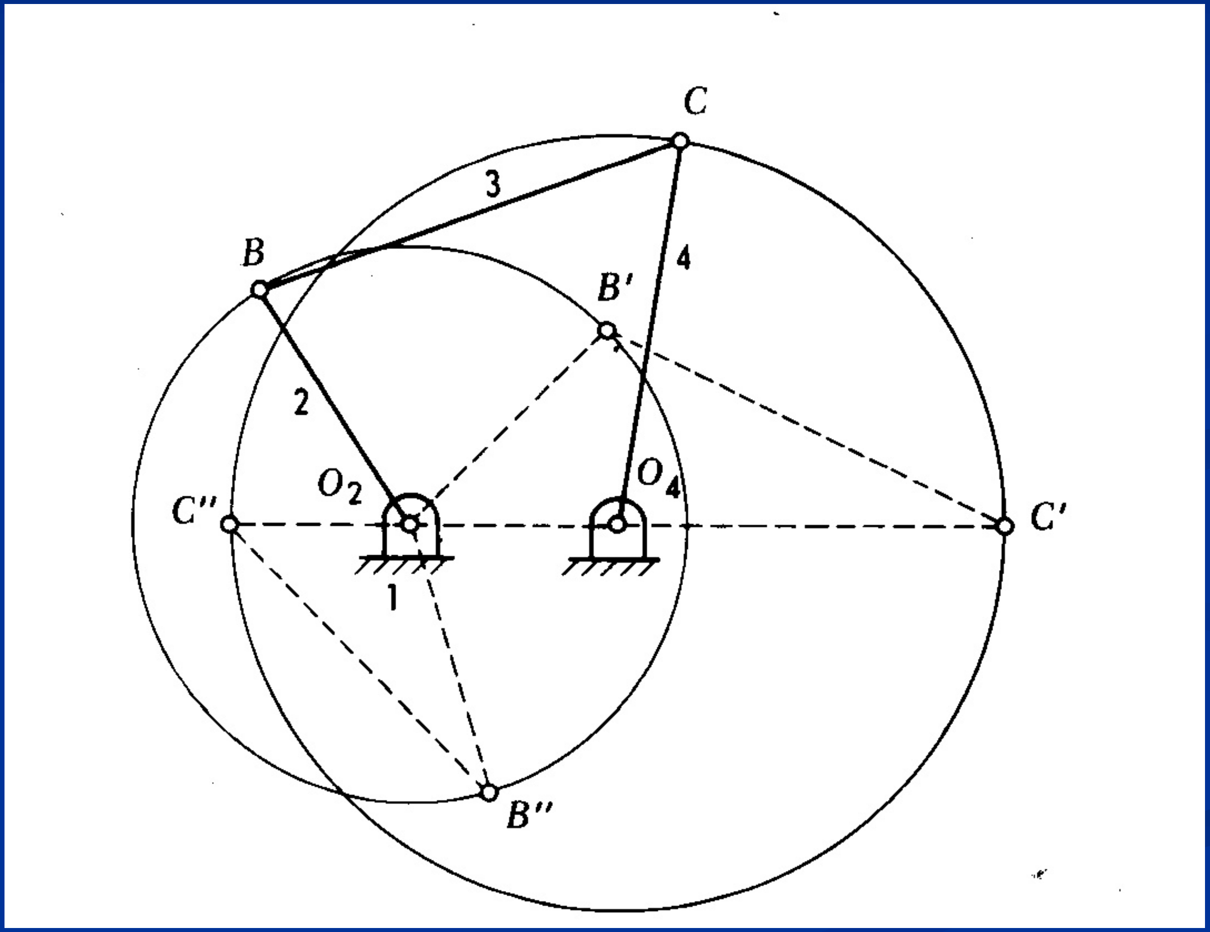
APPLICATION
link-1 fixed-
CRANK-ROCKER MECHANISM
OSCILLATORY MOTION



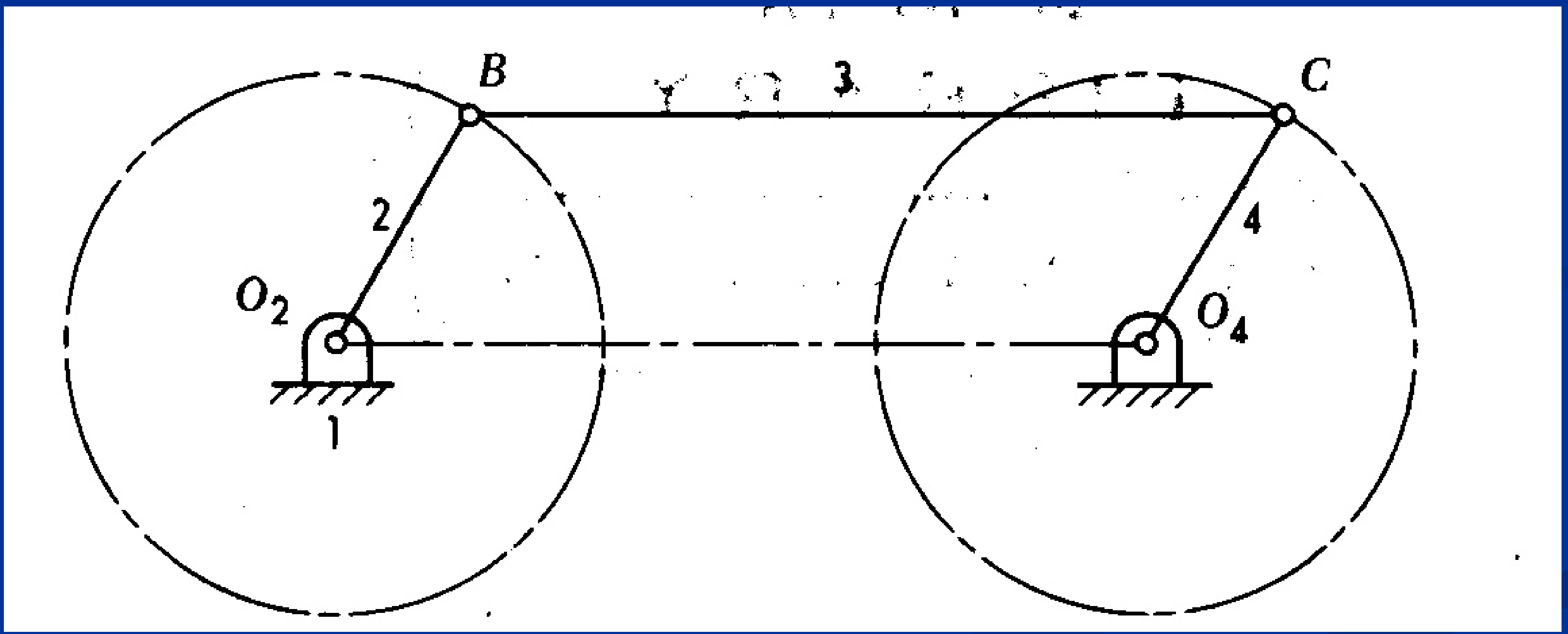
CRANK-ROCKER MECHANISM



Link 2 Fixed- DRAG LINK MECHANISM



Locomotive Wheel - DOUBLE CRANK MECHANISM



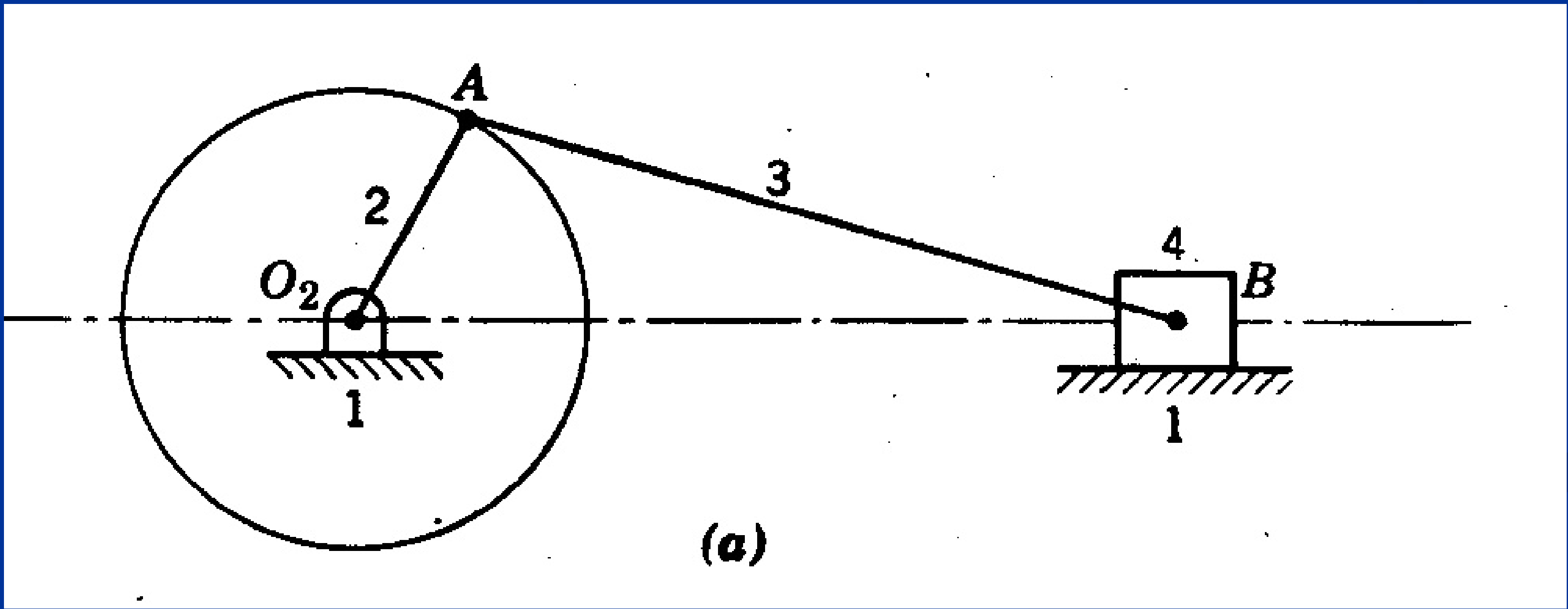
2.SLIDER CRANK CHAIN

Link1=Ground

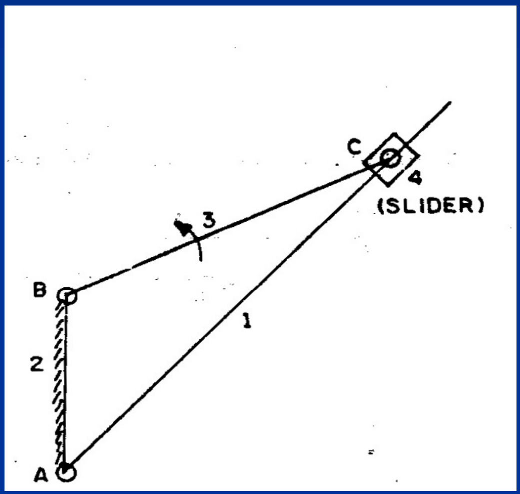
Link2=Crank

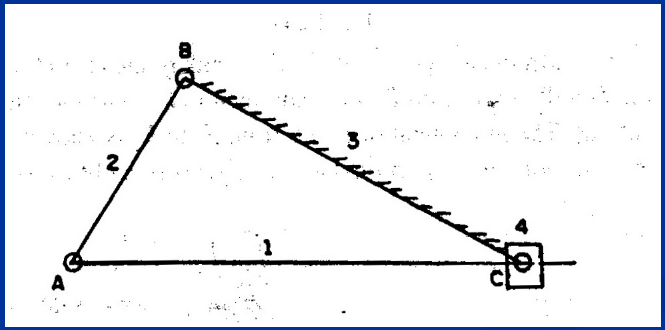
Link3=ConnectingRod

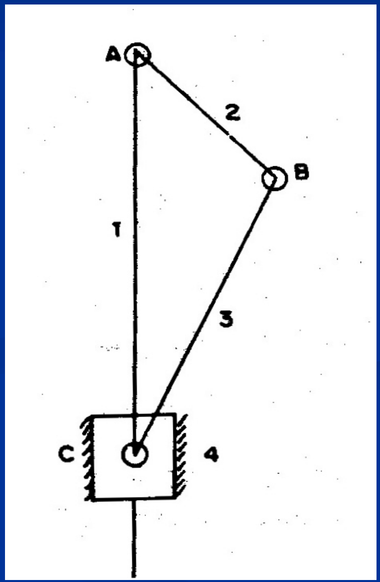
Link4=Slider



Inversions of slider crank chain





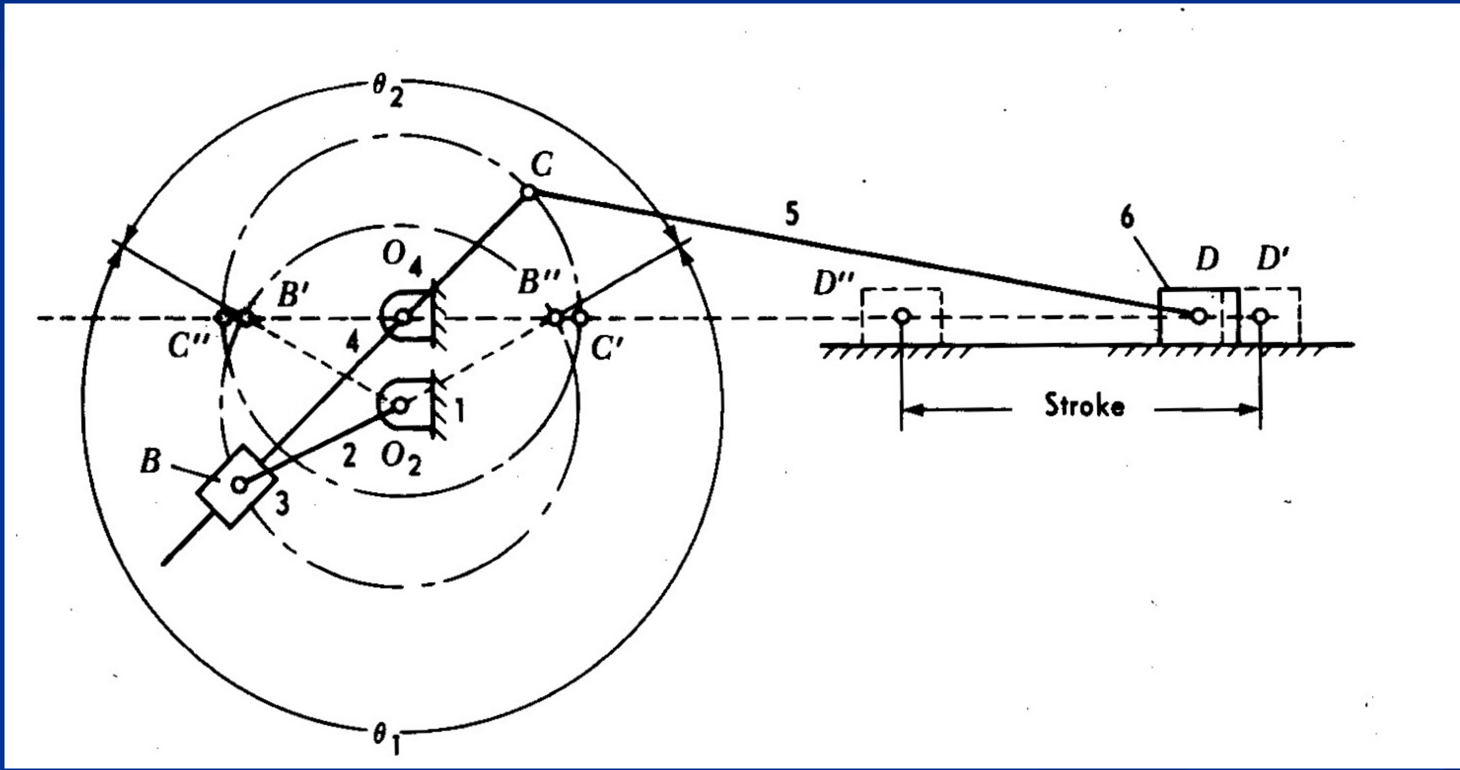


(a) crank fixed
Link 2 fixed

(b) connecting rod fixed
Link 3 fixed

(c) slider fixed
Link 4 fixed

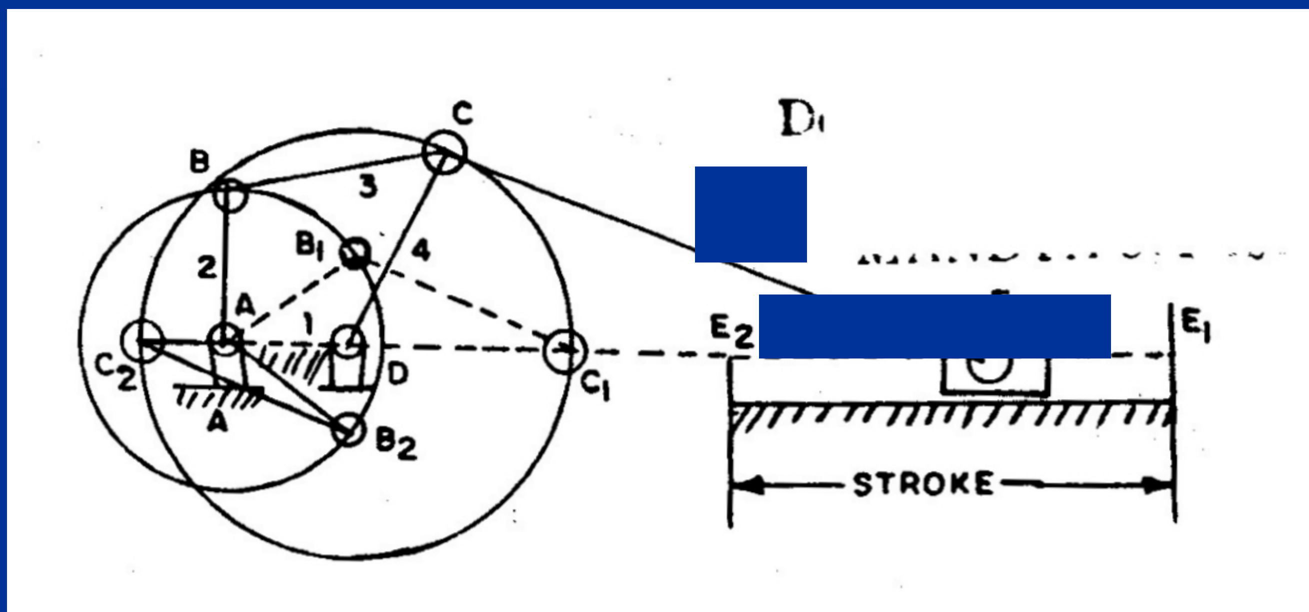
Application
Inversion II – Link 2 Crank fixed
Whitworth quick return motion mechanism



$$\frac{Timeforforwardstroke}{Timeforreturnstroke} = \frac{B'\hat{o}_2 B''}{B''\hat{o}_2 B'} = \frac{\theta_1}{\theta_2}$$

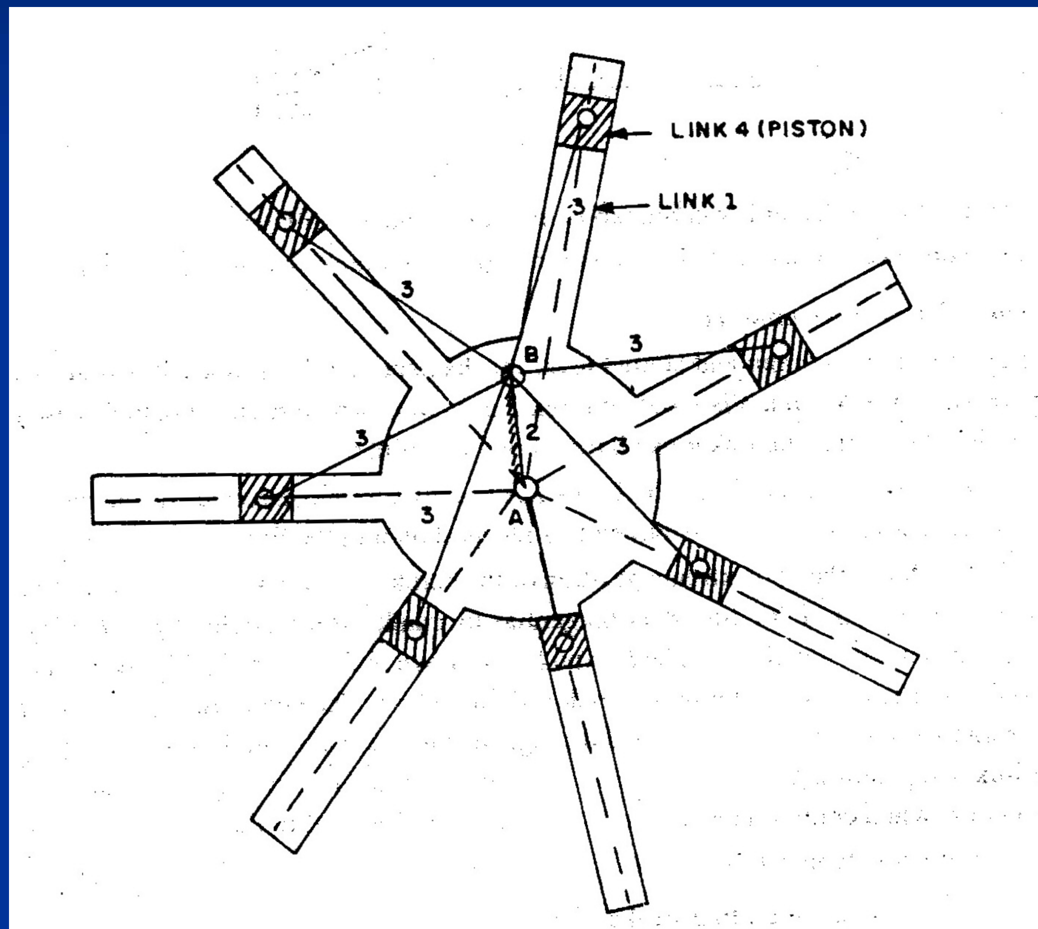
Quick return motion mechanisms

Drag link mechanism

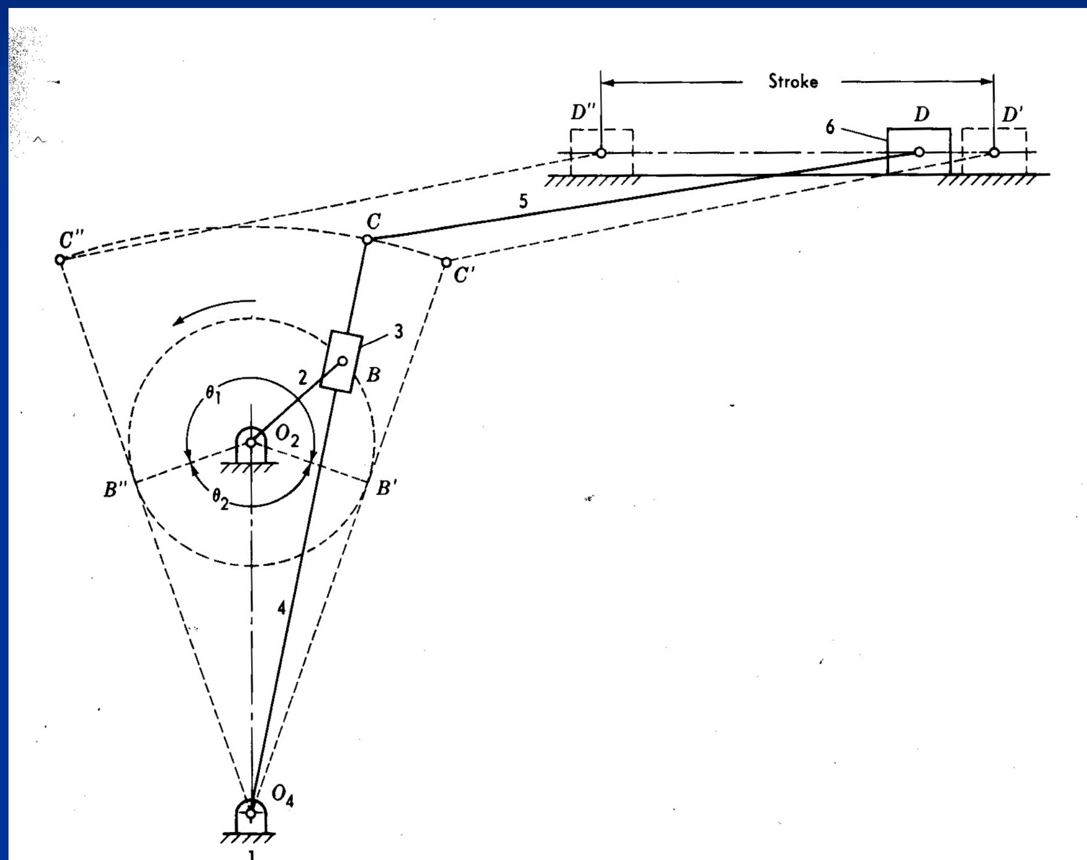


$$\frac{Timeforforwardstroke}{Timeforreturnstroke} = \frac{B_1\hat{A}B_2}{B_2\hat{A}B_1}$$

Rotary engine– II inversion of slider crank mechanism. (crank fixed)

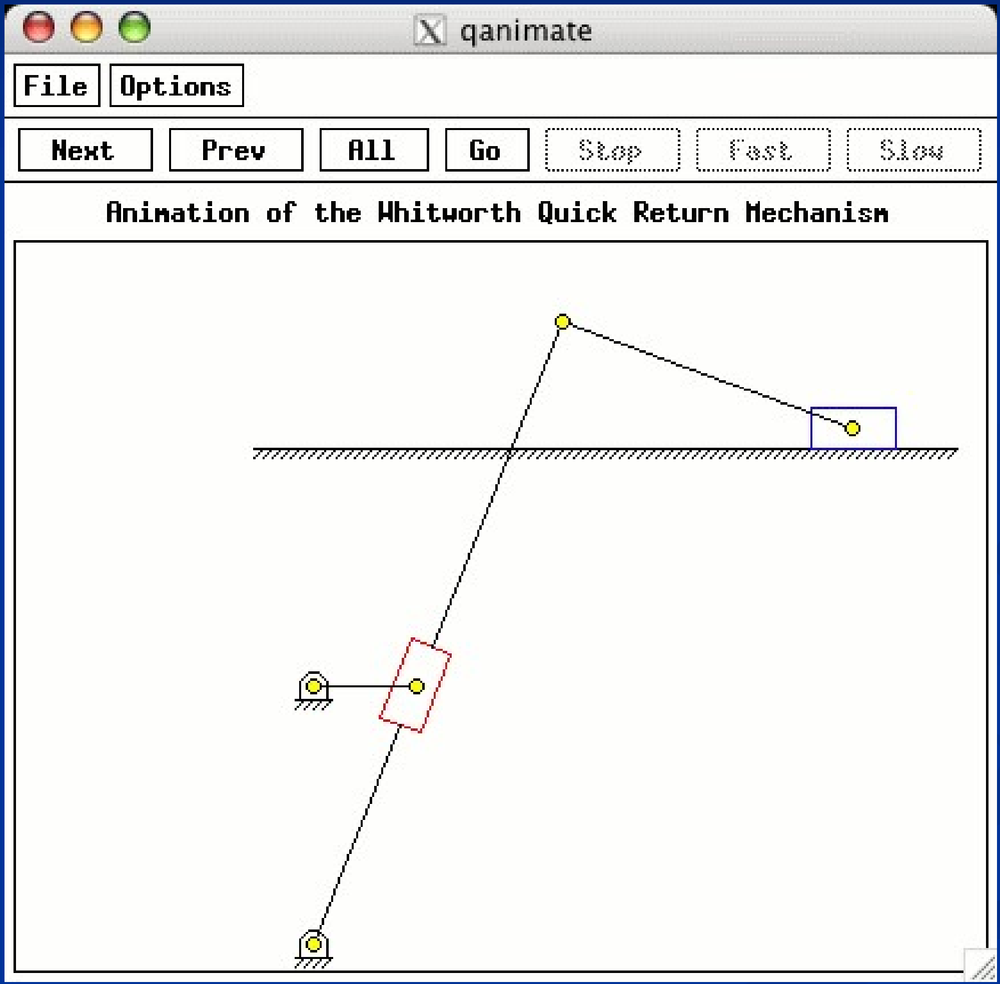


Application
Inversion III -Link 3 Connecting rod fixed
Crank and slotted lever quick return mechanism



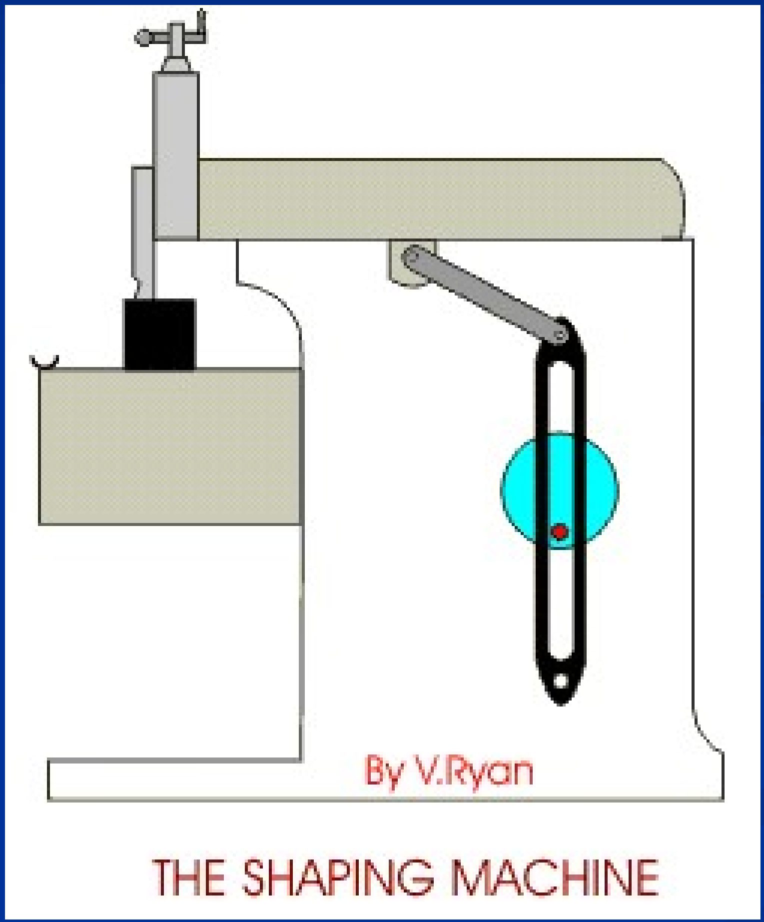
$$\frac{Time for forward stroke}{Time for return stroke} = \frac{B'\hat{o}_2 B''}{B''\hat{o}_2 B'} = \frac{\theta_1}{\theta_2}$$

Crank and slotted lever quick return motion mechanism

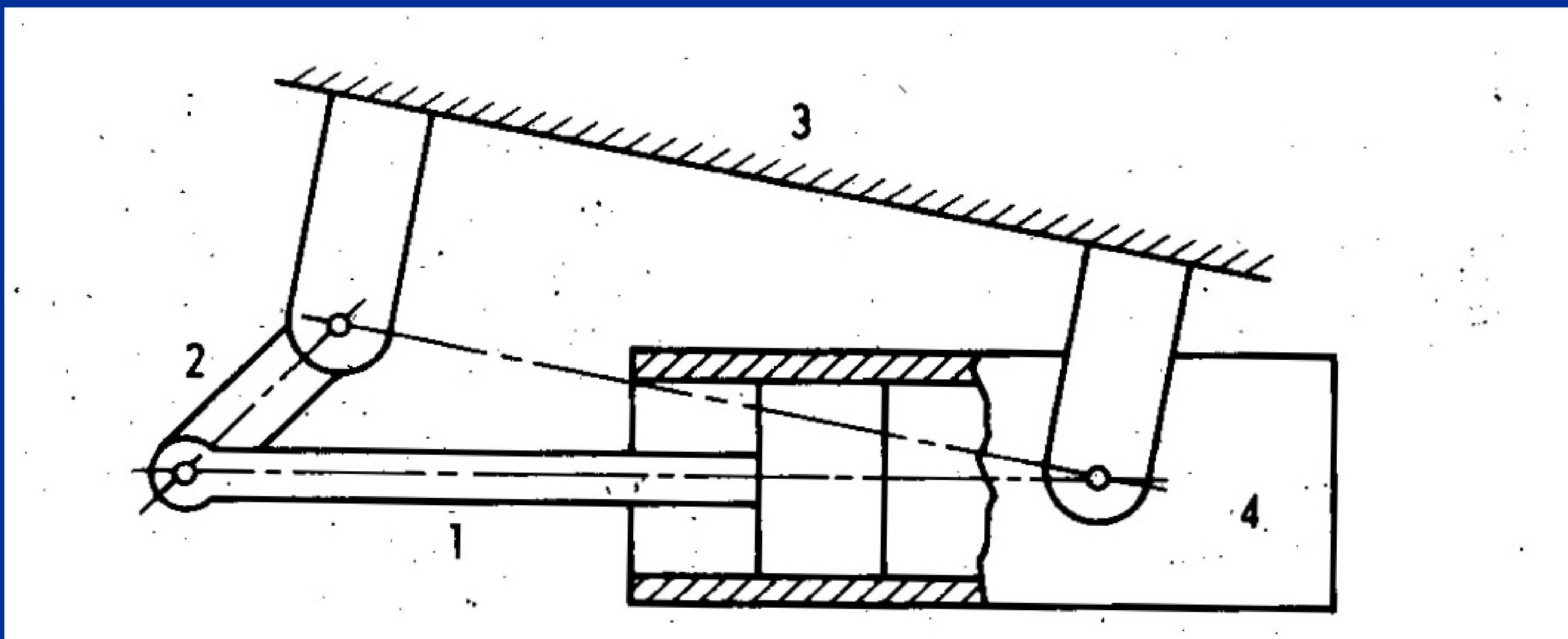


Crank and slotted lever quick return motion mechanism

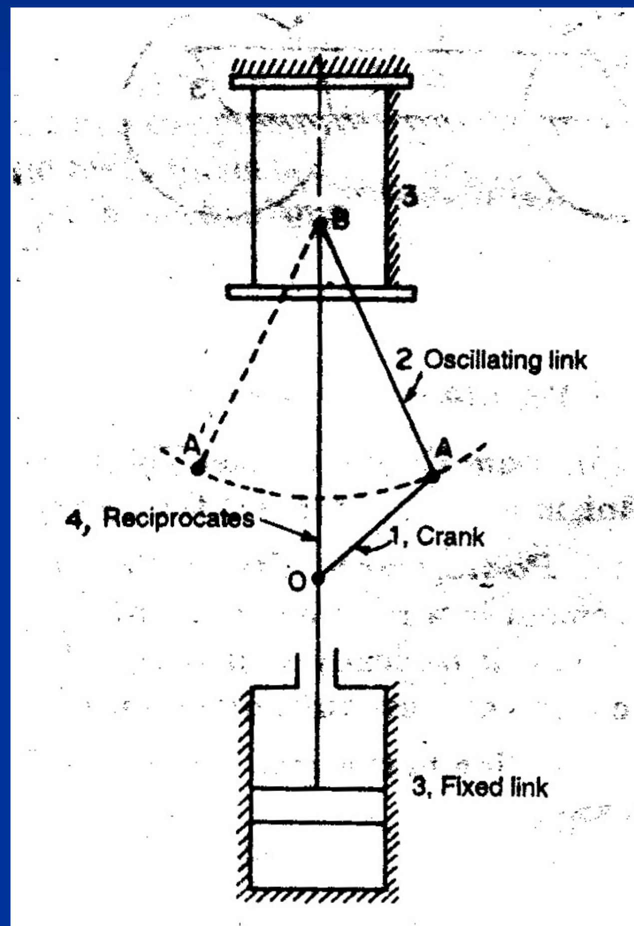
Application of Crank and slotted lever quick return motion mechanism



Oscillating cylinder engine–III inversion of slider crank mechanism (connecting rod fixed)



Application
Inversion IV – Link 4 Slider fixed
Pendulum pump or bull engine



3. DOUBLE SLIDER CRANK CHAIN

It is a kinematic chain consisting of two turning pairs and two sliding pairs.

Link 1 Frame

Link 2 Slider -I

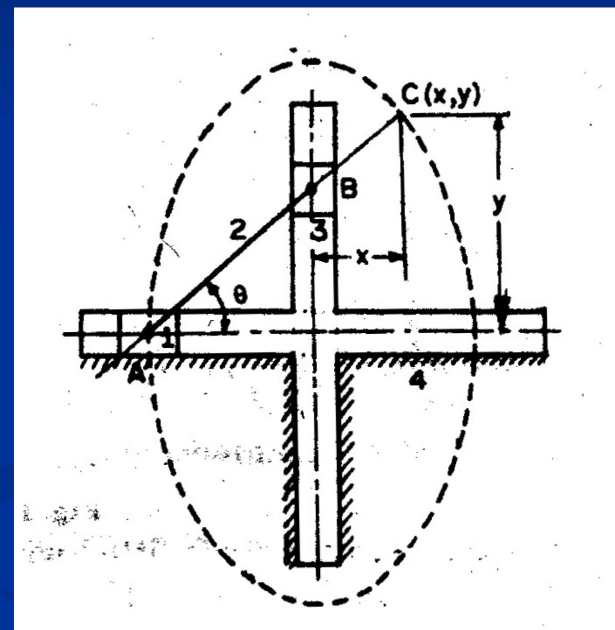
Link 3 Coupler

Link 4 Slider - II

$$\left(\frac{x}{q}\right)^2+\left(\frac{y}{p}\right)^2=\cos^2\theta+\sin^2\theta=1$$

Inversion I – Frame Fixed
Double slider crank mechanism

Elliptical trammel
AC = p and BC = q, then,
x = q.cosθ and y = p.sinθ.
Rearranging,

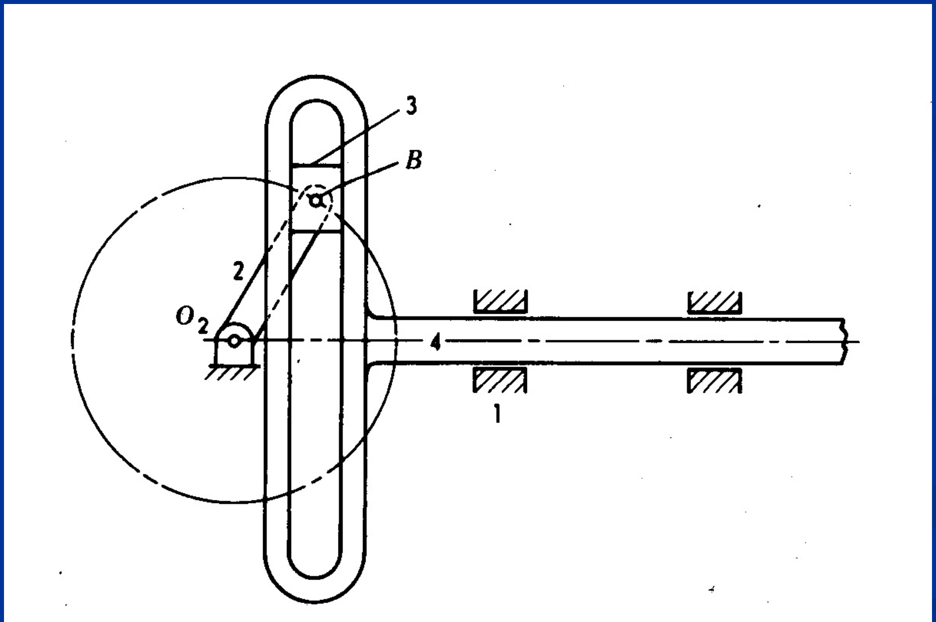


$$\left(\frac{x}{q}\right)^2+\left(\frac{y}{p}\right)^2=\cos^2\theta+\sin^2\theta=1$$

Inversion II – Slider - I Fixed

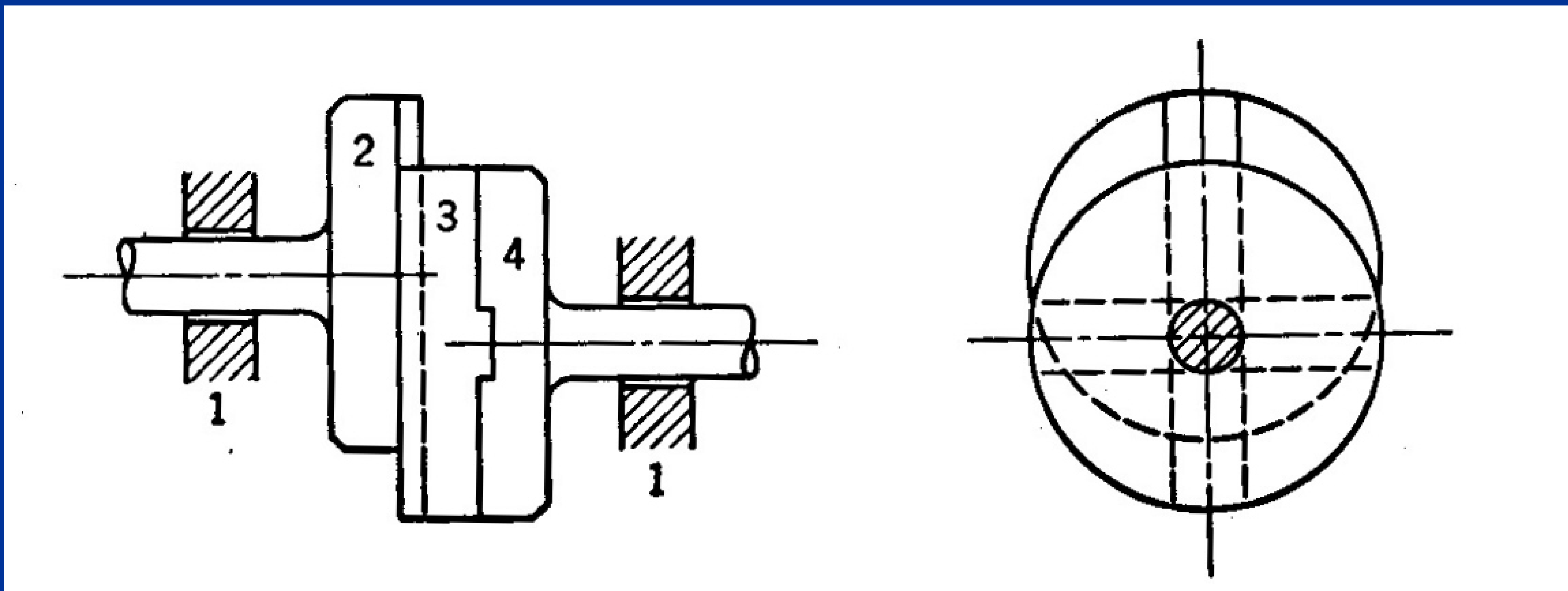
SCOTCH –YOKE MECHANISM

Turning pairs –1&2, 2&3; Sliding pairs – 3&4, 4&1



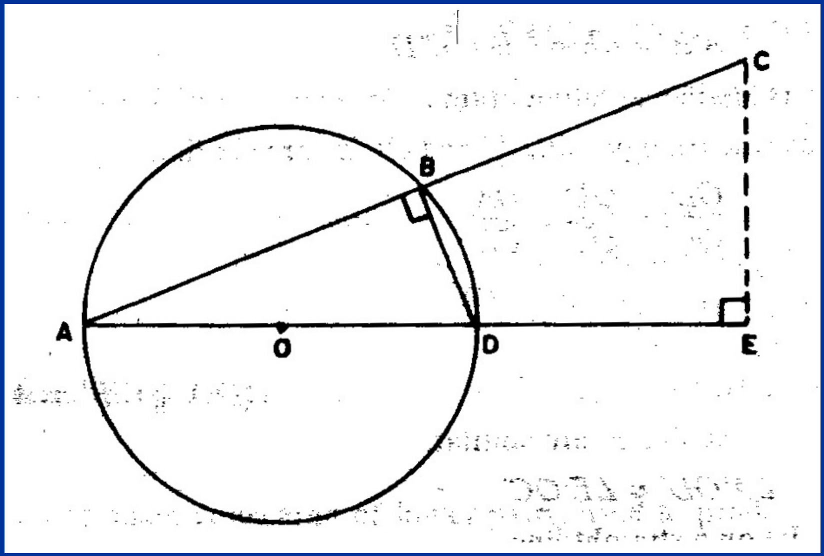
Inversion III – Coupler Fixed

OLDHAM COUPLING



Other Mechanisms
1.Straight line motion mechanisms

Condition for perfect steering



Locus of pt.C will be a straight line, $\perp$ to AE if, $AB \times AC$ is constant.

Proof:

$$\Delta AEC \equiv \Delta ABD$$

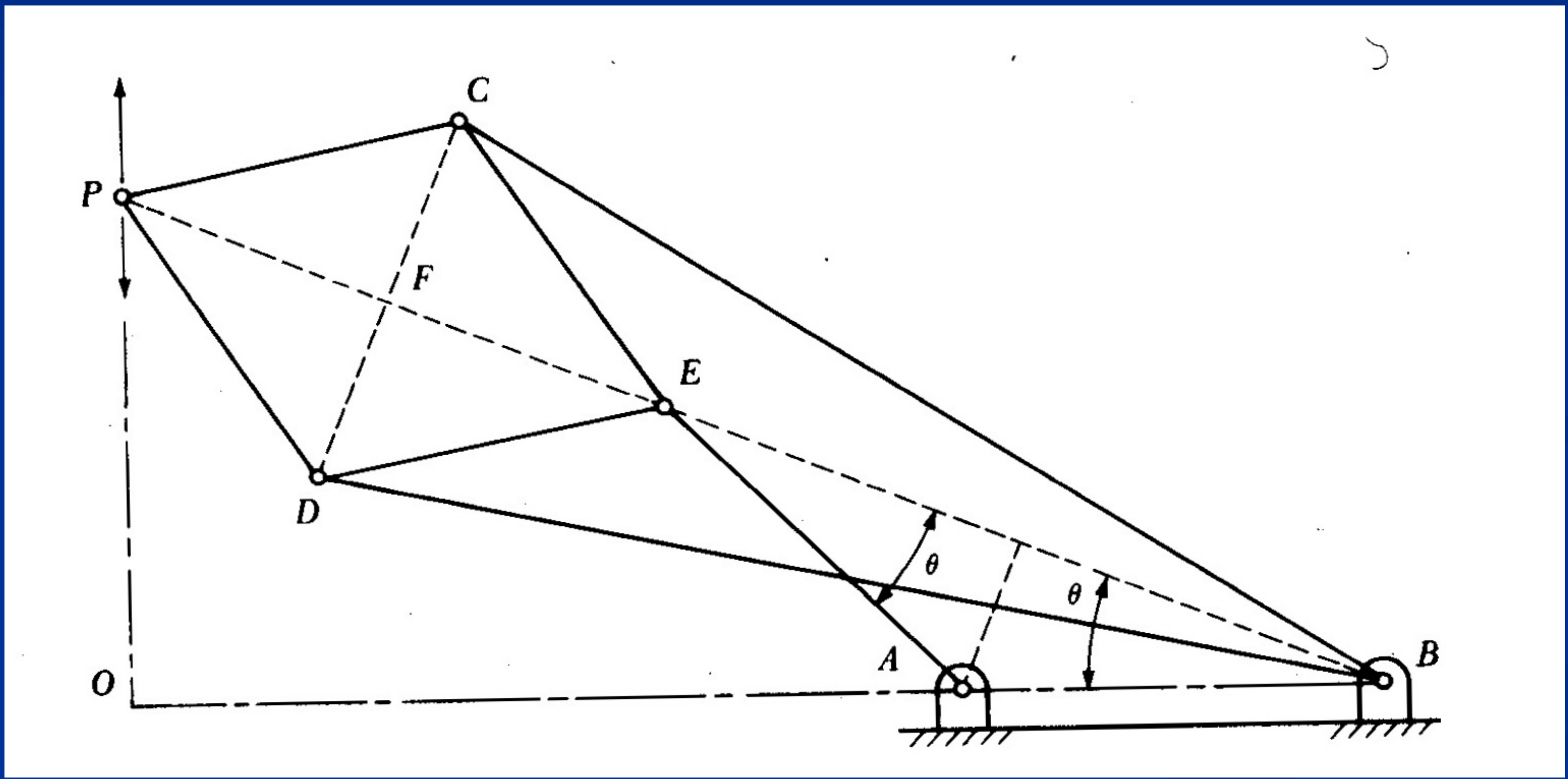
$$\therefore \frac{AD}{AC} = \frac{AB}{AE}$$

$$\therefore AE = \frac{AB \times AC}{AD}$$

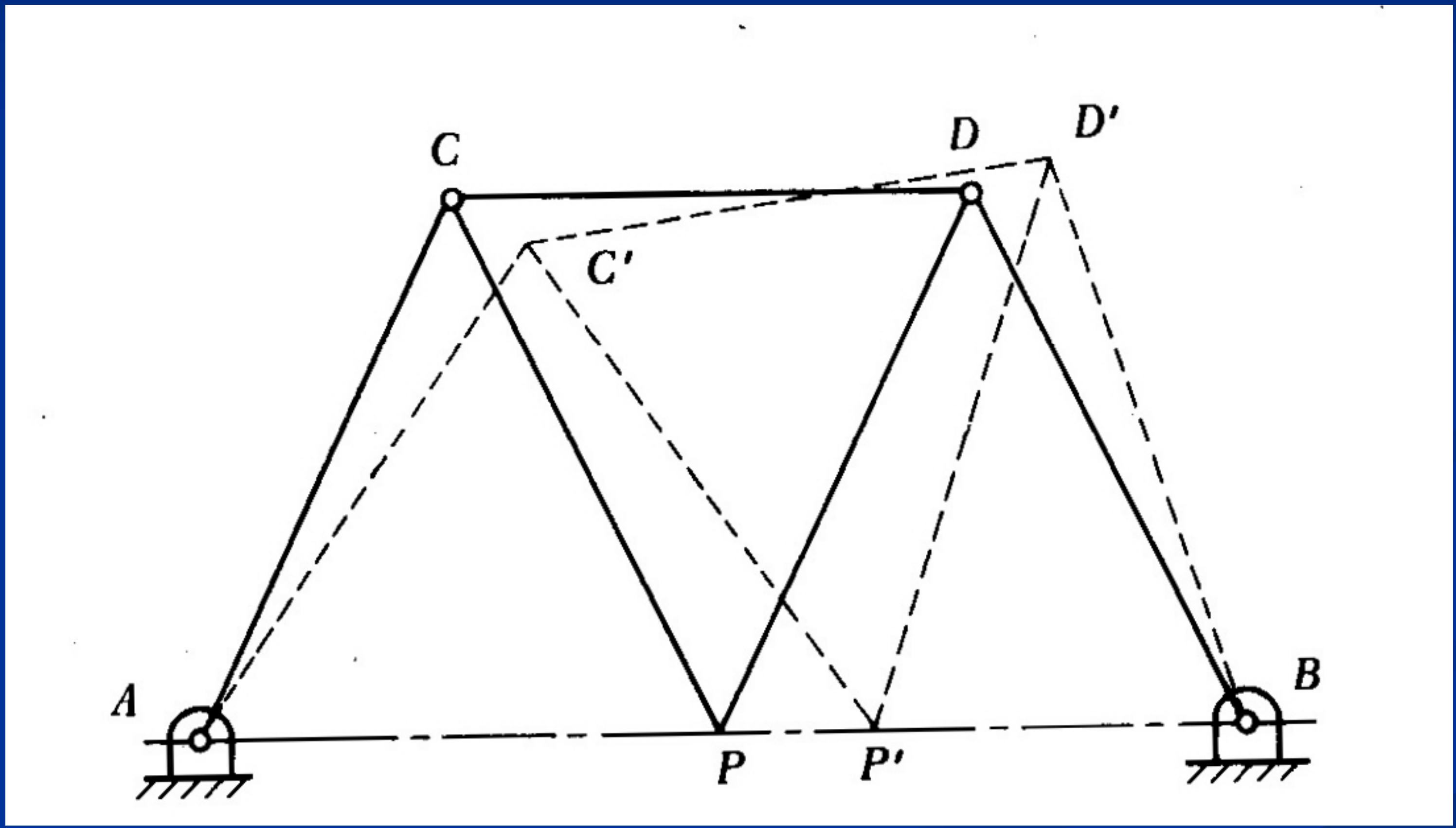
$$but AD = const.$$

$$\therefore AE = const., if AB \times AC = const.$$

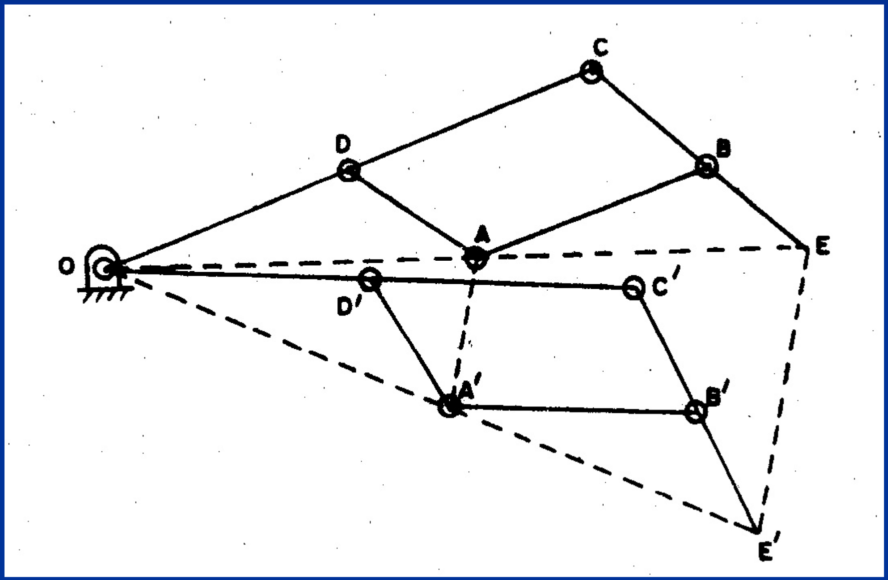
1.a) Peaucellier mechanism

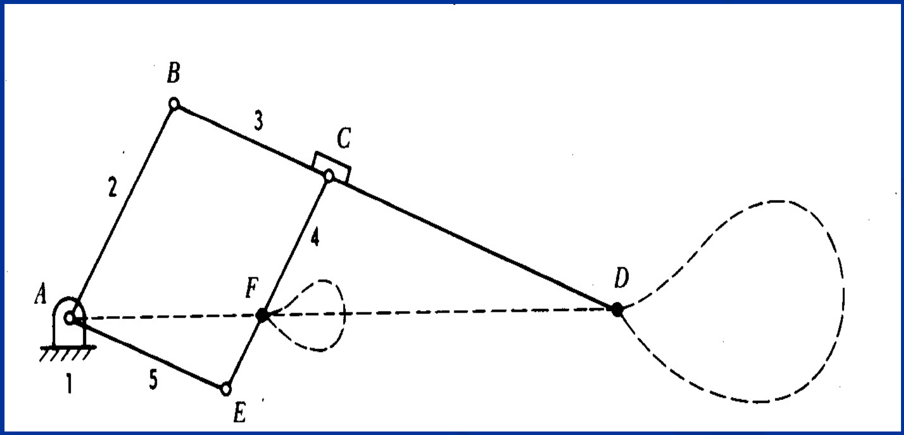


1.b) Robert's mechanism



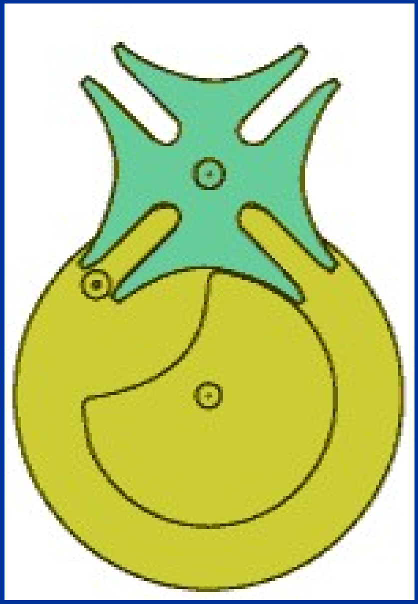
1.c) Pantograph

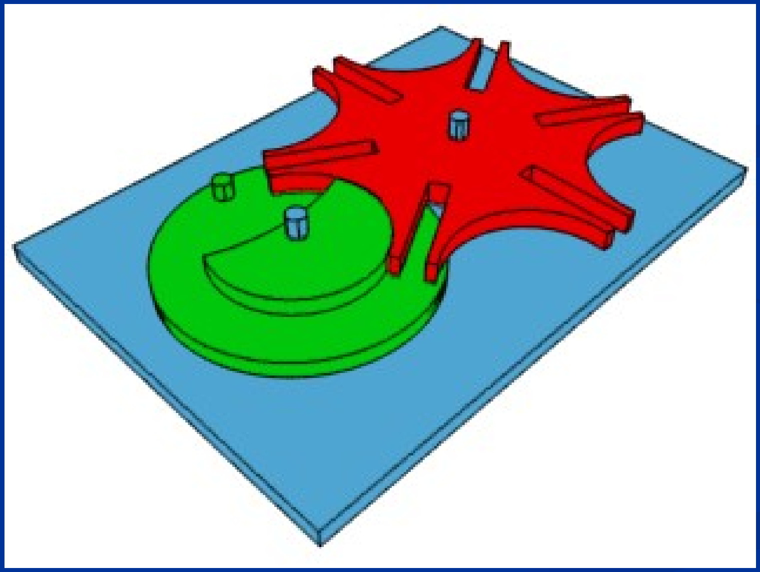


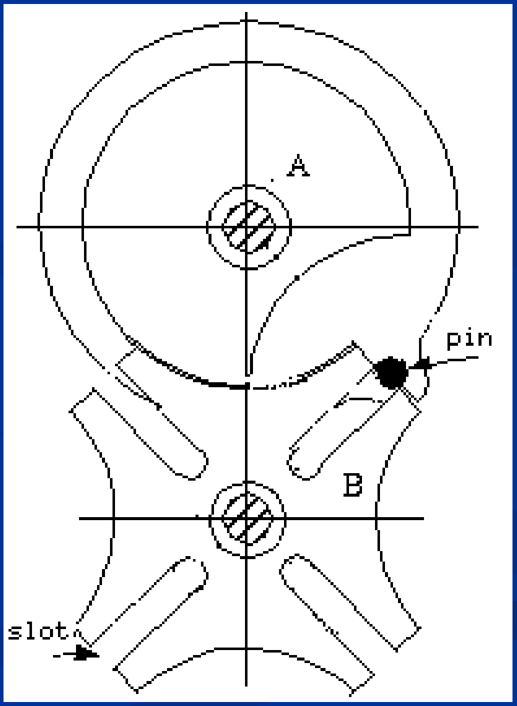


2.Indexing Mechanism

Geneva wheel mechanism

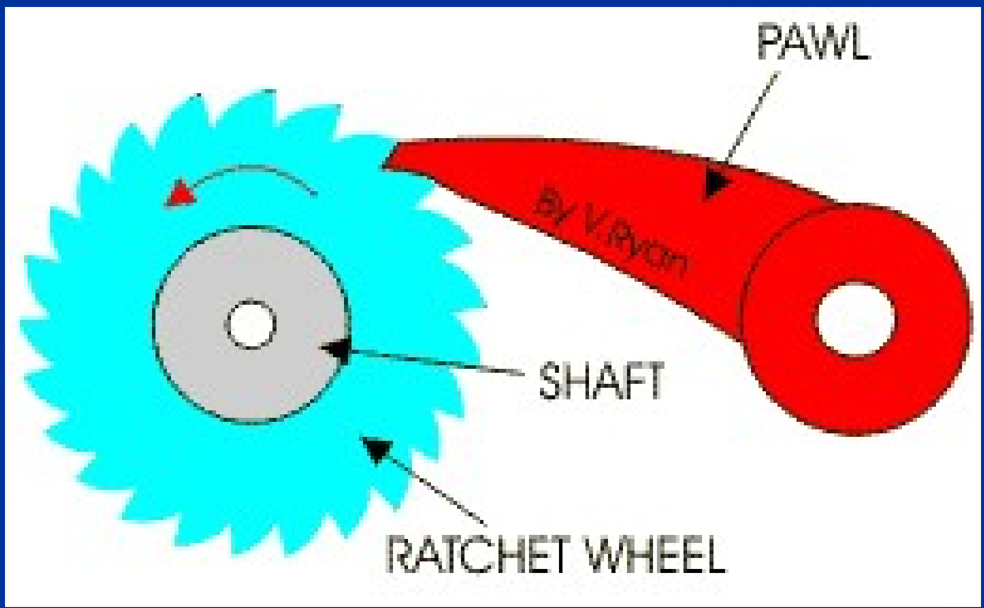


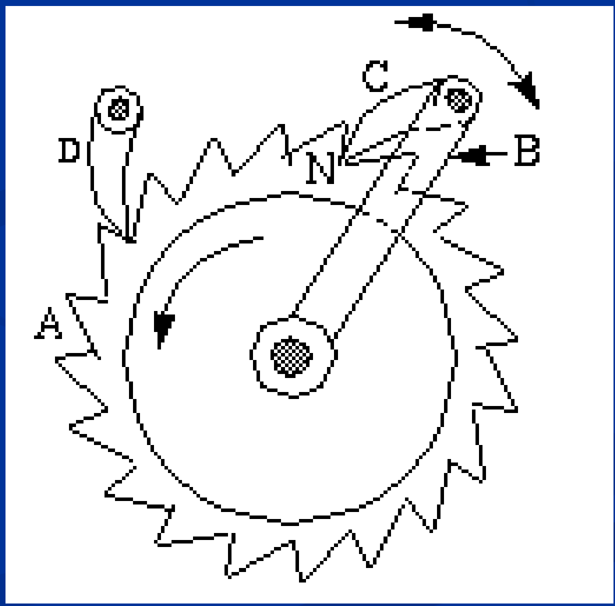




3.Ratchets and Escapements

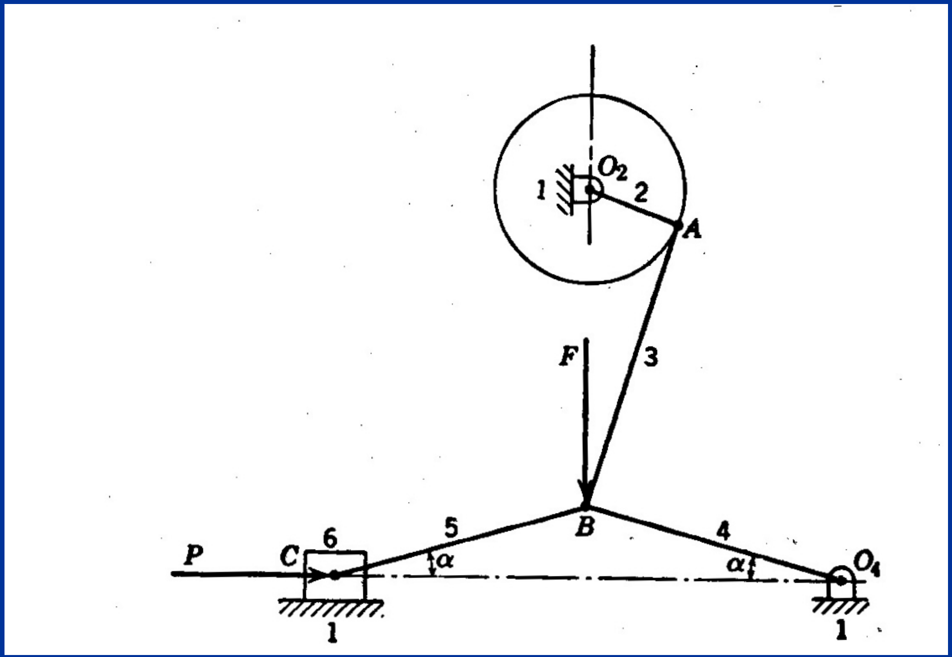
Ratchet and pawl mechanism





Application of Ratchet Pawl mechanism

4. Toggle mechanism



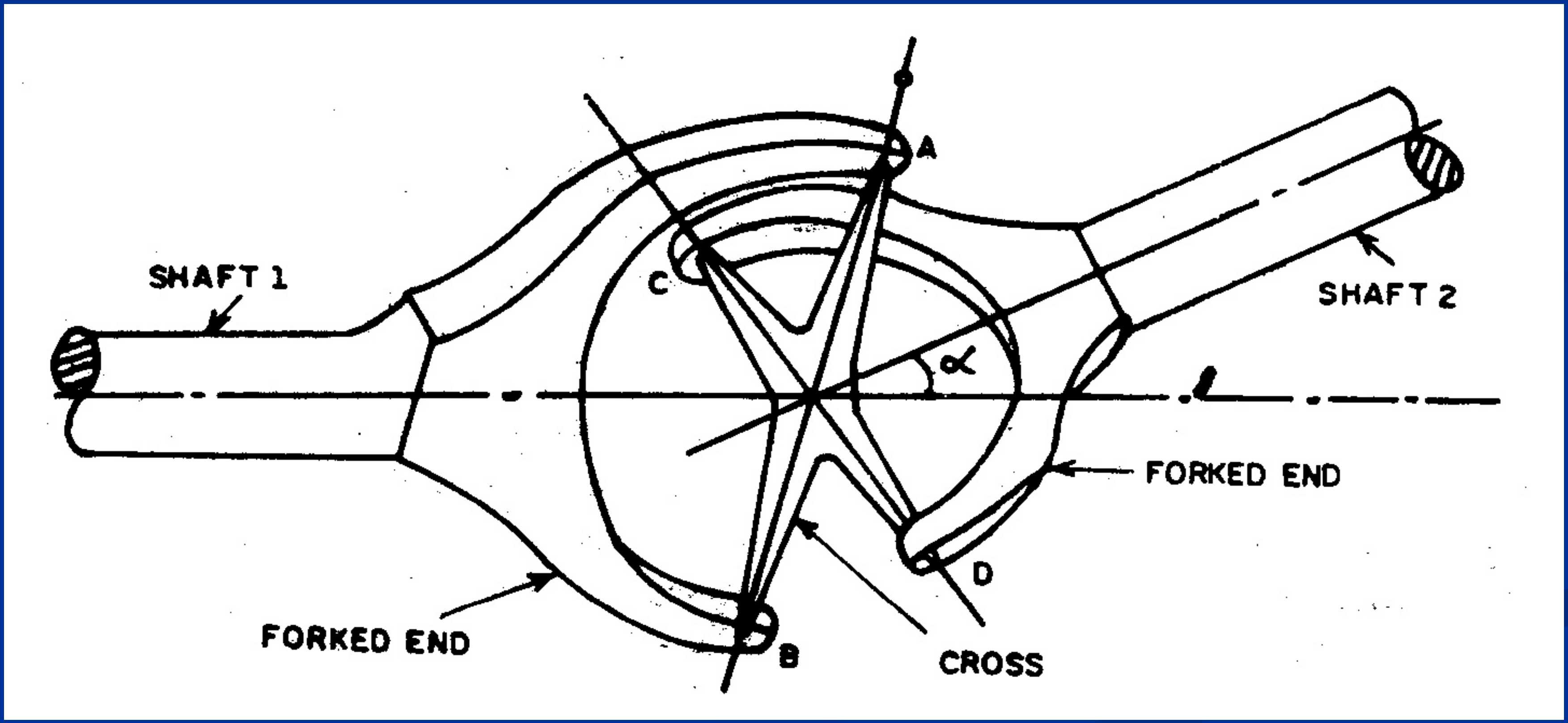
Considering the equilibrium condition of slider 6,

$$\tan\alpha = \frac{F/2}{P}$$

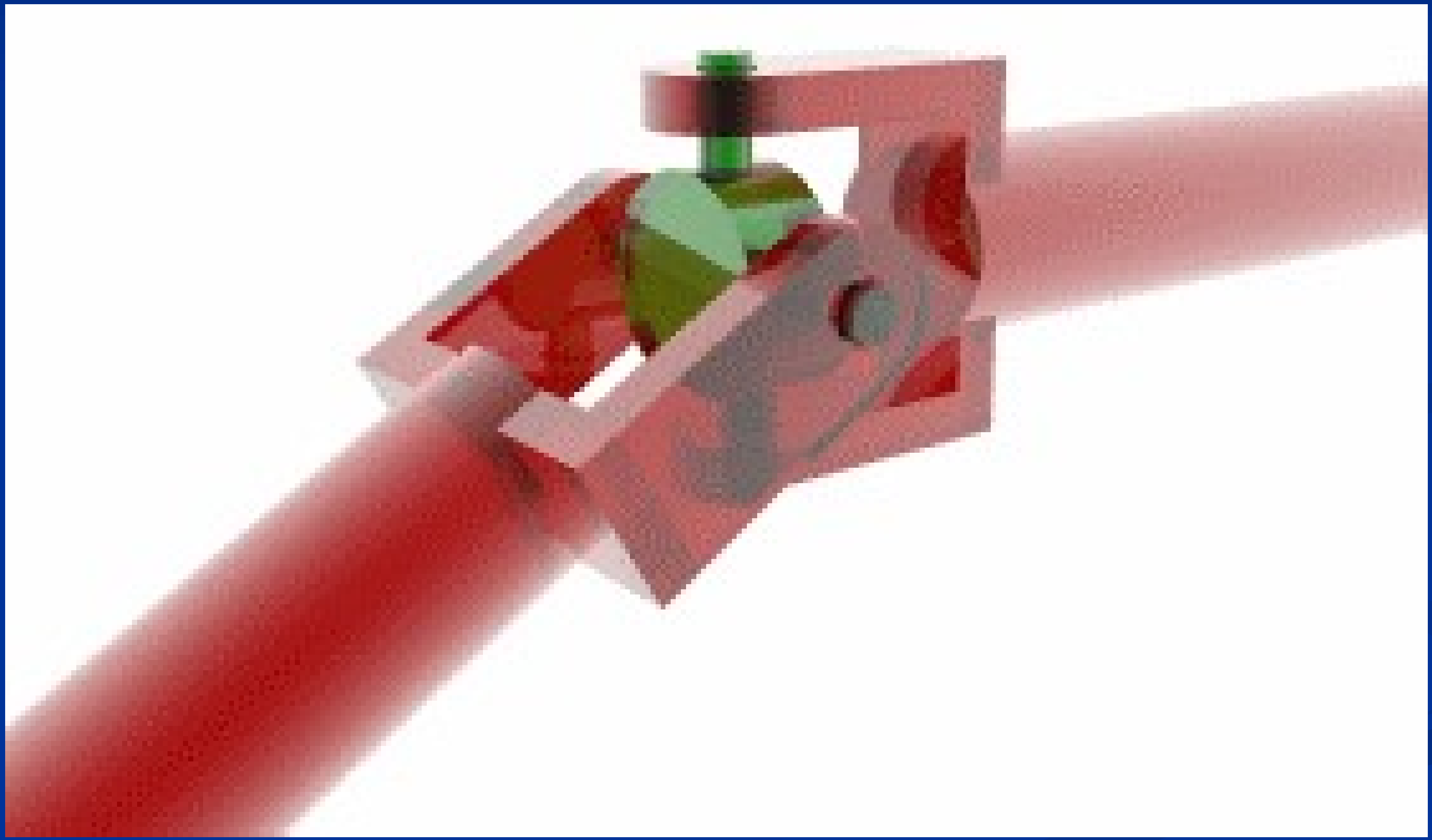
$$\therefore F = 2P\tan\alpha$$

For small angles of α, F is much smaller than P.

5.Hooke's joint

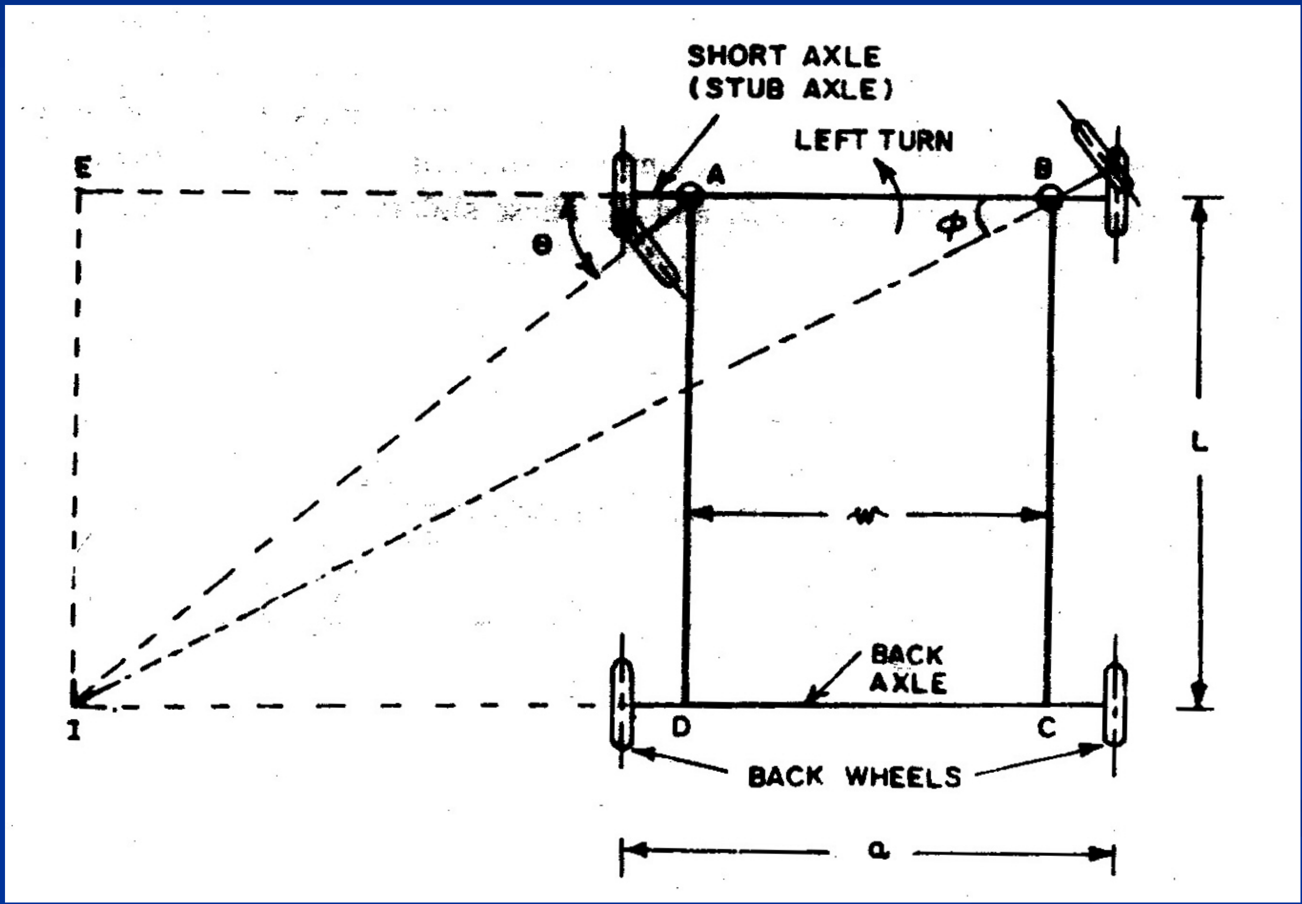


Hooke's joint

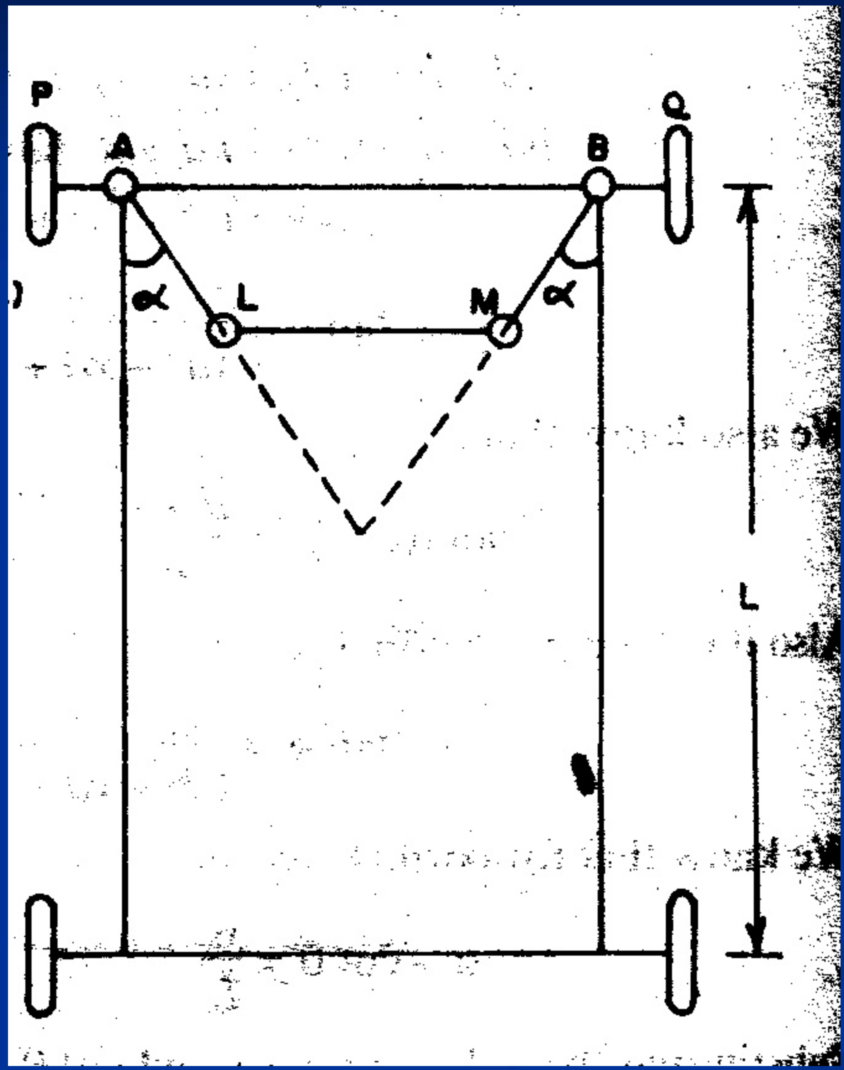


6.Steering gear mechanism

Condition for perfect steering



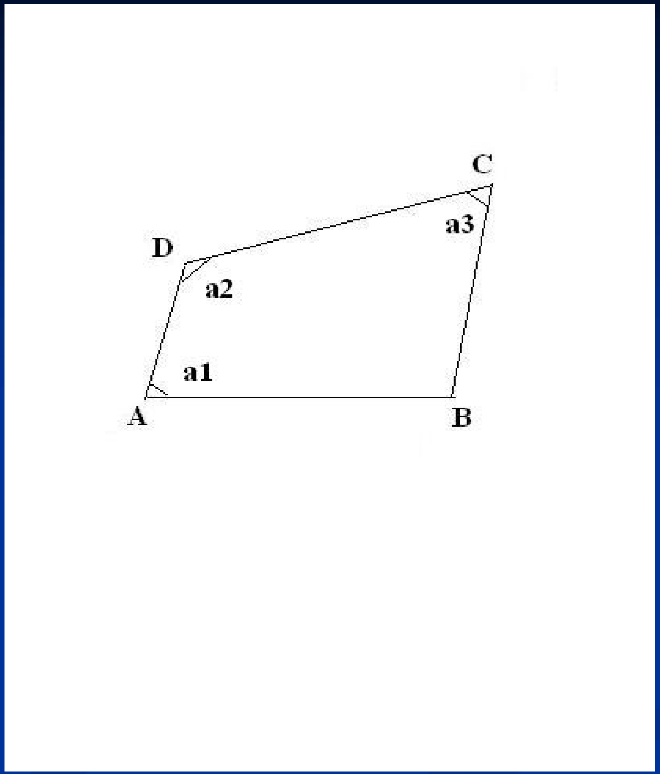
Ackermann steering gear mechanism

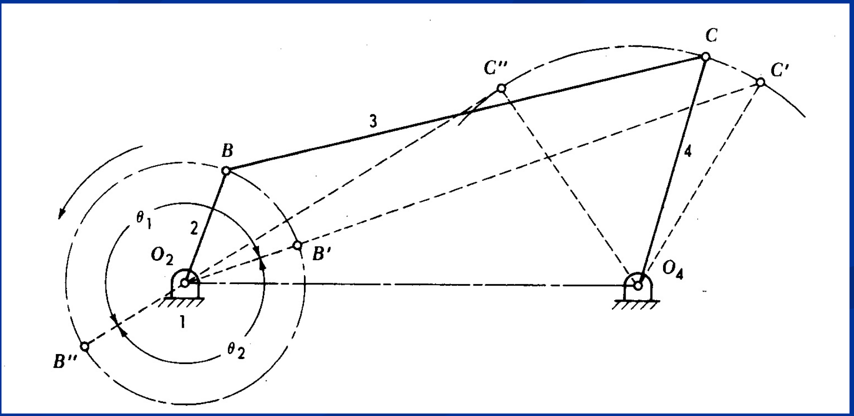


Mechanical Advantage

- Mechanical Advantage of the Mechanism at angle a2 = 0^0 or 180^0

- Extreme position of the linkage is known as toggle positions.





ALL THE BEST