Lecture - 15 ·

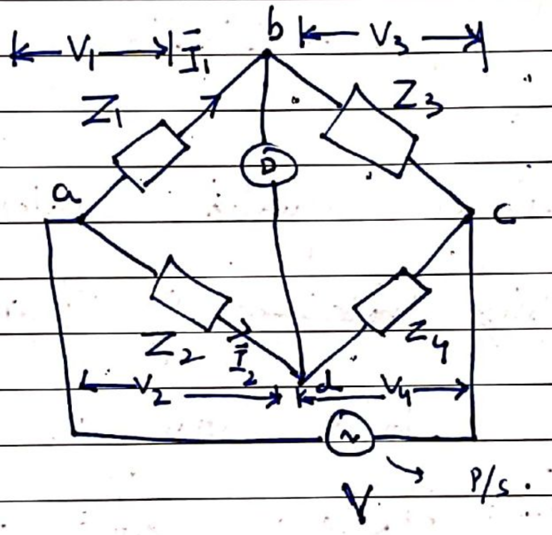
Topic   Date

A.c. Bridges

The most usual method used for precise measurement of self and mutual inductance and capacitance is the method of employing a bridge network.

An a.c. bridge comprises of 4 arms comprising of R, L, & C or their combination (series, parallel) an a.c. supply and a Detector. The Bridge circuits are all, in general the modifications of the original wheatstones Bridge.

Let us consider a generalised Bridge n/w



P/s. →
Detector → G, Head phones, etc;

for Detector to show zero deflection, the P.D across b & d should be same. i.e. both magnitude and phase of voltage at b as well as at d must be same.

General Balance equation :-

under balanced conditions, no current flows through the Detector. This is possible when the voltage drop from a to b and a to d in both magnitude and phase is same. i.e,

Topic Date

$$\bar{I}_1 \bar{Z}_1 = \bar{I}_2 \bar{Z}_2$$

now $\bar{I}_1 = \dfrac{\bar{V}}{\bar{Z}_1+\bar{Z}_3}$ & $\bar{I}_2 = \dfrac{V}{\bar{Z}_2+\bar{Z}_4}$

$$\therefore \frac{Z_1 \cdot V}{Z_1+Z_3} = \frac{V}{Z_2+Z_4} \cdot Z_2$$

or $Z_1(Z_2+Z_4) = Z_2(Z_1+Z_3)$

$$\cancel{Z_1 Z_2} + Z_1 Z_4 = \cancel{Z_1 Z_2} + Z_2 Z_3$$

$$\boxed{Z_1 Z_4 = Z_2 Z_3}$$

The balance equation states that the product of complex impedances of opposite arms is equal.

In polar form, $\bar{Z}_1 = Z_1 \angle \theta_1$, $\bar{Z}_2 = Z_2 \angle \theta_2$

$\bar{Z}_3 = Z_3 \angle \theta_3$, $\bar{Z}_4 = Z_4 \angle \theta_4$

$$Z_1 \angle \theta_1 \cdot Z_4 \angle \theta_4 = Z_2 \angle \theta_2 \cdot Z_3 \angle \theta_3$$

$$Z_1 Z_4 \angle \theta_1+\theta_4 = Z_2 Z_3 \angle \theta_2+\theta_3;$$

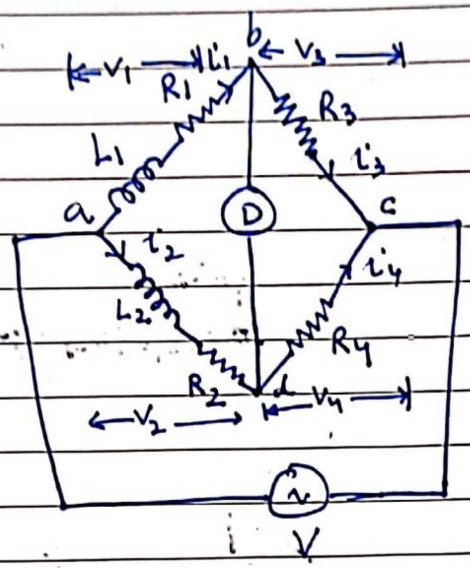
Thus the product of magnitudes of ~~opposite~~ impedance of opposite arms is equal. Also the sum of phases of opposite arms is equal. we have to meet both the conditions.

Citizen

Topic Date

Measurement of Self Inductance

① Maxwells Bridge for measurement of self Inductance



In this bridge an unknown inductance is measured by comparing it with a known self inductance.

L_1 = unknown self Inductance of resistor, R_1
L_2 = Known " " " " R_2
R_3, R_4 = non inductive resistors;
D = Detector

The bridge is balanced by varying L_2 and one of the resistors (R_3, R_4).

under balanced condition;

$$(R_1 + j\omega L_1) R_4 = (R_2 + j\omega L_2) R_3$$

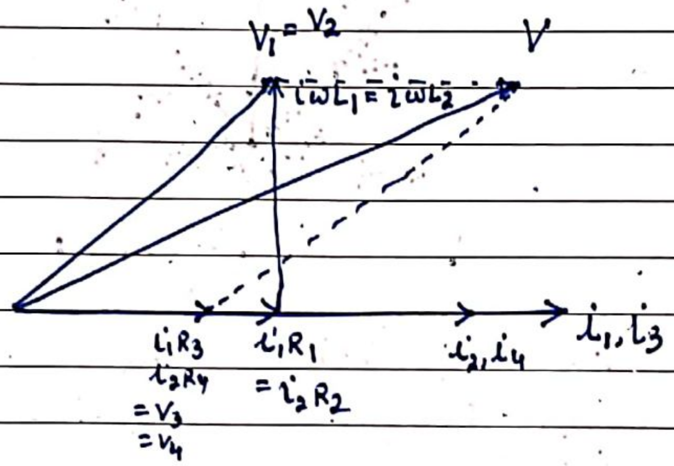
$$R_1 R_4 + j\omega L_1 R_4 = R_2 R_3 + j\omega L_2 R_3$$

equating real and imaginary parts ~~on~~

Topic ______ Date ______

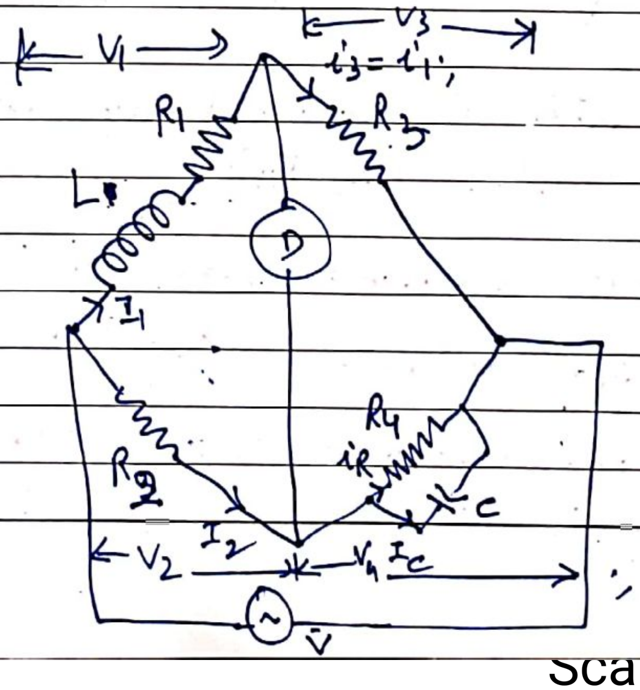
$$R_1 R_4 = R_2 R_3 \quad ; \quad \omega L_1 R_4 = \omega L_2 R_3$$

$$\boxed{\frac{L_1}{L_2} = \frac{R_3}{R_4} = \frac{R_1}{R_2}}$$



② Maxwells Inductance - Capacitance Bridge

In this bridge, we measure self inductance by comparison with a capacitance (standard/variable).



LA5

Topic Date

$L =$ Self inductance to be measured

$R_1 =$ Effective resistance of Inductor

$R_2, R_3, R_4 =$ known non-inductive resistances

$C =$ Standard variable Capacitor ;

Now at Balance :-

$$(R_1 + j\omega L)\left(\frac{R_4}{1+j\omega C R_4}\right) = R_2 R_3$$

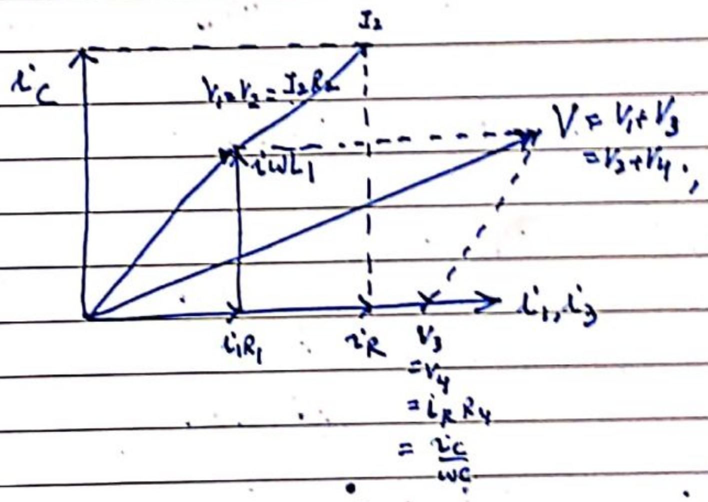
$$\therefore R_1 R_4 + j\omega L R_4 = R_2 R_3 + j\omega C R_2 R_3 R_4$$

Equating real and imaginary parts,

$R_1 R_4 = R_2 R_3$ ie. $\cancel{\omega} L \cancel{R_4} = \cancel{\omega} C R_2 R_3 \cancel{R_4}$

$$\boxed{R_1 = \frac{R_2 R_3}{R_4}} \qquad \boxed{L = C R_2 R_3}$$

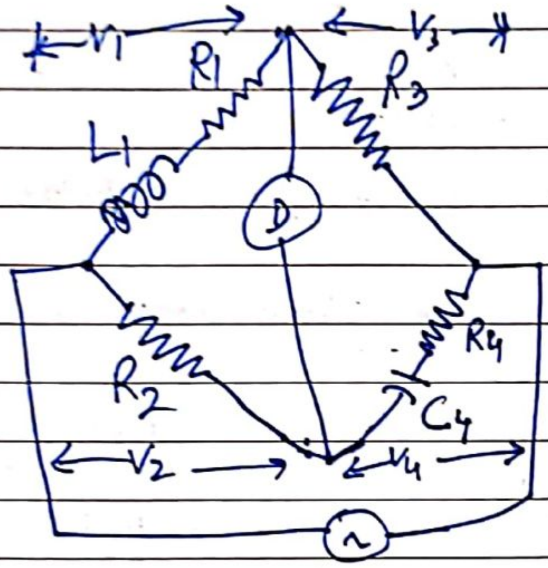
Phasor Diagram under balanced conditions :

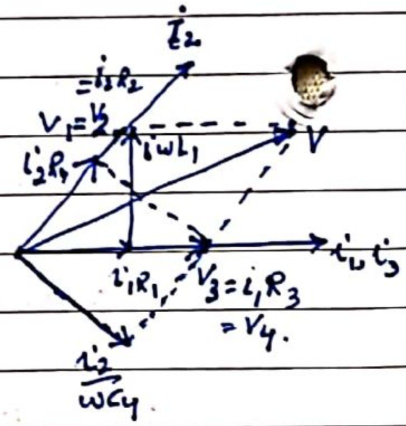


Citizen

Topic Date

(3) Hays Bridge A modification of Maxwells Capacitance bridge where in a series combination of Resistance and Capacitance is used.





At balance; $(R_1 + j\omega L_1)\left(R_4 - \frac{j}{\omega C_4}\right) = R_2 R_3$

$$R_1 R_4 + j\left(\omega L_1 R_4 - \frac{R_1}{\omega C_4}\right) + \frac{L_1}{C_4} = R_2 R_3$$

$$R_1 R_4 + \frac{L_1}{C_4} = R_2 R_3 \quad \text{—— (1)}$$

and $\omega L_1 R_4 = \frac{R_1}{\omega C_4}$

or $L_1 = \frac{R_1}{\omega^2 C_4 R_4}$ —— (2);

unknowns are L_1 & R_1, and we have two equations (1) & (2) to solve L_1 & R_1,

$$R_1 = \frac{\omega^2 R_2 R_3 R_4 C_4^2}{1 + \omega^2 C_4^2 R_4^2}, \qquad L_1 = \frac{R_2 R_3 C_4}{1 + \omega^2 C_4^2 R_4^2},$$

Topic Date

The quality factor of Coil $Q = \frac{\omega L_1}{R_1} = \frac{1}{\omega C_4 R_4}$;

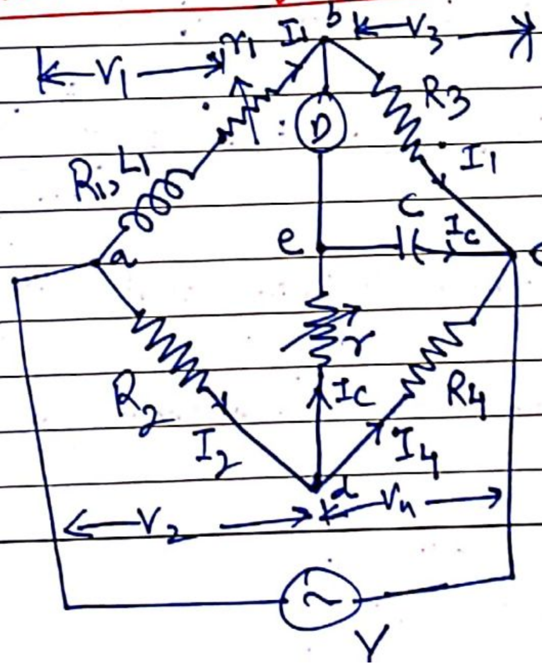
The quality factor, resistance R_1 and inductor L_1, Contain frequency term (ω), Thus frequency of Source must be known.

However, for high Q coils; we have,

$$L_1 = \frac{R_2 R_3 C_4}{1 + \frac{1}{Q^2}}$$, ~~since~~ when Q is large; $\frac{1}{Q^2}$ is very small.

and $\therefore$ $\boxed{L_1 \simeq R_2 R_3 C_4}$ which is same as the result obtained in Maxwells ind-Cap Bridge.

Andersons' Bridge



L_1 = self induct (??)
R_1 = resistance of coil of unknown inductor
r_1 = resistance connected in series with self ind.
r, R_2, R_3, R_4 → known non-inductive resistances.
C = fixed capacitor (Known).

Topic Date

At balance :

$I_1 = I_3$; $I_2 = I_C + I_4$

$I_1 R_3 = \frac{I_C}{j\omega C}$ $\therefore$ $I_C = I_1 j\omega R_3 C$.

Writing the other balance equations :

$V_{ab} = V_{ade}$:

$$I_1(r_1 + R_1 + j\omega L_1) = I_2 R_2 + I_C r \quad \text{--- (1)}$$

$$I_C\left(r + \frac{1}{j\omega C}\right) = (I_2 - I_C) R_4 \quad \text{--- (2)}$$

$V_{dec} = V_{dc}$.

Substitute the value of I_C from above in equations (1) & (2) ; we get .

$$I_1(r_1 + R_1 + j\omega L_1) = I_2 R_2 + I_1 j\omega C r R_3$$

$$\text{or} \quad I_1(r_1 + R_1 + j\omega L_1 - j\omega C R_3 r) = I_2 R_2 \quad \text{--- (3)}$$

$$I_1 j\omega C R_3\left(r + \frac{1}{j\omega C}\right) = (I_2 - I_1 j\omega C R_3) R_4$$

$$I_1\left[j\omega C r R_3 + R_3 + j\omega C R_3 R_4\right] = I_2 R_4 \quad \text{--- (4)}$$

Obtain value of I_2 from (4) & substitute in (3) ,

$$I_1(r_1 + R_1 + j\omega L_1 - j\omega C R_3 r) = \frac{R_2 I_1(j\omega C r R_3 + R_3 + j\omega C R_3 R_4)}{R_4}$$

$$\cancel{I_1}(r_1 + R_1 + j\omega L_1 - j\omega C r R_3) = \cancel{I_1}\left(\frac{R_2 R_3}{R_4} + j\omega C r \frac{R_3 R_2}{R_4} + j\omega C R_2 R_3\right)$$

Equal real and imaginary parts ,

$$r_1 + R_1 = \frac{R_2 R_3}{R_4}$$

$$\boxed{R_1 = \frac{R_2 R_3}{R_4} - r_1}$$

Topic Date

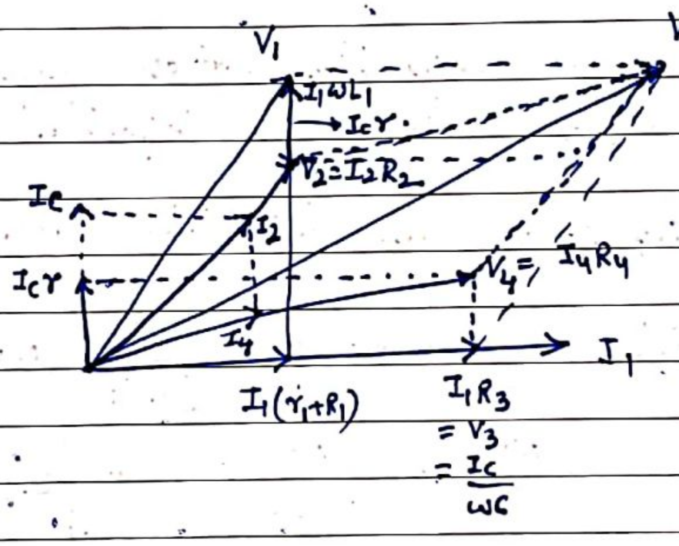
$$\omega L_1 - \omega C r R_3 = \omega C r \frac{R_3 R_2}{R_4} + \omega C R_2 R_3$$

$$L_1 = C r R_3 + C r \frac{R_3 R_2}{R_4} + C R_2 R_3$$

$$= C \cdot \frac{R_3}{R_4}\left[r R_4 + r R_2 + R_2 R_4 \right].$$

$$\boxed{L_1 = C \cdot \frac{R_3}{R_4}\left[r(R_2+R_4) + R_2 R_4 \right]}$$

Phasor Diagram under balanced Conditions.

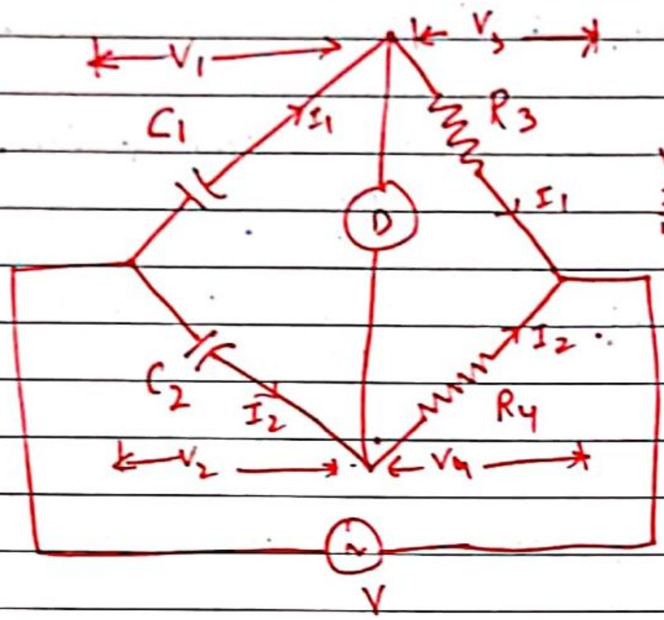


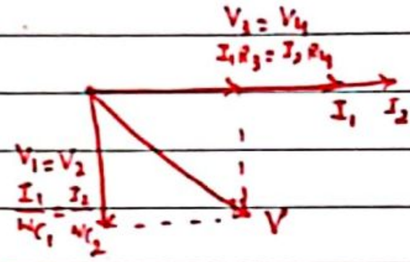
Scanned with CamScan

Topic ______ Date ______

Measurement of Capacitance

① De Sauty's Bridge :-





$C_1 =$ Capacitor whose capacitance is to evaluated.
$C_2 =$ Standard Capacitor

$R_3, R_4 =$ Non inductive resistors.

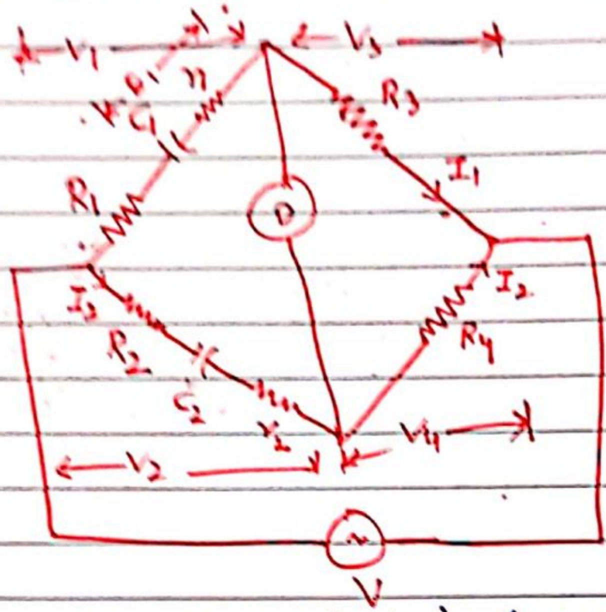
At balance ; $\left(\frac{1}{j\omega C_1}\right) R_4 = \left(\frac{1}{j\omega C_2}\right) R_3$

$$\frac{R_4}{C_1} = \frac{R_3}{C_2} \quad \therefore \boxed{C_1 = \frac{R_4}{R_3} \cdot C_2}$$

The balance can be obtained by varying either R_3 or R_4. The advantage is simplicity. The capacitors have been found to be perfect. However, to consider dielectric loss, we need to modify this bridge by taking into account the dielectric loss.

Topic Date

Grover Bridge



$r_1 \rightarrow$ represents dielectric loss component in C_1.
$r_2 \rightarrow$ " " " " " C_2;

At balance : $\left(R_1 + r_1 + \frac{1}{j\omega C_1}\right) R_4 = \left(R_2 + r_2 + \frac{1}{j\omega C_2}\right) R_3$

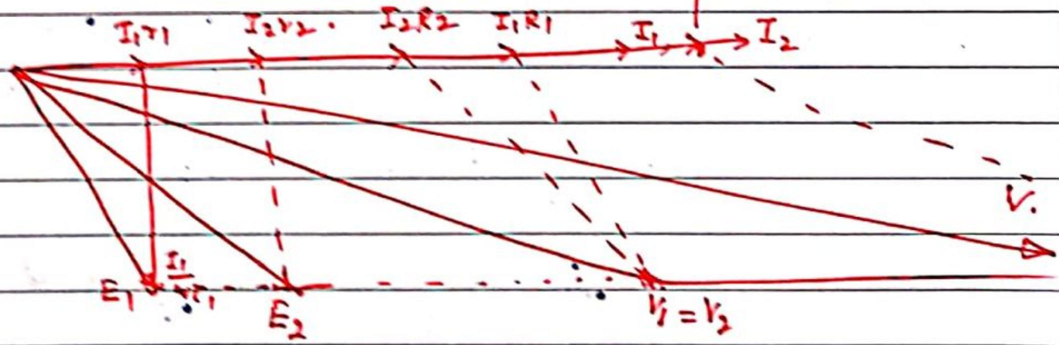
Equate real & imag parts :

$$(R_1 + r_1) R_4 = (R_2 + r_2) R_3$$

$$\frac{R_4}{\omega C_1} = \frac{R_3}{\omega C_2}$$

$$\boxed{\frac{R_4}{R_3} = \frac{C_1}{C_2} = \frac{R_2 + r_2}{R_1 + r_1}}$$

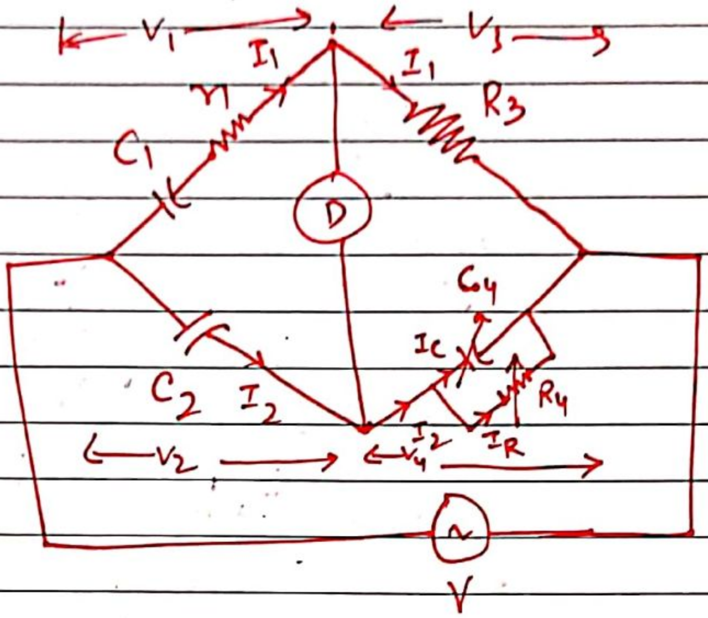
$V_3 = V_4$
$I_1 R_3 = I_2 R_4$

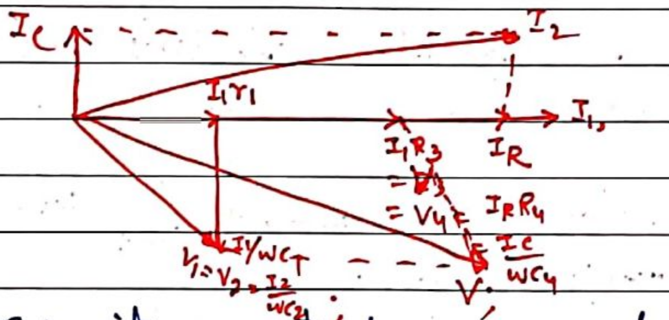


Classmate

Topic Date

③ Schering Bridge





C_1 = Capacitance to be measured.

r_1 = series resistor representing loss in C_1.

C_2 = standard capacitor

R_3 = non-inductive resistor (standard).

C_4 = a variable capacitor (standard).

R_4 = variable non inductive resistor (known).

At balance; $\left(r_1 + \frac{1}{jwC_1}\right)\left(\frac{R_4}{1+jwC_4R_4}\right) = \frac{1}{jwC_2}(R_3)$.

$$\left(r_1 + \frac{1}{jwC_1}\right)R_4 = \frac{-jR_3}{wC_2}\left(1 + jwC_4R_4\right).$$

$$r_1R_4 + \frac{jR_4}{jwC_1} = \frac{-jR_3}{wC_2} + \frac{R_3C_4R_4}{C_2}$$

Topic Date

Equating real and imaginary parts,

$$r_1 \cancel{R_4} = \frac{R_3 . C_4 . \cancel{R_4}}{C_2} \qquad \boxed{r_1 = \frac{R_3 C_4}{C_2}}$$

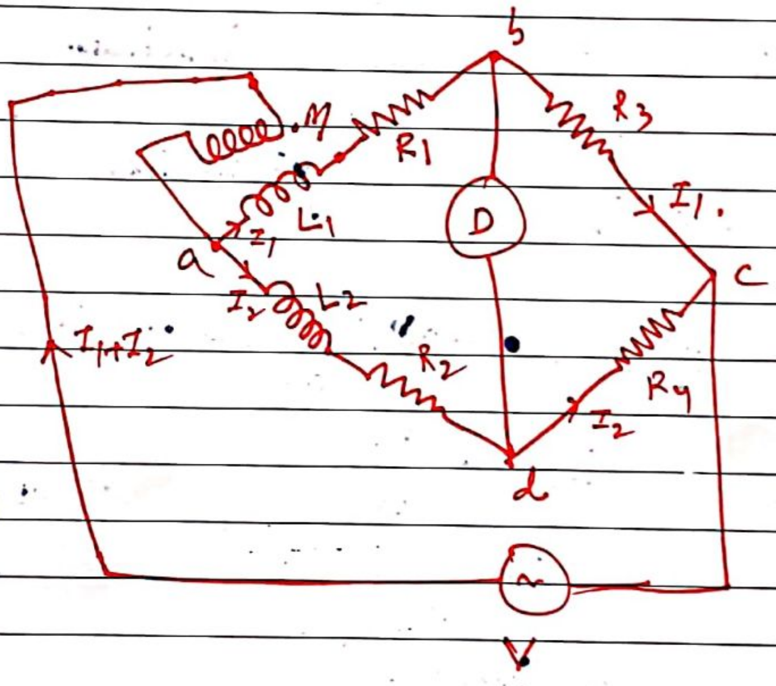
$$\frac{R_4}{\cancel{\omega} C_1} = \frac{R_3}{\cancel{\omega} C_2}$$

$$\boxed{C_1 = \frac{R_4}{R_3} . C_2}$$

---o---

Measurement of Mutual Inductance :-

1. Heaviside's Mutual Inductance Bridge :-



M = unknown mutual Inductance

L_1 = self inductance of secondary of MI.

L_2 = known self inductance

R_1, R_2, R_3, R_4 = non-inductive resistances.

Citizen

Topic ______ Date ______

At balance the voltage drop b/w b & c must equal the V.D b/w d & c.

$$V_{bc} = V_{dc}$$

Also

The voltage drop; V_{abc} must be equal to V_{adc}. Thus we can write;

$$I_1 R_3 = I_2 R_4 \quad \text{——— (1)}$$

$$(I_1 + I_2) j\omega M + I_1 (R_1 + R_3 + j\omega L_1) = I_2 (R_2 + R_4 + j\omega L_2) \quad \text{——— (2)}$$

Eliminate I_1 in (2); by substituting its value $I_1 = \frac{R_4}{R_3} \cdot I_2$; obtained from (1) above;

$$\left(\frac{R_4}{R_3} + 1\right) I_2 j\omega M + \frac{R_4}{R_3} I_2 (R_1 + R_3 + j\omega L_1) = I_2 (R_2 + R_4 + j\omega L_2)$$

equating real and imaginary parts.

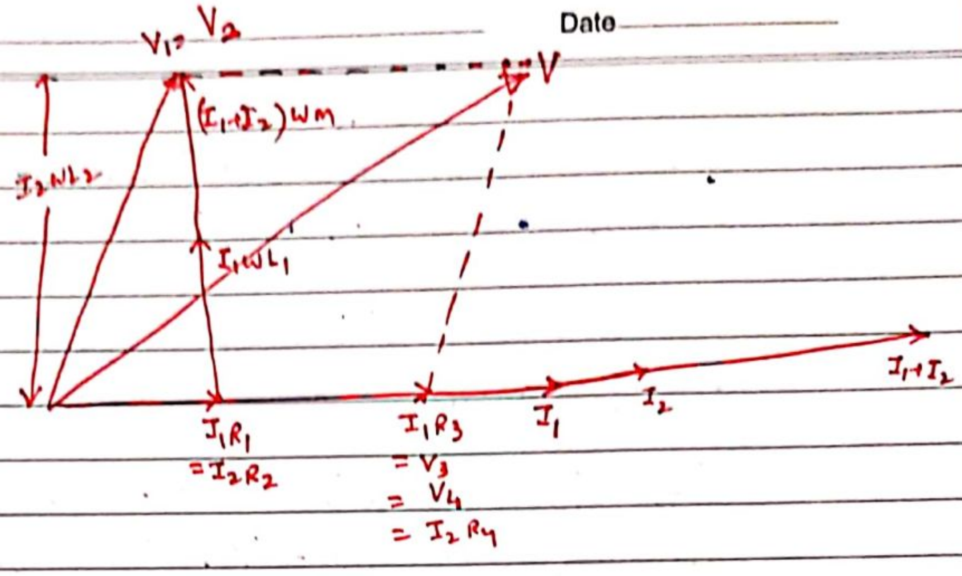
$$\frac{R_1 R_4}{R_3} = R_2 \Rightarrow \boxed{R_1 R_4 = R_2 R_3}$$

$$\omega M \left(\frac{R_4}{R_3} + 1\right) + \frac{\omega L_1 R_4}{R_3} = \omega L_2$$

$$M = \frac{L_2 - \frac{L_1 R_4}{R_3}}{\left(R_4/R_3 + 1\right)} = \frac{L_2 R_3 - R_4 L_1}{R_3 + R_4}$$

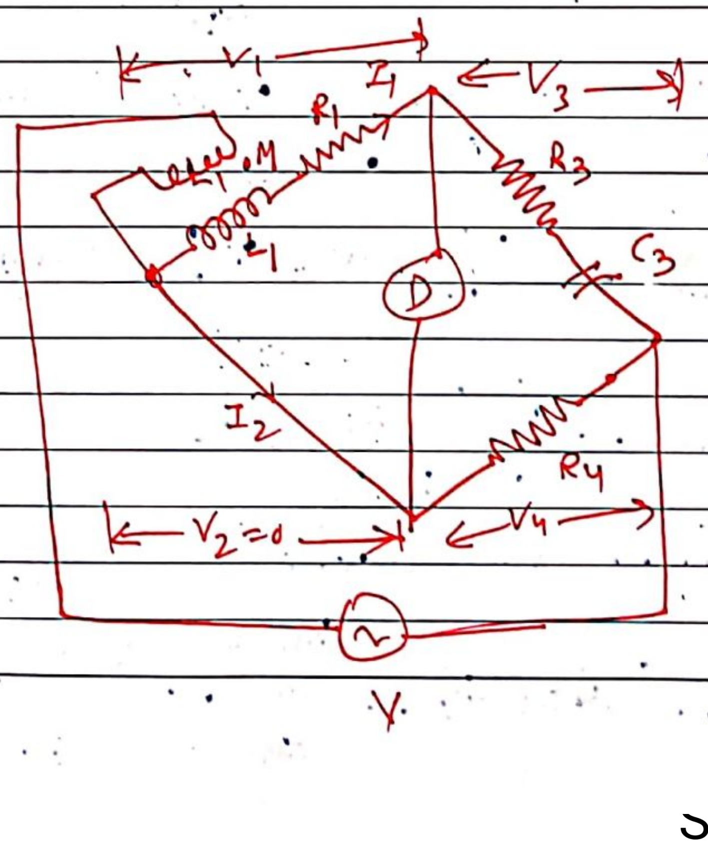
$$\boxed{M = \frac{L_2 R_3 - R_4 L_1}{R_3 + R_4}}$$

Topic Date



2. Carey Foster / Heydweiller Bridge

This Bridge was basically used by Carey Foster for measurement of Capacitance in terms of mutual inductance. It was subsequently modified by Heydweiller for measurement of mutual Inductance in terms of Capacitance.



Citizen

Topic Date

arm ad is short ckted. Thus voltage drop across ad = 0;

under balanced conditions the voltage across arm ab should also be zero. This is achieved by negative coupling for mutual inductance M. i.e. at balance.

$$I_1(R_1+j\omega L_1) - (I_1+I_2)j\omega M = 0 \quad \text{— (1)}$$

Also, $$I_1\left(R_3+\frac{1}{j\omega C_3}\right) = I_2 R_4; \quad \text{— (2)}$$

find value of I_2 from (2) & insert in (1).

$$\cancel{I_1}(R_1+j\omega L_1) - \cancel{I_1}\left(1+\frac{R_3+\frac{1}{j\omega C_3}}{R_4}\right)j\omega M = 0.$$

$$R_1+j\omega L_1 - \left(1+\frac{1+j\omega C_3R_3}{R_4 j\omega C_3}\right)j\omega M = 0.$$

$$\cancel{R_1+j\omega L_1 - \left(\frac{R_4 j\omega C_3+1+j\omega C_3R_3}{R_4 j\omega C_3}\right)\times j\omega M = 0.}$$

$$\cancel{R_1+j\omega L_1 - \left[\frac{R_4 j\omega C_3+1+j\omega C_3 R_3}{R_4 j\omega C_3}\right] = 0.}$$

$$\cancel{R_1+j\omega L_1 - \left(\frac{j\omega R_4 C_3 M}{R_4 C_3}\right)}$$

$$R_1+j\omega L_1 - \frac{R_4 j\omega C_3+1+j\omega C_3R_3}{\cancel{j\omega} R_4 C_3}\cdot\cancel{j\omega}M = 0$$

$$R_1+j\omega L_1 - \frac{j\omega R_4 C_3 M}{R_4 C_3} - \frac{M}{R_4 C_3} - \frac{j\omega C_3 R_3 M}{R_4 C_3}$$

Topic Date

Equating real and imaginary parts.

$$R_1 - \frac{M}{R_4 C_3} = 0 \Rightarrow \boxed{M = R_1 R_4 C_3}$$

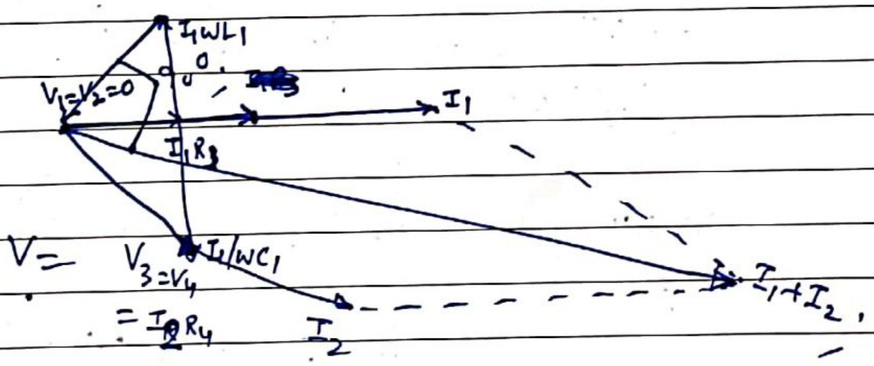
$$\omega L_1 - \omega M - \omega \frac{R_3}{R_4} . M = 0 .$$

$$L_1 = M\left(1 + \frac{R_3}{R_4}\right)$$

$$\boxed{L_1 = \frac{M(R_3+R_4)}{R_4}}$$

$$L_1 = \frac{R_1 \cancel{R_4} C_3 \cdot (R_3+R_4)}{\cancel{R_4}}$$

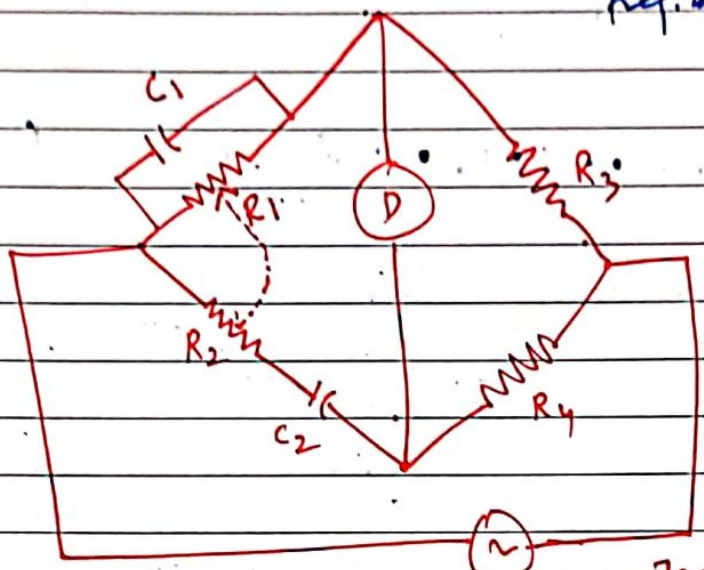
$$\boxed{L_1 = R_1 C_3 (R_3+R_4)}$$



Topic ________ Date ________

Wein Bridge for measurement of Frequency

Freq. range:- 100Hz – 100kHz
Ac → 0.1 to 0.5%



V, ω → frequency;

Under balanced conditions;

$$\left(\frac{R_1}{1+j\omega C_1 R_1}\right) R_4 = \left(R_2 + \frac{1}{j\omega C_2}\right) R_3$$

$$\frac{R_1 R_4}{1+j\omega C_1 R_1} = \left(R_2 - \frac{j}{\omega C_2}\right) R_3$$

$$\frac{R_1 R_4}{R_3} = \left(R_2 - \frac{j}{\omega C_2}\right)\left(1 + j\omega C_1 R_1\right)$$

$$= R_2 + \frac{C_1 R_1}{C_2} + j\omega C_1 R_1 R_2 - \frac{j}{\omega C_2}$$

$$\frac{R_1 R_4}{R_3} = R_2 + \frac{C_1 R_1}{C_2} + j\left(\omega C_1 R_1 R_2 - \frac{1}{\omega C_2}\right)$$

Equating Imaginary part;

$$\omega C_1 R_1 R_2 = \frac{1}{\omega C_2}$$

or $\omega^2 = \dfrac{1}{C_1 R_1 R_2 C_2}$

$\omega = \dfrac{1}{\sqrt{C_1 C_2 R_1 R_2}}$

or $\boxed{f = \dfrac{1}{2\pi\sqrt{C_1 C_2 R_1 R_2}}}$

Citizen