12

Transients in Power Systems

INTRODUCTION

Transients phenomenon is an aperiodic function of time and does not last longer. The duration for which they last is very insignificant as compared with the operating time of the system. Yet they are very important because depending upon the severity of these transients, the system may result into black out in a city, shut down of a plant, fires in some buildings, etc.

The power system can be considered as made up of linear impedance elements of resistance, inductance and capacitance. The circuit is normally energized and carries load until a fault suddenly occurs. The fault, then, corresponds to the closing of a switch (or switches, depending upon the type of fault) in the electrical circuit. The closing of this switch changes the circuit so that a new distribution of currents and voltages is brought about. This redistribution is accompanied in general by a transient period during which the resultant currents and voltages may momentarily be relatively high. It is very important to realize that this redistribution of currents and voltages cannot take place instantaneously for the following reasons:

1. The electromagnetic energy stored by an inductance L is $\frac{1}{2}LI^2$, where I is the instantaneous value of current. Assuming inductance to be constant the change in magnetic energy requires change in current which an inductor is opposed by an e.m.f. of magnitude $L\frac{dI}{dt}$. In order to change the current instantaneously $dt = 0$ and therefore $L\frac{dI}{0}$ is infinity, *i.e.,* to bring about instantaneous change in current the e.m.f. in the inductor should become infinity which is practically not possible and, therefore, it can be said that the change of energy in an inductor is gradual.

2. The electrostatic energy stored by a capacitor C is given by $\frac{1}{2}CV^2$, where V is the instantaneous value of voltage. Assuming capacitance to be constant, the change in energy requires change in voltage across the capacitor.

Since, for a capacitor, $\frac{dV}{dt} = \frac{I}{C}$, to bring instantaneous change in voltage, *i.e.,* for $dt = 0$ the change in current required is infinite which again cannot be achieved in practice and, therefore, it can be said that change in energy in a capacitor is also gradual.

There are only two components L and C in an electrical circuit which store energy and we have seen that the change in energy through these components is gradual and, therefore, the redistribution of energy following a circuit change takes a finite time. The third component, the resistance R, consumes energy. At any time, the principle of conservation of energy in an electrical circuit applies, *i.e.*, the rate of generation of energy is equal to the rate of storage of energy plus the rate of energy consumption.

It is clear that the three simple facts, namely,

1. the current cannot change instantaneously through an inductor,

2. the voltage across a capacitor cannot change instantaneously, and

3. the law of conservation of energy must hold good, are fundamental to the phenomenon of transients in electric power systems.

From the above it can be said that in order to have transients in an electrical system the following requirements should be met:

1. Either inductor or capacitor or both should be present.

2. A sudden change in the form of a fault or any switching operation should take place.

There are two components of voltages in a power system during transient period: (*i*) Fundamental frequency voltages, and (*ii*) natural frequency voltages usually of short duration which are superimposed upon the fundamental frequency voltages. There is third component also known as harmonic voltages resulting from unbalanced currents flowing in rotating machines in which the reactances in the direct and quadrature axes are unequal.

Natural frequency voltages appear immediately after the sudden occurrence of a fault. They simply add to the fundamental frequency voltages. Since resultant voltages are of greater importance from a practical viewpoint it will be preferable to speak of the fundamental frequency and natural frequency components simply as a transient voltage. The transient voltages are affected by the number of connections and the arrangements of the circuits.

Transients in which only one form of energy—storage, magnetic or electric is concerned, are called single energy transients, where both magnetic and electric energies are contained in or accepted by the circuit, double energy transients are involved.

12.1 TRANSIENTS IN SIMPLE CIRCUITS

For analysing circuits for transients we will make use of Laplace transform technique which is more powerful and easy to handle the transient problems than the differential equation technique. We will assume here lumped impedances only. The transients will depend upon the driving source also, *i.e.*, whether it is a d.c. source or an a.c. source. We will begin with simple problems and then go to some complicated problems.

1. D.C. Source

(*a*) *Resistance only* (Fig. 12.1 (*a*)): As soon as the switch *S* is closed, the current in the circuit will be determined according to Ohm's law.

$$I = \frac{V}{R}$$

Now transients will be there in the circuit.

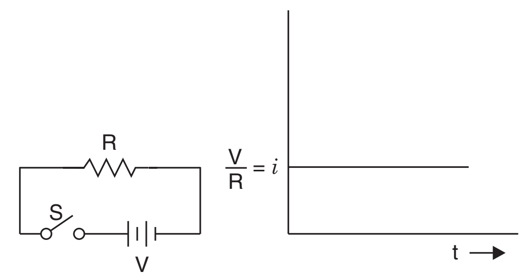

Fig. 12.1(*a*) Resistive circuit.

(*b*) *Inductance only* (Fig. 12.1 (*b*)): When switch *S* is closed, the current in the circuit will be given by

$$I(s) = \frac{V(s)}{Z(s)} = \frac{V}{s} \cdot \frac{1}{Ls} = \frac{V}{L} \cdot \frac{1}{s^2}$$

$$i(t) = \frac{V}{L} t$$

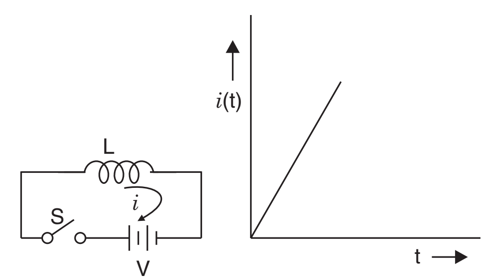

Fig. 12.1 (*b*) Inductive circuit.

This shows that when a pure inductance is switched on to a d.c. source, the current at $t = 0_+$ is zero and this increases linearly with time till for infinite time it becomes infinity. In practice, of course, a choke coil will have some finite resistance, however small; the value of the current will settle down to the value *V*/*R*, where *R* is the resistance of the coil.

(*c*) *Capacitance only* (Fig. 12.1 (*c*)): When switch *S* is closed, the current in the circuit is given by

$$I(s) = \frac{V(s)}{Z(s)} = \frac{V}{s} \, . \, Cs = VC$$

which is an impulse of strength (magnitude) *VC*.

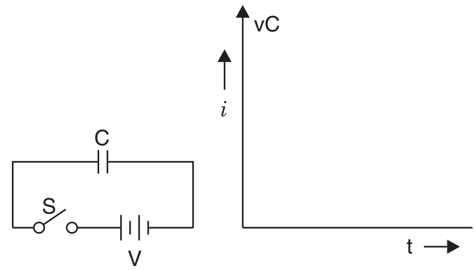

Fig. 12.1 (*c*) Capacitive circuit.

(*d*) *R-L circuit* (Fig. 12.1 (*d*)): When switch *S* is closed, the current in the circuit is given by

$$I(s) = \frac{V(s)}{Z(s)} = \frac{V}{s} \cdot \frac{1}{R + Ls} = \frac{V}{s} \cdot \frac{1/L}{s + R/L}$$

$$= \frac{V}{L}\left[\frac{1}{s} - \frac{1}{s + R/L}\right] \cdot \frac{L}{R}$$

$$= \frac{V}{R}\left[\frac{1}{s} - \frac{1}{s + R/L}\right]$$

$$i(t) = \frac{V}{R}\left[1 - \exp\left(-\frac{R}{L}t\right)\right]$$

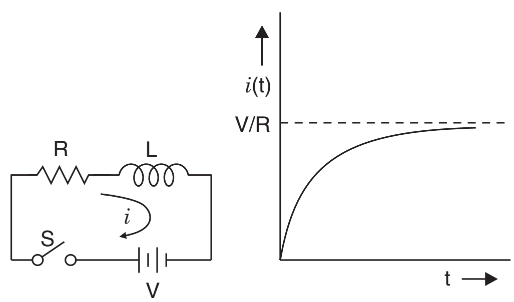

Fig. 12.1(*d*) *R-L* circuit.

The variation of current is shown in Fig. 12.1(*d*). It can be seen from the expression that the current will reach *V*/*R* value after infinite time. Also it can be seen that the inductor just after closing of the switch behaves as an open circuit and that is why the current at $t = 0_+$ is zero. When $t = L/R$,

$$i(t) = \frac{V}{R}\left(I - \frac{1}{e}\right)$$

$$= I_m\left(I - \frac{1}{e}\right)$$

$$= 0.632\ I_m$$

At time $t = L/R$, the current in the circuit is 63.2% of the maximum value reached in the circuit. This time in seconds is called the time-constant of the circuit. The larger the value of inductance in the circuit as compared with resistance the slower will be the build up of current in the circuit. The energy stored in the inductor under steady state condition will be $\frac{1}{2}LI_m^2$, where $I_m = V/R$.

(*e*) *R-C circuit* (Fig. 12.1(*e*)): After the switch *S* is closed, current in the circuit is given by

$$I(s) = \frac{V(s)}{Z(s)} = \frac{V}{s} \cdot \frac{1}{R + 1/Cs}$$

$$= \frac{V}{s} \cdot \frac{(1/RC)Cs}{s + 1/RC} = \frac{V}{R} \cdot \frac{1}{s + 1/RC}$$

$$i(t) = \frac{V}{R} \cdot e^{-t/CR}$$

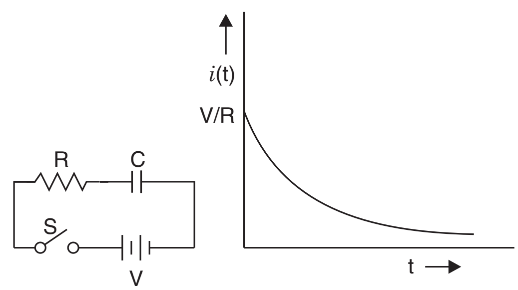

Fig. 12.1(*e*) *R-C* circuit.

It is seen that at $t = 0$, the capacitor acts as a short-circuit to the d.c. source and the current is *V/R* limited only by the resistance of the circuit. At $t = \infty$ the current in the circuit is zero and the capacitor is charged to a voltage *V*. The energy stored by the capacitor is $\frac{1}{2}CV^2$.

(*f*) *R-L-C circuit* (Fig. 12.1(*f*)): After the switch *S* is closed, the current in the circuit is given by

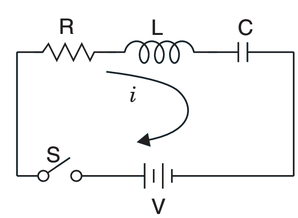

Fig. 12.1(*f*) *R-L-C* circuit.

$$I(s) = \frac{V}{s} \cdot \frac{1}{R + Ls + 1Cs}$$

$$= \frac{V}{s} \cdot \frac{Cs}{RCs + LCs^2 + 1}$$

$$= \frac{V}{s} \cdot \frac{1/L}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

$$= \frac{V}{L} \cdot \frac{1}{\left\{s + \left(\frac{R}{2L} - \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}}\right)\right\}\left\{s + \left(\frac{R}{2L} + \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}}\right)\right\}}$$

Let $\frac{R}{2L} = a$ and $\sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}} = b$; then

$$I(s) = \frac{V}{L} \cdot \frac{1}{(s + a - b)(s + a + b)}$$

$$= \frac{V}{2bL}\left\{\frac{1}{(s+a-b)} - \frac{1}{(s+a+b)}\right\}$$

$$i(t) = \frac{V}{2bL}\{e^{-(a-b)t} - e^{-(a+b)t}\}$$

There are three conditions based on the value of b:

(*i*) If $\frac{R^2}{4L^2} > \frac{1}{LC}$, b is real.

(*ii*) If $\frac{R^2}{4L^2} = \frac{1}{LC}$, b is zero.

(*iii*) If $\frac{R^2}{4L^2} < \frac{1}{LC}$, b is imaginary.

Case I: When b is real.

The expression for current will be

$$i(t) = \frac{V}{2\sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}}\,.\,L}\left[\exp\left\{-\left(\frac{R}{2L} + \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}}\right)t\right\} - \exp\left\{-\left(\frac{R}{2L} - \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}}\right)t\right\}\right]$$

and the variation of current is given in Fig. 12.1(*g*).

Case II: When $b = 0$.

The expression for current becomes

$$i(t) = \frac{V}{2bL}\{e^{-at} - e^{-at}\}$$ which is indeterminate.

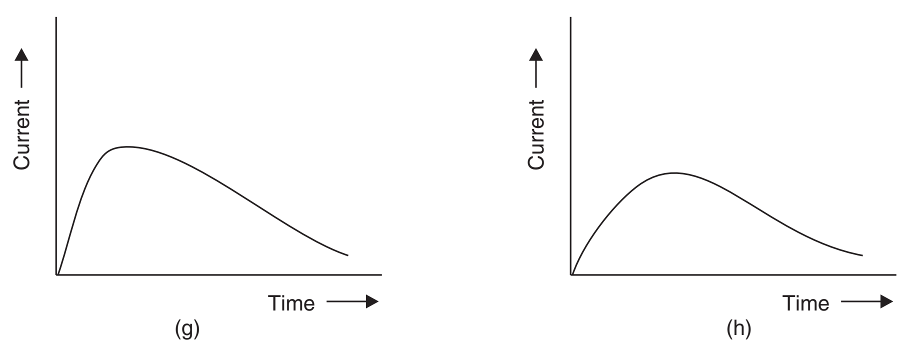

Fig. 12.1(*g*) Waveform when *b* is real (*h*) Waveform when *b* = 0.

Therefore, differentiating $i(t)$ with respect to b gives

$$i(t) = \frac{V}{2L}\,.\,t\{e^{-(a-b)t} + e^{-(a+b)t}\}$$

Now at $b = 0$

$$i(t) = \frac{V}{L}te^{-at} = \frac{V}{L}te^{-(R/2L)t}$$

The variation of current is given in Fig. 12.1(*h*).

Case III: When b is imaginary.

$$i(t) = \frac{V}{2bL}\{e^{-at}\,.\,e^{jkt} - e^{-at}\,.\,e^{-jkt}\} = \frac{V}{2bL}\,e^{-at}\,.\,2\sin kt$$

$$= \frac{V}{2L\sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}}}\,e^{-at}\,.\,2\sin\left(\sqrt{-\frac{R^2}{4L^2} + \frac{1}{LC}}\right)t$$

The wave shape of the current is shown in Fig. 12.1(i).

When b is positive or zero, the variation of current is non-oscillatory whereas it is oscillatory when b is imaginary. Because of the presence of the capacitance, the current in all the three cases dies down to zero value with d.c. source in the circuit.

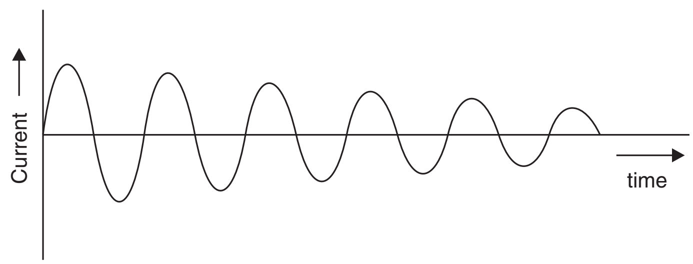

Fig. 12.1(i) Waveform when b is imaginary.

2. A.C. Source

R-L circuit (Fig. 12.2): When switch S is closed, the current in the circuit is given by

$$I(s) = \frac{V(s)}{Z(s)} = V_m\left\{\frac{\omega\cos\phi}{s^2+\omega^2} + \frac{s\sin\phi}{s^2+\omega^2}\right\}.\frac{1}{R+Ls}$$

$$= \frac{V_m}{L}\left\{\frac{\omega\cos\phi}{s^2+\omega^2} + \frac{s\sin\phi}{s^2+\omega^2}\right\}.\frac{1}{s+R/L}$$

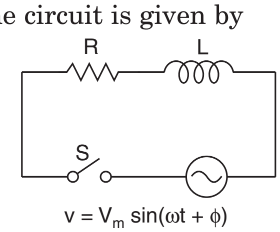

Fig. 12.2 *R-L* circuit connected to an a.c. source.

Let $\frac{R}{L} = a$; then

$$I(s) = \frac{V_m}{L}\left\{\frac{\omega\cos\phi}{(s+a)(s^2+\omega^2)} + \frac{s\sin\phi}{(s+a)(s^2+\omega^2)}\right\}$$

Now
$$\frac{1}{(s+a)(s^2+\omega^2)} = \frac{1}{(a^2+\omega^2)}\left\{\frac{1}{s+a} + \frac{a}{s^2+\omega^2} - \frac{s}{s^2+\omega^2}\right\}$$

and
$$\frac{s}{(s+a)(s^2+\omega^2)} = \frac{1}{(a^2+\omega^2)}\left\{\frac{as}{s^2+\omega^2} + \frac{\omega^2}{s^2+\omega^2} - \frac{a}{s+a}\right\}$$

Therefore
$$\mathcal{L}^{-1}I(s) = \frac{V_m}{(a^2+\omega^2)L}\left[\omega\cos\phi\left\{e^{-at} + \frac{a}{\omega}\sin\omega t - \cos\omega t\right\}\right.$$
$$\left. + \sin\phi\{a\cos\omega t + \omega\sin\omega t - ae^{-at}\}\right]$$

The equation can be further simplified to

$$i(t) = \frac{V_m}{L\sqrt{a^2 + \omega^2}} \{\sin(\omega t + \phi - \theta) - \sin(\phi - \theta)\, e^{-at}\}$$

$$= \frac{V_m}{(R^2 + \omega^2 L^2)^{1/2}} \{\sin(\omega t + \phi - \theta) - \sin(\phi - \theta)\, e^{-at}\}$$

where $\theta = \tan^{-1} \dfrac{\omega L}{R}$.

The variation of current is shown in Fig. 12.3.

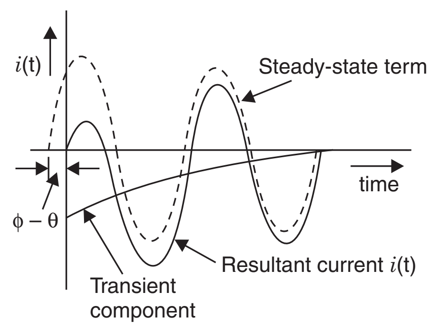

Fig. 12.3 Asymmetrical alternating current.

The first term in the expression above is the steady state sinusoidal variation and the second term is the transient part of it which vanishes theoretically after infinite time. But practically, it vanishes very quickly after two or three cycles. The transient decay as is seen depends upon the time constant $\dfrac{1}{a} = \dfrac{L}{R}$ of the circuit. Also at $t = 0$ it can be seen that the transient component equals the steady state component and since the transient component is negative the net current is zero at $t = 0$. It can be seen that the transient component will be zero in case the switching on of the voltage wave is done when $\theta = \phi$, *i.e.*, when the wave is passing through an angle $\phi = \tan^{-1} \dfrac{\omega L}{R}$. This is the situation when we have no transient even though the circuit contains inductance and there is switching operation also. On the other hand if $\phi - \theta = \pm \pi/2$, the transient term will have its maximum value and the first peak of the resulting current will be twice the peak value of the sinusoidal steady state component.

12.2 3-PHASE SUDDEN SHORT CIRCUIT OF AN ALTERNATOR

The study of 3-phase short circuit of an alternator is almost the same as the previous article except for the fact that in the previous case we assumed the voltage source to be of constant magnitude; here in this case the flux linkages vary and therefore the source is of varying magnitude. This being a 3-phase circuit, the switching angles in the different phases are 120° apart. So there is a good chance that the conditions of $\phi - \theta = \pm \pi/2$ may occur where the d.c.

decaying component may have its maximum value at $t = 0$ and the total current in some phase may be twice the peak value of the steady state current.

Whenever a 3-phase short circuit occurs at the terminals of an alternator, the current in the armature circuit increases suddenly to a large value and since the resistance of the circuit then is small as compared to its reactance, the current is highly lagging and the p.f. is approximately zero. Due to this sudden switching, as analysed in the previous section, there are two components of currents:

(*i*) a.c. component,

(*ii*) d.c. component (decaying).

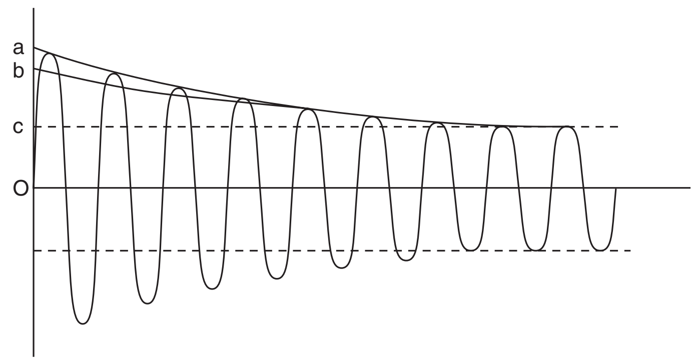

Fig. 12.4 The oscillogram of current variation as a function of time after a 3-phase fault takes place at the terminals of an alternator. The d.c. component is not shown. *Oa*—Subtransient current; *Ob*—Transient current; and *Oc*—Steady state current.

The current oscillogram is shown in Fig. 12.4. The rotor rotates at zero speed with respect to the field due to a.c. component of current in the stator whereas it rotates at synchronous speed with respect to the field due to the d.c. component of current in the stator conductors. The rotor winding acts as the secondary of a transformer for which the primary is the stator winding. Similarly in case the rotor has the damper winding fixed on its poles, the whole system will work as a three winding transformer in which stator is the primary and the rotor field winding and damper windings form the secondaries of the transformer. It is to be noted that the transformer action is there with respect to the d.c. component of current only. The a.c. component of current being highly lagging tries to demagnetise *i.e.*, reduce the flux in air gap. This reduction of flux from the instant of short circuit to the steady state operation cannot take place instantaneously because of the large amount of energy stored by the inductance of the corresponding system. So this change in flux is slow and depends upon the time-constant of the system. In order to balance the suddenly increased demagnetising m.m.f. of the armature current, the exciting current, *i.e.*, the field winding current must increase in the same direction of flow as before the fault. This happens due to the transformer action. At the same time, the current in the damper and the eddy currents in the adjacent metal parts increase in obedience to Lenz's law, thus assisting the rotor field winding to sustain the flux in the air gap.

At the instant of the short-circuit there is mutual coupling between the stator winding, rotor winding and the damper winding and the equivalent circuit is represented in Fig. 12.5(*a*).

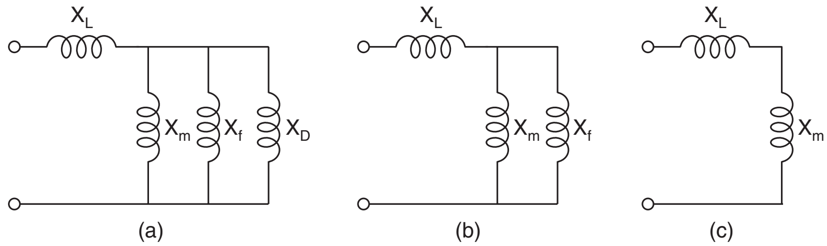

Fig. 12.5 Equivalent circuit of an alternator under (*a*) Subtransient; (*b*) Transient; and (*c*) Steady state conditions.

Since the equivalent resistance of the damper winding when referred to the stator is more as compared to the rotor winding, the time constant of damper winding is smaller than the rotor field winding. Therefore, the effect of damper winding and the eddy current in the pole faces disappears after the first few cycles. Accordingly, the equivalent circuit after first few cycles reduces to the one shown in Fig. 12.5(*b*). After a few more cycles depending upon the time constant of the field winding the effect of the d.c. component will die down and steady state conditions will prevail for which the equivalent circuit is shown in Fig. 12.5(*c*).

The reactance in the initial stage corresponding to Fig. 12.5(*a*) is called the subtransient reactance; corresponding to Fig. 12.5(*b*) it is called as the transient reactance and the steady state reactance is the synchronous reactance (Fig. 12.5(*c*)). It can be seen from the equivalent circuit that the inductance increases as from the initial stage to the final steady state *i.e.*, synchronous reactance > transient reactance > subtransient reactance.

12.3 THE RESTRIKING VOLTAGE AFTER REMOVAL OF SHORT CIRCUIT

The system (Fig. 12.6) consists of an alternator connected to a busbar. The load is removed after a short circuit occurs. It is required to determine the voltage across the circuit breaker during the opening period.

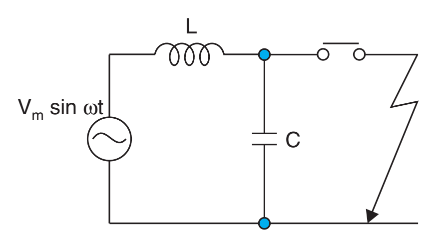

Fig. 12.6 Equivalent circuit to determine the restriking voltage.

The generator is represented by a constant voltage source behind the internal inductance L. The capacitance to ground of the busbars, the bushings etc., is lumped and is represented by C. The following assumptions are made, in addition, for the analysis of the system:

(*i*) The fault is a solid one *i.e.*, there is no arcing.

(*ii*) The magnitude of the positive sequence impedance is assumed to be constant for the period in which the overvoltage is to be determined.

(*iii*) The effects of saturation and corona are neglected which will tend to reduce the overvoltages.

(*iv*) The charging current of the transmission line before the fault, and load currents are neglected.

(*v*) The current interruption takes place at current zero when the voltage passes through maximum value.

(*vi*) The system is assumed to be lossless.

The method used for analysis is known as current cancellation method which means the voltage across the C.B. contact after it opens is the product of the current during the fault and the impedance of the network between the circuit breaker contacts shorting the voltage sources.

$$\text{The fault current} = I(s) = \frac{V(s)}{Z(s)}$$

$$= \frac{V_m}{s} \cdot \frac{1}{Ls}$$

Here we have taken V_m, instead of $V_m \sin \omega t$, because the fault interruption takes place at current zero when the voltage is passing through maximum value V_m.

Now the impedance between the circuit breaker contacts after shortcircuiting the voltage source will be the impedance of the parallel combination of L and C, *i.e.*,

$$Z_0(s) = \frac{Ls \,.\, 1/Cs}{Ls + 1/Cs} = \frac{Ls}{LCs^2 + 1} = \frac{s/C}{s^2 + 1/LC}$$

$$v(s) = I(s)Z_0(s) = \frac{V_m}{s} \cdot \frac{1}{sL} \cdot \frac{s/C}{s^2 + 1/LC}$$

$$= \frac{V_m}{s} \cdot \frac{1}{LC} \cdot \frac{1}{s^2 + 1/LC} = V_m \left[\frac{1}{s} - \frac{s}{s^2 + 1/LC} \right]$$

$$v(t) = V_m [1 - \cos \omega_0 t]$$

where $\omega_0 = \dfrac{1}{\sqrt{LC}}$ or $f_0 = \dfrac{1}{2\pi \sqrt{LC}}$

f_0 is the natural frequency of oscillation.

This variation of voltage across the circuit breaker is shown in Fig. 12.7. The voltage $v(t)$ is called the restriking voltage and it has its first peak value when

$$\omega_0 t = \pi$$

or $$\frac{1}{\sqrt{LC}} t = \pi$$

or $$t = \pi \sqrt{LC}$$

and the value is $2V_m$.

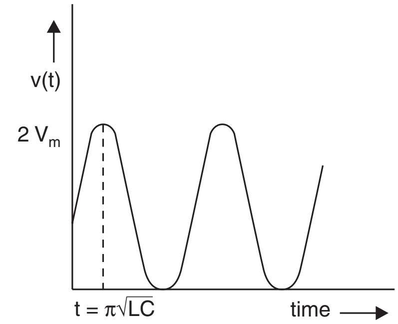

Fig. 12.7 Restriking voltage.

At $t = 0$ the value of the voltage is zero.

This type of transient is known as single frequency or energy transient.

Double Frequency Transient

The simplest circuit to demonstrate the double frequency transients is given in Fig. 12.8. Here L_1 and C_1 are the inductance and stray capacitance on the source side of the breaker and L_2 and C_2 on the load side.

When the circuit breaker operates, the load is completely isolated from the generator and the

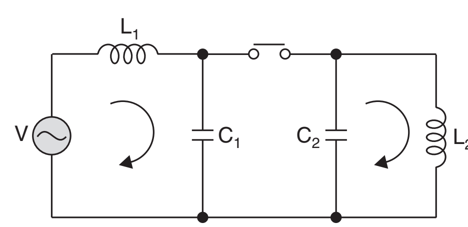

Fig. 12.8 Circuit with double frequency transients.

two halves of the circuit behave independently. Before the switch operates, the voltage across the capacitors is given by

$$V_c = V \cdot \frac{L_2}{L_1 + L_2}$$

Normally $L_2 > L_1$ and therefore the capacitor voltage is a little less than the source voltage at any time. When the current passes through zero value, the voltage is at its maximum. When the circuit breaker interrupts the current at its zero, the capacitor C_2 will oscillate with L_2 at a natural frequency of

$$f_2 = \frac{1}{2\pi\sqrt{L_2 C_2}}$$

and C_1 will oscillate with L_1 at a natural frequency

$$f_1 = \frac{1}{2\pi\sqrt{L_1 C_1}}$$

So opening of the switch will result in double frequency transients in this circuit.

12.4 TRAVELLING WAVES ON TRANSMISSION LINES

So far, we have analysed the transient behaviour of various circuits with lumped parameters. However, there are some parts of a power system where this approach is inadequate. The most obvious example is the transmission line. Here the parameters L, C and R are uniformly distributed over the length of the line. For steady state operation of the line the transmission lines could be represented by lumped parameters but for the transient behaviour of the lines they must be represented by their actual circuits *i.e.*, distributed parameters. We say that for a 50 Hz supply and short transmission line the sending end current equals the receiving end current and the change in voltage from sending end to receiving end is smooth. This is not so when transmission line is subjected to a transient.

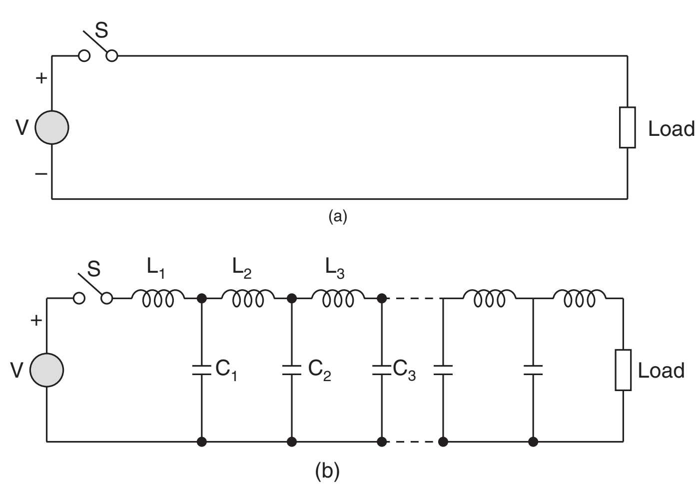

Fig. 12.9 (*a*) Long transmission line, (*b*) Equivalent π-section of a long transmission line.

To understand the travelling wave phenomenon over transmission line consider Fig. 12.9 (*a*). The line is assumed to be lossless. Let L and C be the inductance and capacitance respectively per unit length of the line. The line has been represented in Fig. 12.9 (*b*) by a large number of L and C π-sections. The closing of the switch is similar to opening the valve at the end of a channel, thereby admitting water to the channel from some reservoir behind. When the valve is opened the channel does not get filled up instantaneously. We observe the water advancing down the channel. At any instant the channel ahead of the wave front is dry while that behind is filled with water to the capacity. Similarly, when the switch S is closed the voltage does not appear instantaneously at the other end. When switch S is closed, the inductance L_1 acts as an open circuit and C_1 as short circuit instantaneously. The same instant the next section cannot be charged because the voltage across the capacitor C_1 is zero. So unless the capacitor C_1 is charged to some value whatsoever, charging of the capacitor C_2 through L_2 is not possible which, of course, will take some finite time. The same argument applies to the third section, fourth section and so on. So we see that the voltage at the successive sections builds up gradually. This gradual build up of voltage over the transmission line conductors can be regarded as though a voltage wave is travelling from one end to the other end and the gradual charging of the capacitances is due to the associated current wave.

Now it is desired to find out expressions for the relation between the voltage and current waves travelling over the transmission lines and their velocity of propagation.

Suppose that the wave after time t has travelled through a distance x. Since we have assumed lossless lines whatever is the value of voltage and current waves at the start, they remain same throughout the travel. Consider a distance dx which is travelled by the waves in time dt. The electrostatic flux is associated with the voltage wave and the electromagnetic flux with the current wave. The electrostatic flux which is equal to the charge between the conductors of the line up to a distance x is given by

$$q = VCx \tag{12.1}$$

The current in the conductor is determined by the rate at which the charge flows into and out of the line.

$$I = \frac{dq}{dt} = VC\,\frac{dx}{dt} \tag{12.2}$$

Here dx/dt is the velocity of the travelling wave over the line conductor and let this be represented by v. Then

$$I = VCv \tag{12.3}$$

Similarly the electromagnetic flux linkages created around the conductors due to the current flowing in them up to a distance of x is given by

$$\psi = ILx \tag{12.4}$$

The voltage is the rate at which the flux linkages link around the conductor

$$V = IL\,\frac{dx}{dt} = ILv \tag{12.5}$$

Dividing equation (12.5) by (12.3), we get

$$\frac{V}{I} = \frac{ILv}{VCv} = \frac{I}{V} \cdot \frac{L}{C}$$

or
$$\frac{V^2}{I^2} = \frac{L}{C}$$

or
$$\frac{V}{I} = \sqrt{\frac{L}{C}} = Z_n \quad (12.6)$$

The expression is a ratio of voltage to current which has the dimensions of impedance and is therefore here designated as surge impedance of the line. It is also known as the natural impedance because this impedance has nothing to do with the load impedance. It is purely a characteristic of the transmission line. The value of this impedance is about 400 ohms for overhead transmission lines and 40 ohms for cables.

Next, multiplying equations (12.3) with (12.5), we get

$$VI = VCv \cdot ILv = VILCv^2$$

or
$$v^2 = \frac{1}{LC}$$

or
$$v = \frac{1}{\sqrt{LC}} \quad (12.7)$$

Now expressions for L and C for overhead lines are

$$L = 2 \times 10^{-7} \ln \frac{d}{r} \text{ H/metre}$$

$$C = \frac{2\pi\varepsilon}{\ln \dfrac{d}{r}} \text{ F/metre}$$

Substituting these values in equation (12.7), the velocity of propagation of the wave

$$v = \frac{1}{\left(2 \times 10^{-7} \ln \dfrac{d}{r} \cdot \dfrac{2\pi\varepsilon}{\ln d/r}\right)^{1/2}}$$

$$= \frac{1}{\sqrt{4\pi\varepsilon \,.\, 10^{-7}}} = \frac{1}{\sqrt{4\pi \,.\, \dfrac{1}{36\pi} \times 10^{-9} \times 10^{-7}}}$$

$$= 3 \times 10^8 \text{ metres/sec.}$$

This is the velocity of light. This means the velocity of propagation of the travelling waves over the overhead transmission lines equals the velocity of light. In actual practice because of the resistance and leakance of the lines the velocity of the travelling wave is slightly less than the velocity of light. Normally a velocity of approximately 250 m/μ sec is assumed. It can be seen from the expression that the velocity of these waves over the cables will be smaller than over the overhead lines because of the permittivity term in the denominator.

Since $\varepsilon = \varepsilon_0 \varepsilon_r$ for overhead lines $\varepsilon_r = 1$ whereas for cables where the conductor is surrounded by some dielectric material for which $\varepsilon_r > 1$, the term ε is greater for cables than for overhead lines and therefore the velocity of the waves over the cables is smaller than over the overhead lines.

Let us study the behaviour of these lines to the travelling waves when they reach the other end of the lines or whenever they see a change in the impedance (impedance other than characteristic impedance of the line).

Open-End Line

Consider a line with the receiving end open-circuited as shown in Fig. 12.10.

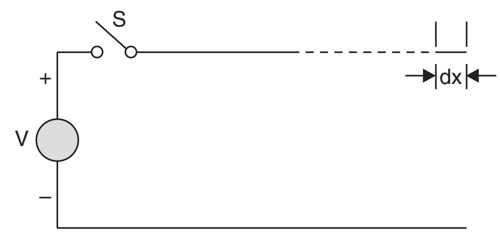

Fig. 12.10 Case of an open-ended line.

When switch S is closed, a voltage and current wave of magnitudes V and I respectively travel towards the open-end. These waves are related by the equation:

$$\frac{V}{I} = Z$$

where Z is the characteristic impedance of the line. Consider the last element dx of the line, because, it is here where the wave is going to see a change in impedance, an impedance different from Z (infinite impedance as the line is open-ended).

The electromagnetic energy stored by the element dx is given by $\frac{1}{2}LdxI^2$ and electrostatic energy in the element dx, $\frac{1}{2}CdxV^2$. Since the current at the open-end is zero, the electromagnetic energy vanishes and is transformed into electrostatic energy. As a result, let the change in voltage be e; then

$$\frac{1}{2}LdxI^2 = \frac{1}{2}Cdxe^2$$

or $$\left(\frac{e}{I}\right)^2 = \frac{L}{C}$$

or $$e = IZ = V$$

This means the potential of the open-end is raised by V volts; therefore, the total potential of the open-end when the wave reaches this end is

$$V + V = 2V$$

The wave that starts travelling over the line when the switch S is closed, could be considered as the incident wave and after the wave reaches the open-end, the rise in potential V could be considered due to a wave which is reflected at the open-end and actual voltage at the open-end could be considered as the refracted or transmitted wave and is thus

Refracted wave = Incident wave + Reflected wave

We have seen that for an open-end line a travelling wave is reflected back with positive sign and coefficient of reflection as unity.

Let us see now about the current wave.

As soon as the incident current wave I reaches the open-end, the current at the open end is zero, this could be explained by saying that a current wave of I magnitude travels back over the transmission line. This means for an open-end line, a current wave is reflected with negative sign and coefficient of reflection unity. The variation of current and voltage waves over the line is explained in Fig. 12.11.

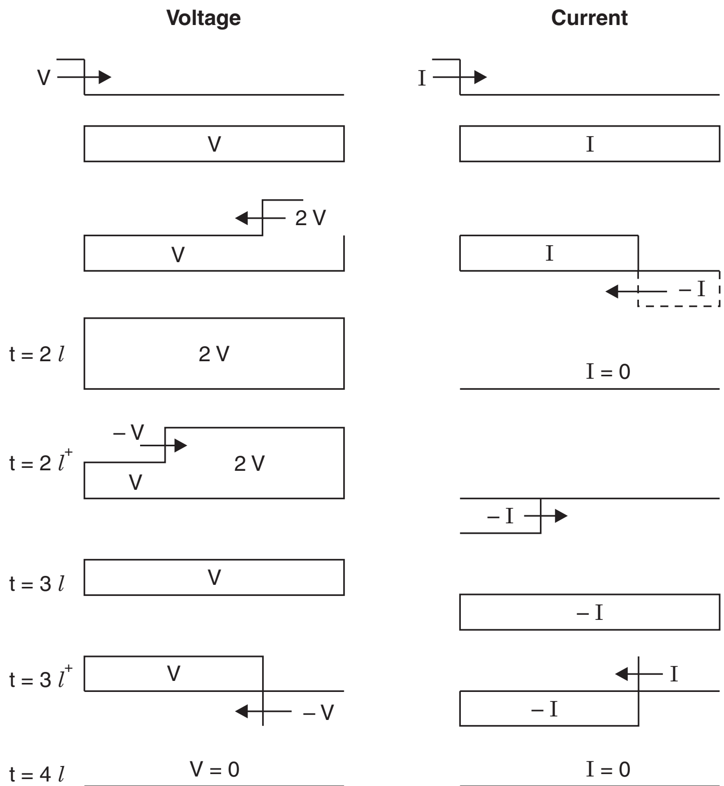

Fig. 12.11 Variation of voltage and current in an open-ended line.

After the voltage and current waves are reflected back from the open-end, they reach the source end, the voltage over the line becomes $2V$ and the current is zero. The voltage at source end cannot be more than the source voltage V therefore a voltage wave of $-V$ and current wave of $-I$ is reflected back into the line (Fig. 12.11). It can be seen that after the waves have travelled through a distance of $4l$, where l is the length of the line, they would have wiped out both the current and voltage waves, leaving the line momentarily in its original state. The above cycle repeats itself.

Short-circuited Line

Consider the line with receiving end short-circuited as shown in Fig. 12.12.

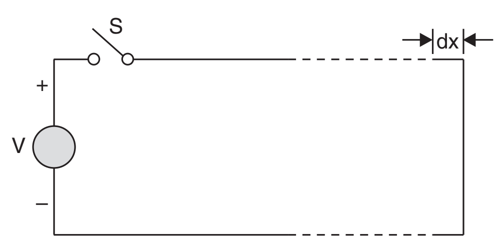

Fig. 12.12 Case of a short-circuited line.

When switch S is closed, a voltage wave of magnitude V and current wave of magnitude I start travelling towards the shorted end. Consider again the last element dx, where the electrostatic energy stored by the element is $\frac{1}{2}CdxV^2$ and electromagnetic energy $\frac{1}{2}LdxI^2$. Since the voltage at the shorted end is zero, the electrostatic energy vanishes and is transformed into electromagnetic energy. As a result, let the change in the current be i; then

$$\frac{1}{2}CdxV^2 = \frac{1}{2}Ldxi^2$$

or $$V = iZ$$

or $$i = \frac{V}{Z} = I$$

This means the increase in current is I amperes. As a result the total current at the shorted end, when the current wave reaches the end is $(I + I) = 2I$ amperes. This could be considered due to a reflected current wave of magnitude I amperes. Therefore for a short-circuited end the current wave is reflected back with positive sign and coefficient of reflection as unity. Since the voltage at the shorted end is zero, a voltage wave of $-V$ could be considered to have been reflected back into the line, *i.e.*, the current wave in case of short-circuited end is reflected back with positive sign and with coefficient of reflection as unity, whereas the voltage wave is reflected back with negative sign and unity coefficient of reflection. The variation of voltage and current over the line is explained in Fig. 12.13.

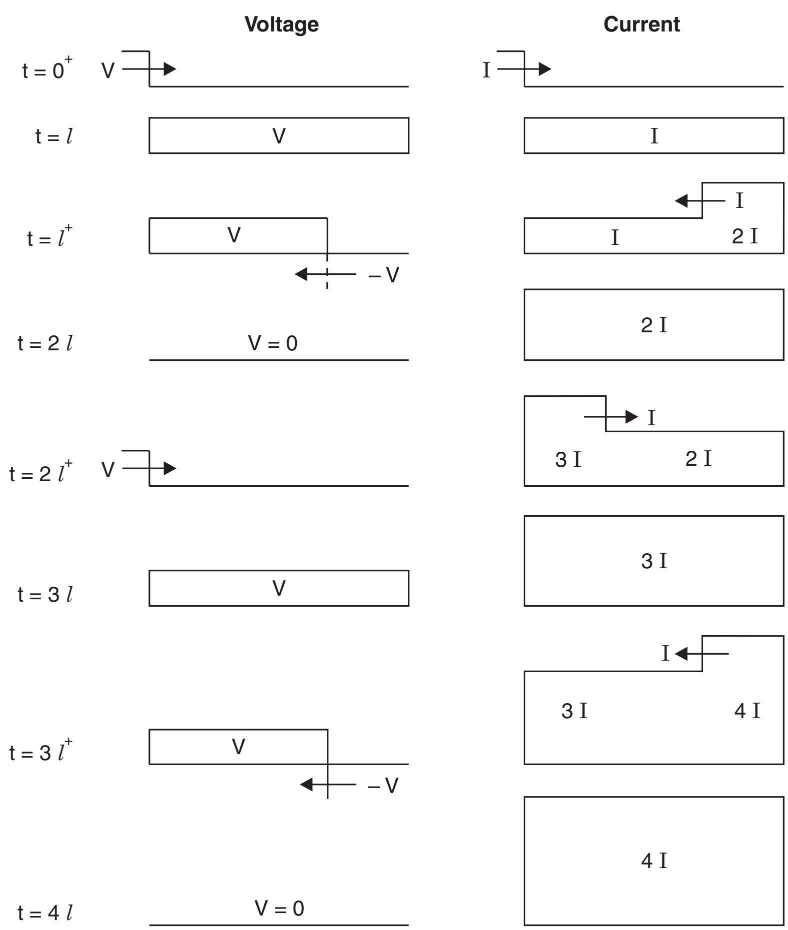

Fig. 12.13 Variation of voltage and current in a short ended line.

It is seen from above that the voltage wave periodically reduces to zero after it has travelled through a distance of twice the length of the line whereas after each reflection at either end the current is built up by an amount $V/Z_n = I$. Theoretically, the reflection will be infinite and therefore the current will reach infinite value. But practically in an actual system the current will be limited by the resistance of the line and the final value of the current will be $I' = V/R$, where R is the resistance of transmission line.

Line Terminated Through a Resistance

Let Z be the surge impedance of the line terminated through a resistance R (Fig. 12.14). It has been seen in the previous sections that whatever be the value of the terminating impedance whether it is open or short circuited, one of the two voltage or current waves is reflected back with negative sign. Also, since the reflected wave travels along the overhead line or over the line along which the incident wave travelled, therefore, the following relation holds good for reflected voltage and current waves.

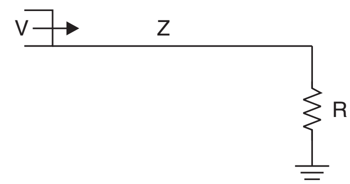

Fig. 12.14 Line terminated through a resistance.

$$I' = -\frac{V'}{Z}$$

where V' and I' are the reflected voltage and current waves. Also,

Refracted or transmitted wave = Incident wave + Reflected wave

Let V'' and I'' be the refracted voltage and current waves into the resistor R, when the incident waves V and I reach the resistance R. The following relations hold good:

$$I = \frac{V}{Z}$$

$$I' = -\frac{V'}{Z}$$

$$I'' = \frac{V''}{R}$$

Since $I'' = I + I'$ and $V'' = V + V'$, using these relations, we have

$$\frac{V''}{R} = \frac{V}{Z} - \frac{V'}{Z} \quad (12.8)$$

$$= \frac{V}{Z} - \frac{V'' - V}{Z} = \frac{2V}{Z} - \frac{V''}{Z}$$

or
$$V'' = \frac{2VR}{Z + R} \quad (12.9)$$

and current
$$I'' = \frac{2V}{R + Z} = \frac{V}{Z} \cdot \frac{2Z}{R + Z} = I \cdot \frac{2Z}{R + Z} \quad (12.10)$$

Similarly substituting for V'' in terms of $(V + V')$, equation (12.8) becomes

$$\frac{V + V'}{R} = \frac{V}{Z} - \frac{V'}{Z}$$

or $$V' = V \cdot \frac{R - Z}{R + Z} \tag{12.11}$$

and $$I' = -\frac{V'}{Z} = -\frac{V}{Z} \cdot \frac{(R - Z)}{R + Z} \tag{12.12}$$

From the relations above, the coefficient of refraction for current waves

$$= \frac{2Z}{R + Z}$$

and for voltage waves $$= \frac{2R}{R + Z}$$

Similarly, the coefficient of reflection for current waves

$$= -\frac{R - Z}{R + Z}$$

and for voltage waves $$= +\frac{R - Z}{R + Z}$$

Now the two extreme cases can be derived out of this general expression. For open circuit,

$$R \to \infty$$

and coefficient of refraction for current waves

$$\frac{2Z}{\infty + Z} = 0$$

and coefficient of refraction for voltage waves

$$= \frac{2R}{R + Z} = \frac{2}{1 + Z/R} = \frac{2}{1 + Z/\infty} = 2$$

Similarly, coefficient of reflection for current waves

$$= -\frac{R - Z}{R + Z} = -\frac{1 - Z/R}{1 + Z/R} = -1$$

and coefficient of reflection for voltage waves

$$= \frac{R - Z}{R + Z} = 1$$

Similarly, to find out the coefficients of reflection and refraction for current and voltage waves for the short circuit case, the value of $R = 0$ is to be substituted in the corresponding relations as derived in this section.

It is, therefore, seen here that whenever a travelling wave looks into a change in impedance, it suffers reflection and refraction. It is shown below that in case $Z = R$ *i.e.*, the line is terminated through a resistance whose value equals the surge impedance of the line (*i.e.*, no change in the impedance) there will be no reflection and the wave will enter fully into the resistance, *i.e.*, the coefficient of refraction will be unity whereas the coefficient of reflection will be zero.

When $R = Z$, substituting this, the coefficient of reflection for current wave

$$= -\frac{R-Z}{R+Z} = \frac{Z-Z}{Z+Z} = 0$$

and for voltage wave
$$= \frac{R-Z}{R+Z} = 0$$

The coefficient of refraction for current wave

$$= \frac{2Z}{R+Z} = \frac{2Z}{2Z} = 1$$

and for voltage wave
$$= \frac{2R}{R+Z} = 1$$

It is seen that when a transmission line is terminated through a resistance equal to its surge impedance the wave does not suffer reflection and, therefore, such lines could be said to be of infinite length. Such lines are also called as matched lines and the load corresponding to this is known as surge impedance loading or natural impedance loading. Detailed idea about this kind of loading is given in Chapter 4.

Line Connected to a Cable

A wave travels over the line and enters the cable (Fig. 12.15). Since the wave looks into a different impedance, it suffers reflection and refraction at the junction and the refracted voltage wave is given by

$$V'' = V \cdot \frac{2Z_2}{Z_1 + Z_2}$$

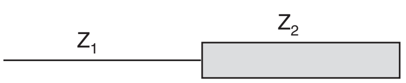

Fig. 12.15 Line connected to a cable.

The other waves can be obtained by using the relations (12.10–12.12). The impedance of the overhead line and cable are approximately 400 ohms and 40 ohms respectively. With these values it can be seen that the voltage entering the cable will be

$$V'' = V \cdot \frac{2 \times 40}{40 + 400} = \frac{2}{11} V$$

i.e., it is about 20% of the incident voltage V. It is for this reason that an overhead line is terminated near a station by connecting the station equipment to the overhead line through a short length of underground cable. Besides the reduction in the magnitude of the voltage wave, the steepness is also reduced because of the capacitance of the cable. This is explained in the next section. The reduction in steepness is very important because this is one of the factors for reducing the voltage distribution along the windings of the equipment. While connecting the overhead line to a station equipment through a cable it is important to note that the length of the cable should not be very short (should not be shorter than the expected length of the wave) otherwise successive reflections at the junction may result in piling up of voltage and the voltage at the junction may reach the incident voltage.

Reflection and Refraction at a T-junction

A voltage wave V is travelling over the line with surge impedance Z_1 as shown in Fig. 12.16. When it reaches the junction, it looks a change in impedance and, therefore, suffers reflection and refraction. Let V_2'', I_2'' and V_3'', I_3'' be the voltages and currents in the lines having surge impedances Z_2 and Z_3 respectively. Since Z_2 and Z_3 form a parallel path as far as the surge wave is concerned, $V_2'' = V_3'' = V''$. Therefore, the following relations hold good:

$$V + V' = V''$$

$$I = \frac{V}{Z_1}, I' = -\frac{V'}{Z_1}$$

$$I_2'' = \frac{V''}{Z_2} \text{ and } I_3'' = \frac{V''}{Z_3}$$

and $$I + I' = I_2'' + I_3'' \qquad (12.13)$$

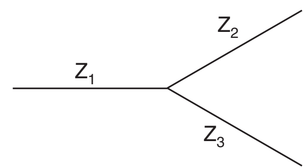

Fig. 12.16 A bifurcated line.

Substituting in equation (12.13) the values of currents

$$\frac{V}{Z_1} - \frac{V'}{Z_1} = \frac{V''}{Z_2} + \frac{V''}{Z_3}$$

Substituting for $V' = V'' - V$,

$$\frac{V}{Z_1} - \frac{V'' - V}{Z_1} = \frac{V''}{Z_2} + \frac{V''}{Z_3}$$

$$\frac{2V}{Z_1} = V''\left[\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}\right]$$

or $$V'' = \frac{2V/Z_1}{\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}} \qquad (12.14)$$

Similarly other quantities can be derived.

Example 12.1: A 3-phase transmission line has conductors 1.5 cms in diameter spaced 1 metre apart in equilateral formation. The resistance and leakance are negligible. Calculate (*i*) the natural impedance of the line, (*ii*) the line current if a voltage wave of 11 kV travels along the line, (*iii*) the rate of energy absorption, the rate of reflection and the state and the form of reflection if the line is terminated through a star connected load of 1000 ohm per phase, (*iv*) the value of the terminating resistance for no reflection and (*v*) the amount of reflected and transmitted power if the line is connected to a cable extension with inductance and capacitance per phase per cm of 0.5×10^{-8} H and 1×10^{-6} μF respectively.

Solution: The inductance per unit length

$$= 2 \times 10^{-7} \ln \frac{d}{r} \text{ H/metre}$$

$$= 2 \times 10^{-7} \ln \frac{100}{0.75}$$

$$= 2 \times 10^{-7} \ln 133.3$$

$$= 2 \times 10^{-7} \times 4.89$$

$$= 9.78 \times 10^{-7} \text{ H/m}$$

The capacitance per phase per unit length

$$= \frac{2\pi\varepsilon}{\ln d/r} \text{ F/metre}$$

$$= \frac{2\pi \times 10^{-9}}{36\pi \ln d/r}$$

$$= \frac{1}{18} \times \frac{10^{-9}}{4.89} = 1.136 \times 10^{-11}$$

$\therefore$ The natural impedance $= \sqrt{\frac{L}{C}}$ ohms

$$= \sqrt{\frac{9.78 \times 10^{-7}}{1.136 \times 10^{-11}}} = 294\ \Omega. \quad \textbf{Ans.}$$

(*ii*) The line current $= \frac{11000}{\sqrt{3} \times 294} = 21.6$ amps. **Ans.**

(*iii*) Since the terminating resistance is of higher value as compared to the value of the surge impedance of the line, the reflection is with a positive sign.

The voltage across the terminating resistance

$$E'' = \frac{2Z_2 E}{Z_1 + Z_2}$$

where Z_1 = line surge impedance, Z_2 = terminating impedance, and E = incident voltage.

$$E'' = 2 \times \frac{11000}{\sqrt{3}} \frac{1000}{1294} = 9.8 \text{ kV}$$

$\therefore$ The rate of power consumption $= \frac{3E''^2}{R}$ MW

$$= \frac{3 \times 9.8 \times 9.8}{1000} \times 1000 \text{ kW}$$

$= 288$ kW. **Ans.**

The reflected voltage $\quad E' = \frac{Z_2 - Z_1}{Z_2 + Z_1} E = \frac{1000 - 294}{1294} \times \frac{11}{\sqrt{3}}$ kV

$$= \frac{706}{1294} \times \frac{11}{\sqrt{3}} = 3.465 \text{ kV}$$

$\therefore$ The rate of reflected energy $= \frac{3 \times 3.465^2}{294} \times 1000$ kW

$= 121.8$ kW. **Ans.**

(*iv*) In order that the incident wave when reaches the terminating resistance, does not suffer reflection, the terminating resistance should be equal to the surge impedance of the line, *i.e.*, 294 ohms. **Ans.**

(*v*) The surge impedance of the cable $= \sqrt{\frac{L}{C}} = \sqrt{\frac{0.5 \times 10^{-8}}{10^{-12}}}$

$= 70.7$ ohm

The refracted voltage $= \frac{2 \times 70.7}{294 + 70.7} \times \frac{11}{\sqrt{3}}$

$$= \frac{2 \times 70.7 \times 11}{\sqrt{3} \times 364.7} = 2.46 \text{ kV}$$

The reflected voltage $= \dfrac{70.7 - 294}{364.7} \times \dfrac{11}{\sqrt{3}}$

$$= \frac{-223.3 \times 11}{\sqrt{3} \times 364.7} = -3.9 \text{ kV}$$

∴ The refracted and reflected powers are respectively.

$$\frac{3 \times 2.46^2}{70.7} \times 1000 = 256 \text{ kW} \quad \text{and} \quad \frac{3 \times 3.9^2}{294} \times 1000 = 155 \text{ kW. } \textbf{Ans.}$$

Example 12.2: A surge of 15 kV magnitude travels along a cable towards its junction with an overhead line. The inductance and capacitance of the cable and overhead line are respectively 0.3 mH, 0.4 μF and 1.5 mH, 0.012 μF per km. Find the voltage rise at the junction due to the surge.

Solution: In this problem the surge travels from the cable towards the overhead line and hence there will be positive voltage reflection at the junction.

The natural impedance of the cable $= \sqrt{\dfrac{0.3 \times 10^{-3}}{0.4 \times 10^{-6}}}$

$$= \sqrt{\frac{3 \times 10^{-4}}{0.4 \times 10^{-6}}} = 27.38$$

The natural impedance of the line $= \sqrt{\dfrac{1.5 \times 10^{-3}}{0.012 \times 10^{-6}}}$

$$= \sqrt{\frac{1.5 \times 10^{-3}}{0.12 \times 10^{-7}}} = 353 \text{ ohms.}$$

The voltage rise at the junction is the voltage transmitted into the overhead line as the voltage is zero before the surge reaches the junction.

$$E'' = \frac{2 \times 353 \times 15}{353 + 27} = \frac{2 \times 353 \times 15}{380} = 27.87 \text{ kV. } \textbf{Ans.}$$

Example 12.3: A surge of 100 kV travelling in a line of natural impedance 600 ohms arrives at a junction with two lines of impedances 800 ohms and 200 ohms respectively. Find the surge voltages and currents transmitted into each branch line.

Solution: The problem deals with a reflection at a T-joint. The various natural impedances are: Z_1 = 600 ohms, Z_2 = 800 ohms, Z_3 = 200 ohms. The surge magnitude is 100 kV.

The surge as it reaches the joint suffers reflection and here the two lines are in parallel; therefore, the transmitted voltage will have the same magnitude and is given by

$$E'' = \frac{2E/Z_1}{\dfrac{1}{Z_1} + \dfrac{1}{Z_2} + \dfrac{1}{Z_3}} = \frac{2 \times 100/600}{\dfrac{1}{600} + \dfrac{1}{800} + \dfrac{1}{200}}$$

$$= \frac{0.333}{(1.67 + 1.25 + 5.0) \times 10^{-3}} = \frac{0.333 \times 10^3}{7.92}$$

$$= \frac{33.3}{7.92} \times 10 = 42.04 \text{ kV.} \quad \textbf{Ans.}$$

The transmitted current in line $Z_2 = \frac{42.04 \times 1000}{800}$ amps = 52.55 amps. **Ans.**

The transmitted current in line $Z_3 = \frac{42.04 \times 1000}{200}$ amps = 210.2 amps. **Ans.**

Line Terminated Through a Capacitance

We consider here that a d.c. surge of infinite length travels over the line of surge impedance Z and is incident on the capacitor as shown in Fig. 12.17. We are interested in finding out the voltage across the capacitor *i.e.*, the refracted voltage. The refracted voltage, using equation (12.9),

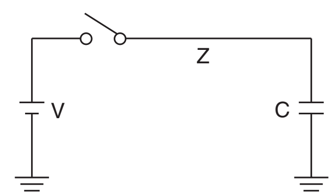

Fig. 12.17 Line terminated through a capacitance.

$$V''(s) = \frac{2.1/Cs}{Z + 1/Cs} \cdot \frac{V}{s} = \frac{2V}{s} \cdot \frac{1}{ZCs + 1}$$

$$= \frac{2V}{s} \cdot \frac{1/ZC}{s + 1/ZC} = 2V \left[\frac{1}{s} - \frac{1}{s + 1/ZC} \right]$$

$$v''(t) = 2V[1 - e^{-t/ZC}] \tag{12.15}$$

The variation of voltage is shown in Fig. 12.18(*b*).

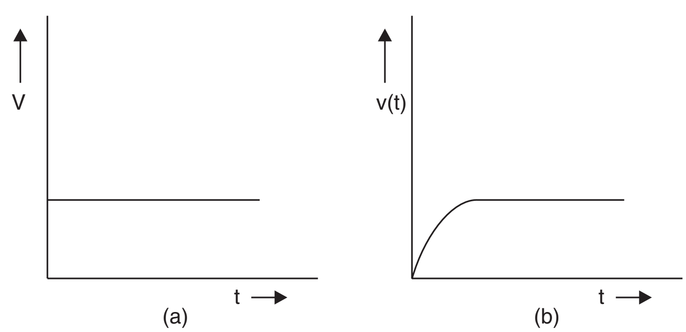

Fig. 12.18 (*a*) Incident voltage and (*b*) Voltage across capacitor.

It is to be noted that since terminating impedance is not a transmission line, therefore, $V''(s)$ is not a travelling wave but it is the voltage across the capacitor C.

Capacitor Connection at a T

The voltage across capacitor is given by the equation

$$V''(s) = \frac{2V/Z_1 s}{\frac{1}{Z_1} + \frac{1}{Z_2} + Cs} = \frac{2VZ_2}{s} \cdot \frac{(1/Z_1 Z_2 C)}{\frac{(Z_1 + Z_2)}{Z_1 Z_2 C} + s}$$

$$= \frac{2V}{sZ_1C} \cdot \frac{1}{s + \frac{Z_1 + Z_2}{Z_1Z_2C}}$$

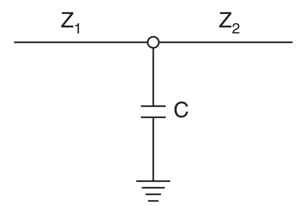

Fig. 12.19 Capacitor connected at *T*.

Let $\frac{Z_1 + Z_2}{Z_1Z_2C} = \alpha$; then

$$V''(s) = \frac{2V}{s} \cdot \frac{1/Z_1C}{s + \alpha}$$

or

$$V''(s) = \frac{2V}{s} \cdot \frac{Z_2}{Z_1 + Z_2} \cdot \frac{(Z_1 + Z_2)/Z_1Z_2C}{(s + \alpha)}$$

$$= \frac{2V}{s} \cdot \frac{Z_2}{Z_1 + Z_2} \cdot \frac{\alpha}{s + \alpha} = \frac{2VZ_2}{Z_1 + Z_2}\left[\frac{1}{s} - \frac{1}{s + \alpha}\right]$$

or

$$v''(t) = \frac{2V \,.\, Z_2}{Z_1 + Z_2}\left[1 - \exp\left(-\frac{Z_1 + Z_2}{Z_1Z_2C}\,t\right)\right] \quad (12.16)$$

The variation of the wave is shown in Fig. 12.20.

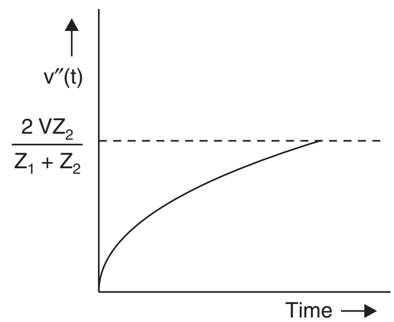

Fig. 12.20 Variation of voltage across the capacitor.

We have assumed in the derivation of the expression for voltage across the capacitor in the previous section that the travelling surge is of infinite length. Let us now derive the expression when the surge is of finite duration say τ (Fig. 12.21). Also, let the magnitude of this wave be V units. The wave could be decomposed into two waves.

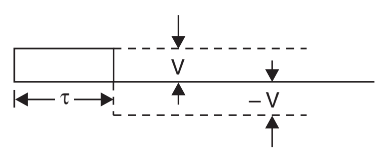

Fig. 12.21 Surge of finite length τ.

Here

$$f(t) = Vu(t) - Vu(t - \tau)$$

$$Vu(t - \tau) = V \text{ for } t \geq \tau$$

$$= 0 \text{ for } t < \tau$$

With this, voltage across the capacitor is given by

$$V''(s) = \mathcal{L}\{f(t)\} \,.\, \frac{2/Z_1}{\frac{1}{Z_1} + \frac{1}{Z_2} + Cs} = \frac{2V/Z_1s}{\frac{1}{Z_1} + \frac{1}{Z_2} + Cs} - \frac{(2V/Z_1s)\,.\,e^{-\tau s}}{\frac{1}{Z_1} + \frac{1}{Z_2} + Cs}$$

$$v''(t) = 2V \cdot \frac{Z_2}{Z_1+Z_2}\left[1-\exp\left(-\frac{Z_1+Z_2}{Z_1Z_2C}\,t\right)\right]$$
$$-\frac{2VZ_2}{Z_1+Z_2}\left[1-\exp\left\{-\frac{Z_1+Z_2}{Z_1Z_2C}\,(t-\tau)\right\}\right]$$

The variation of voltage is shown in Fig. 12.22.

Thus for time $0 < t < \tau$ only the first term in the expression is active and for $t \geq \tau$ both the terms are active. The rise in voltage is maximum at $t = \tau$ when the value will be

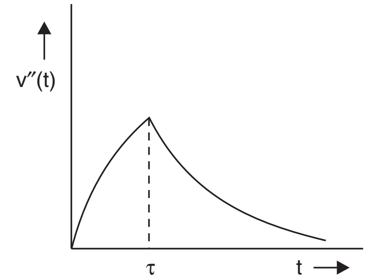

Fig. 12.22 Variation of voltage across the capacitor with finite duration incident surge.

$$v''(t) = \frac{2VZ_2}{Z_1+Z_2}\left[1-\exp\left(-\frac{Z_1+Z_2}{Z_1Z_2C}\,\tau\right)\right]-\frac{2VZ_2}{Z_1+Z_2}\;[1-e^0]$$
$$= \frac{2VZ_2}{Z_1+Z_2}\left[1-\exp\left(-\frac{Z_1+Z_2}{Z_1Z_2C}\,\tau\right)\right] \qquad (12.17)$$

It is, therefore, clear that the attenuation in the magnitude of voltage for a short wave is much more rapid than for long wave.

We have seen that the effect of a shunt capacitor is to reduce the steepness and magnitude of the wave reaching an equipment. Since an inductor is dual to a capacitor, an inductor in series of the lines should give the same effect.

Example 12.4: A 500 kV 2 μ sec rectangular surge on a line having a surge impedance of 350 ohms approaches a station at which the concentrated earth capacitance is 3000 pF. Determine the maximum value of the transmitted wave.

Solution: The diagram corresponding to the problem is as follows:

The maximum value of voltage will be

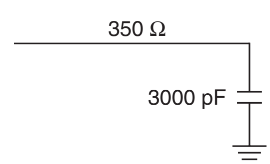

Fig. E.12.4

$$E'' = 2E\left[1-\exp\left(-\frac{\tau}{ZC}\right)\right]$$
$$= 2 \times 500\left[1-\exp\left(-\frac{2\times10^{-6}\times10^{12}}{350\times3000}\right)\right]$$
$$= 2 \times 500\left[1-\exp\left(-\frac{2\times10^{3}}{350\times3}\right)\right]$$
$$= 2 \times 500[1-e^{-1.9}]$$
$$= 2 \times 500[1-0.15]$$
$$= 850 \text{ kV.} \quad \textbf{Ans.}$$

Example 12.5: An inductance of 800 μH connects two sections of a transmission line each having a surge impedance of 350 ohms. A 500 kV 2 μs rectangular surge travels along the line towards the inductance. Determine the maximum value of the transmitted wave.

Solution: The maximum value of the transmitted surge is given by

$$E'' = E\left[1 - \exp\left(-\frac{2Z}{L}\tau\right)\right]$$

$$= 500\left[1 - \exp\left(-\frac{2 \times 350}{800} \times 2\right)\right]$$

$$= 500[1 - e^{-0.875 \times 2}]$$

$$= 500[1 - e^{-1.750}]$$

$$= 500[1 - 0.173]$$

$$= 413.5 \text{ kV.} \quad \textbf{Ans.}$$

12.5 ATTENUATION OF TRAVELLING WAVES

Let R, L, C and G be the resistance, inductance, capacitance and conductance respectively per unit length of a line (Fig. 12.23). Let the value of voltage and current waves at $x = 0$ be V_0 and I_0. Our objective is to find the values of voltage and current waves when they have travelled through a distance of x units over the overhead line. Let the time taken be t units when voltage and current waves are V and I respectively. To travel a distance of dx, let the time taken be dt. The equivalent circuit for the differential length dx of the line is shown in Fig. 12.24.

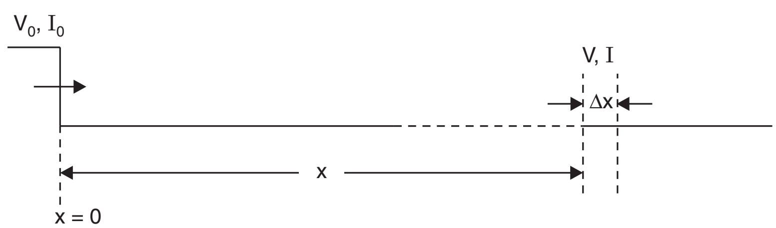

Fig. 12.23 Travelling wave on a lossy line.

The power loss in the differential element is

$$dp = I^2R\,dx + V^2G\,dx \qquad (12.18)$$

Also power at a distance $x \cdot VI = p = I^2Z_n$

Differential power, $\quad dp = -\,2IZ_n dI \qquad (12.19)$

where Z_n is the natural impedance of the line. Here negative sign has been assigned as there is reduction in power as the wave travels with time.

Equating the equations (12.18) and (12.19),

$$-\,2IZ_n dI = I^2R\,dx + V^2G\,dx$$

$$= I^2R\,dx + I^2Z_n^2G\,dx$$

or

$$dI = -\frac{I(R + GZ_n^2)}{2Z_n}\,dx$$

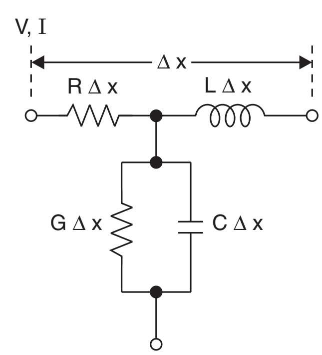

Fig. 12.24 Differential element of transmission line.

or
$$\frac{dI}{I} = -\frac{(R + GZ_n^2)}{2Z_n}\,dx$$

or
$$\ln I = -\left(\frac{R + GZ_n^2}{2Z_n}\right)x + A$$

At $x = 0$, $I = I_0$, $\therefore \quad A = \ln I_0$.

or
$$\ln \frac{I}{I_0} = -\frac{R + GZ_n^2}{2Z_n}\,x = -\,ax$$

where $a = \dfrac{R + GZ_n^2}{2Z_n}$.

$$\therefore \qquad I = I_0 e^{-ax}. \qquad (12.20)$$

Similarly it can be proved that $V = V_0 e^{-ax}$. This shows that the current and voltage waves get attenuated exponentially as they travel over the line and the magnitude of attenuation depends upon the parameters of the line. Since the value of resistance depends not only on the size of the conductors but also on the shape and length of the waves. An empirical relation due to Foust and Menger takes into account the shape and length of the wave for calculating the voltage and current at any point on the line after it has travelled through a distance x units and is given as

$$V = \frac{V_0}{1 + KxV_0} \qquad (12.21)$$

where x is in kms, V and V_0 are in kV and K is the attenuation constant, of value

$K = 0.00037$ for chopped waves
$= 0.00019$ of short-waves
$= 0.0001$ for long-waves.

Example 12.6: A travelling wave of 50 kV enters an overhead line of surge impedance 400 ohms and conductor resistance 6 ohm per km. Determine (*i*) the value of the voltage wave when it has travelled through a distance of 50 km, and (*ii*) the power loss and the heat loss of the wave during the time required to traverse this distance. Neglect the losses in the insulation and assume a wave velocity of 3×10^5 km per second. Determine the corresponding values for a cable having surge impedance of 40 ohms and relative permittivity 4.

Solution: (*i*) Since the line has some specific resistance, the wave as it travels gets attenuated in magnitude.

The magnitude of the wave is given by

$$e = e_0 \varepsilon^{-1/2(R/Z + GZ)x}$$

where e = the value of voltage when travelled through a distance of x kilometres, R, G the resistance and leakance per kilometre length of the line and Z is the surge impedance, e_0 = initial magnitude of the surge voltage, ε the Naperian base.

Here in this problem $e_0 = 50$ kV, $x = 50$ km, $R = 6$ ohm and $Z = 400$ ohm and $G = 0.0$ mhos.

Substituting these values,

$$e = 50\varepsilon^{-1/2}\left(\frac{6}{400}\times 50\right) = 50 \times \varepsilon^{-0.375} = 50 \times 0.69 = 34.5 \text{ kV}$$

(*ii*) The power loss is the instantaneous quantity and is required to be calculated when the wave travels the distance of 50 km where the voltage magnitude is 34.5 kV.

$$\text{The power loss} = \frac{34.5 \times 34.5}{400} \times 1000 \text{ kW} = 2975 \text{ kW}$$

The heat loss is the integrated value of power over the distance (or time) the wave has travelled.

$$\text{Heat loss} = \int_0^t ei\, dt$$

Now $e = e_0\varepsilon^{-1/2}\ \dfrac{Rx}{Z}$ and similarly, $i = i_0\varepsilon^{-1/2}\ \dfrac{R}{Z}x$. Now,

$$x = vt$$

$$\therefore \qquad e = e_0 \,.\, \varepsilon^{-1/2}\ \frac{R}{Z}vt \text{ and } i = i_0\varepsilon^{-1/2}\frac{R}{Z}vt$$

Substituting these values, we get

$$\text{Heat loss} = \int_0^t e_0 i_0\, \varepsilon^{-(R/Z)vt}\, dt$$

where v = the velocity of the wave

$$t = \frac{x}{v} = \frac{50}{3\times 10^5} = 16.67 \times 10^{-5} \text{ sec}$$

and

$$i_0 = \frac{e_0}{Z} = \frac{50\times 1000}{4000} = 125 \text{ amps.}$$

$$\therefore \qquad \text{Heat loss} = -\int_0^{16.67\times 10^{-5}} 50\times 125\varepsilon^{-(R/Z)vt}\, dt$$

$$= -\,50 \times 125 \times \frac{400}{6\times 3\times 10^5}\ [e^{-0.75} - 1]$$

$$= 0.736 \text{ kJ} \quad \text{or} \quad 176 \text{ cal.}$$ **Ans.**

12.6 CAPACITANCE SWITCHING

The switching of a capacitance such as disconnecting a line or a cable or a bank of capacitor poses serious problems in power systems in terms of abnormally high voltages across the circuit breaker contacts. Under this situation the current leads the voltage by about 90°. Assuming that the current interruption takes place when it is passing through zero value the capacitor will be charged to maximum voltage. Since the capacitor is now isolated from the source, it retains its charge as shown in Fig. 12.25 (*c*) and because of trapping of this charge, half a cycle after the current zero the voltage across the circuit breaker contact is 2*V* which may prove to be dangerous and may result in the circuit breaker restrike. This is equivalent to closing the switch suddenly which will result into oscillations in the circuit at the natural frequency

$$f = \frac{1}{2\pi\sqrt{LC}}$$

The circuit condition corresponds to Fig. 12.6. The only difference between the two circuits is that whereas in Fig. 12.25 the capacitor is charged to a voltage V, in Fig. 12.6 it is assumed to be without charge. Therefore, the voltage across the capacitor reaches $3V$. Since the source voltage is V, the voltage across the breaker contacts after another half cycle will be $4V$ which may cause another restrike. This phenomenon may theoretically continue indefinitely, increasing the voltage by successive increments of $2V$. This may result into an external flashover or the failure of the capacitor. This is due to the inability of the circuit breaker to provide sufficient dielectric strength to the contacts to avoid restrikes after they are opened first.

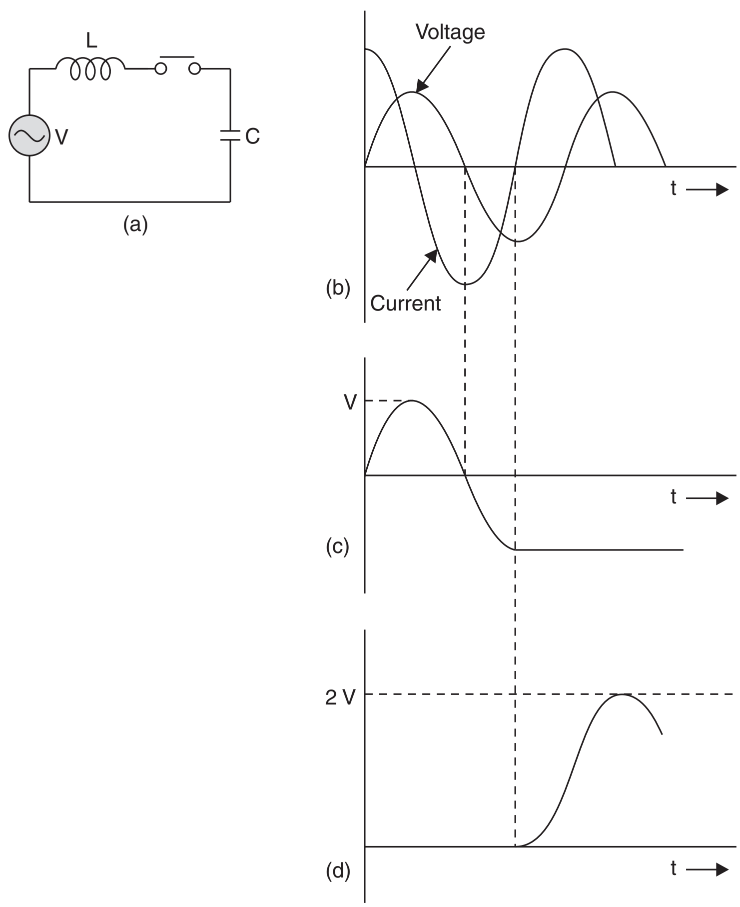

Fig. 12.25 (*a*) Equivalent circuit for capacitor switching; (*b*) System voltage and current; (*c*) Capacitor voltage; (*d*) Voltage across the switch.

This problem is practically solved by using air blast circuit breakers or multibreak breakers.

12.7 OVERVOLTAGE DUE TO ARCING GROUND

Figure 12.26 shows a 3-phase system with isolated neutral. The shunt capacitances are also shown. Under balanced conditions and complete transposed transmission lines, the potential of the neutral is near the ground potential and the currents in various phases through the shunt capacitors are leading their corresponding voltages by 90°. They are displaced from each other by 120° so that the net sum of the three currents is zero (Fig. 12.26(*b*)). Say there is line-to-ground fault on one of the three phases (say phase '*c*'). The voltage across the shunt capacitor of that phase reduces to zero whereas those of the healthy phases become line-to-line voltages and now they are displaced by 60° rather than 120°. The net charging current now is three times the phase current under balanced conditions (Fig. 12.26(*c*)). These currents flow through the fault and the windings of the alternator. The magnitude of this current is often sufficient to sustain an arc and, therefore, we have an arcing ground. This could be due to a flashover of a support insulator. Here this flashover acts as a switch. If the arc extinguishes when the current is passing through zero value, the capacitors in phases *a* and *b* are charged to line voltages. The voltage across the line and the grounded points of the post insulator will be the superposition of the capacitor voltage and the generator voltage and this voltage may be good enough to cause flashover which is equivalent to restrike in a circuit breaker. Because of the presence of the inductance of the generator winding, the capacitances will form an oscillatory circuit and these oscillations may build up to still higher voltages and the arc may reignite causing further transient disturbances which may finally lead to complete rupture of the post insulators.

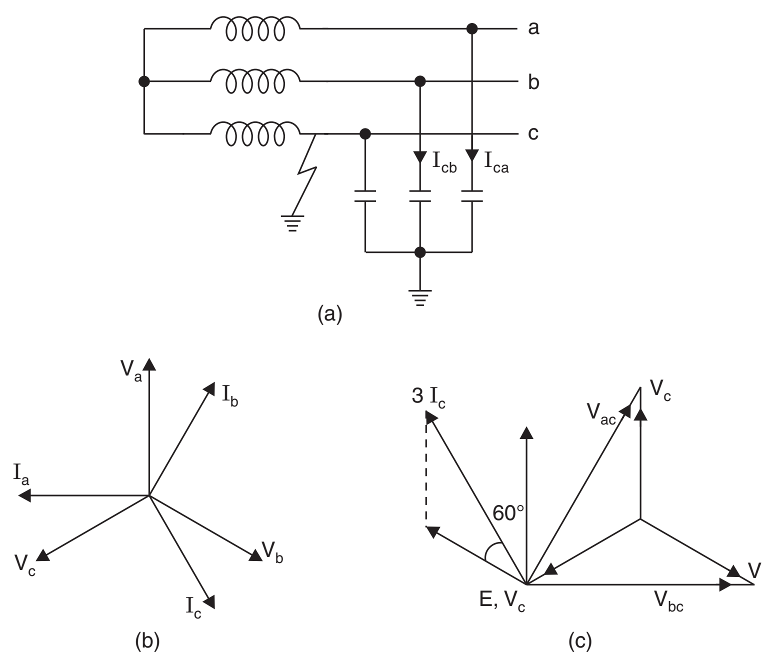

Fig. 12.26 (*a*) 3-phase system with isolated neutral; (*b*) Phasor diagram under healthy condition; (*c*) Phasor diagram under faulted condition.

12.8 LIGHTNING PHENOMENON

Lightning has been a source of wonder to mankind for thousands of years. Schonland points out that any real scientific search for the first time was made into the phenomenon of lightning by Franklin in 18th century.

Before going into the various theories explaining the charge formation in a thunder cloud and the mechanism of lightning, it is desirable to review some of the accepted facts concerning the thunder cloud and the associated phenomenon.

1. The height of the cloud base above the surrounding ground level may vary from 500 to 30,000 ft. The charged centres which are responsible for lightning are in the range of 1000 to 5000 ft.

2. The maximum charge on a cloud is of the order of 10 coulombs which is built up exponentially over a period of perhaps many seconds or even minutes.

3. The maximum potential of a cloud lies approximately within the range of 10 MV to 100 MV.

4. The energy in a lightning stroke may be of the order of 250 kWhr.

5. Raindrops:

(*a*) Raindrops elongate and become unstable under an electric field, the limiting diameter being 0.3 cm in a field of 10 kV/cm.

(*b*) A free falling raindrop attains a constant velocity with respect to the air depending upon its size. This velocity is 800 cm/sec for drops of the size 0.25 cm dia. and is zero for spray. This means that in case the air currents are moving upwards with a velocity greater than 800 cm/sec, no rain drop can fall.

(*c*) Falling raindrops greater than 0.5 cm in dia become unstable and break up into smaller drops.

(*d*) When a drop is broken up by air currents, the water particles become positively charged and the air negatively charged.

(*e*) When an ice crystal strikes with air currents, the ice crystal is negatively charged and the air positively charged.

Wilson's Theory of Charge Separation

Wilson's theory is based on the assumption that a large number of ions are present in the atmosphere. Many of these ions attach themselves to small dust particles and water particles. It also assumes that an electric field exists in the earth's atmosphere during fair weather which is directed downwards towards the earth (Fig. 12.27(*a*)). The intensity of the field is approximately 1 volt/cm at the surface of the earth and decreases gradually with height so that at 30,000 ft it is only about 0.02 V/cm. A relatively large raindrop (0.1 cm radius) falling in this field becomes polarized, the upper side acquires a negative charge and the lower side a positive charge. Subsequently, the lower part of the drop attracts –ve charges from the atmosphere which are available in abundance in the atmosphere leaving a preponderance of positive charges in the air. The upwards motion of air currents tends to carry up the top of the cloud, the +ve air and smaller drops that the wind can blow against gravity. Meanwhile the falling heavier raindrops which are negatively charged settle on the base of the cloud. It is to be noted that the selective action of capturing –ve charges from the atmosphere by the lower surface of the drop is possible. No such selective action occurs at the upper surface. Thus in the original system, both the positive and negative charges which were mixed up, producing essentially a neutral space charge, are now separated. Thus according to Wilson's theory since larger negatively charged drops settle on the base of the cloud and smaller positively charged drops settle on the uper positions of the cloud, the lower base of the cloud is negatively charged and the upper region is positively charged (Fig. 12.27(*b*)).

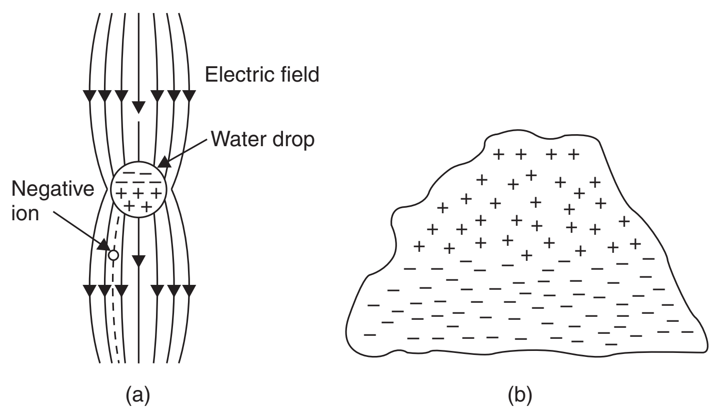

Fig. 12.27 (*a*) Capture of negative ions by large falling drop; (*b*) Charge separation in a thunder cloud according to Wilson's theory.

Simpson's and Scarse Theory

Simpson's theory is based on the temperature variations in the various regions of the cloud. When water droplets are broken due to air currents, water droplets acquire positive charges whereas the air is negatively charged. Also when ice crystals strike with air, the air is positively charged and the crystals are negatively charged. The theory is explained with the help of Fig. 12.28.

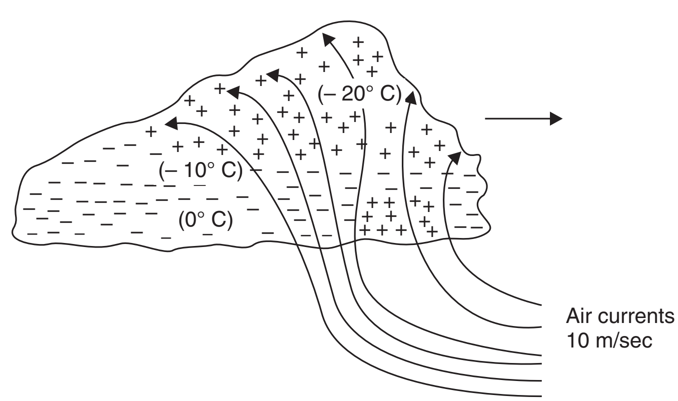

Fig. 12.28 Charge generation and separation in a thunder cloud according to Simpson's theory.

Let the cloud move in the direction from left to right as shown by the arrow. The air currents are also shown in the diagram. If the velocity of the air currents is about 10 m/sec in the base of the cloud, these air currents when collide with the water particles in the base of the cloud, the water drops are broken and carried upwards unless they combine together and fall down in a pocket as shown by a pocket of positive charges (right to bottom region in Fig. 12.28). With the collision of water particles we know the air is negatively charged and the water particles positively charged. These negative charges in the air are immediately absorbed by the cloud particles which are carried away upwards with the air currents. The air currents go still higher in the cloud where the moisture freezes into ice crystals. The air currents when collide with ice crystals the air is positively charged and it goes in the upper region of cloud whereas the negatively charged ice crystals drift gently down in the lower region of the cloud. This is how the charge is separated in a thundercloud. Once the charge separation is complete, the conditions are now set for a lightning stroke.

Mechanism of Lightning Stroke

Lightning phenomenon is the discharge of the cloud to the ground. The cloud and the ground form two plates of a gigantic capacitor and the dielectric medium is air. Since the lower part of the cloud is negatively charged, the earth is positively charged by induction. Lightning discharge will require the puncture of the air between the cloud and the earth. For breakdown of air at STP condition the electric field required is 30 kV/cm peak. But in a cloud where the moisture content in the air is large and also because of the high altitude (lower pressure) it is seen that for breakdown of air the electric field required is only 10 kV/cm. The mechanism of lightning discharge is best explained with the help of Fig. 12.29.

After a gradient of approximately 10 kV/cm is set up in the cloud, the air surrounding gets ionized. At this a streamer (Fig. 12.29(*a*)) starts from the cloud towards the earth which cannot be detected with the naked eye; only a spot travelling is detected. The current in the streamer is of the order of 100 amperes and the speed of the streamer is 0.5 ft/μ sec. This streamer is known as pilot streamer because this leads to the lightning phenomenon. Depending upon the state of ionization of the air surrounding the streamer, it is branched to several paths and this is known as stepped leader (Fig. 12.29(*b*)). The leader steps are of the order of 50 m in length and are accomplished in about a microsecond. The charge is brought from the cloud through the already ionized path to these pauses. The air surrounding these pauses is again ionized and the leader in this way reaches the earth (Fig. 12.29(*c*)).

Once the stepped leader has made contact with the earth it is believed that a power return stroke (Fig. 12.29(*c*)) moves very fast up towards the cloud through the already ionized path by the leader. This streamer is very intense where the current varies between 1000 amps and 200,000 amps and the speed is about 10% that of light. It is here where the –ve charge of the cloud is being neutralized by the positive induced charge on the earth (Fig. 12.29(*d*)). It is this instant which gives rise to lightning flash which we observe with our naked eye. There may be another cell of charges in the cloud near the neutralized charged cell. This charged cell will try to neutralize through this ionised path. This streamer is known as dart leader Fig. 12.29(*e*). The velocity of the dart leader is about 3% of the velocity of light. The effect of the dart leader is much more severe than that of the return stroke.

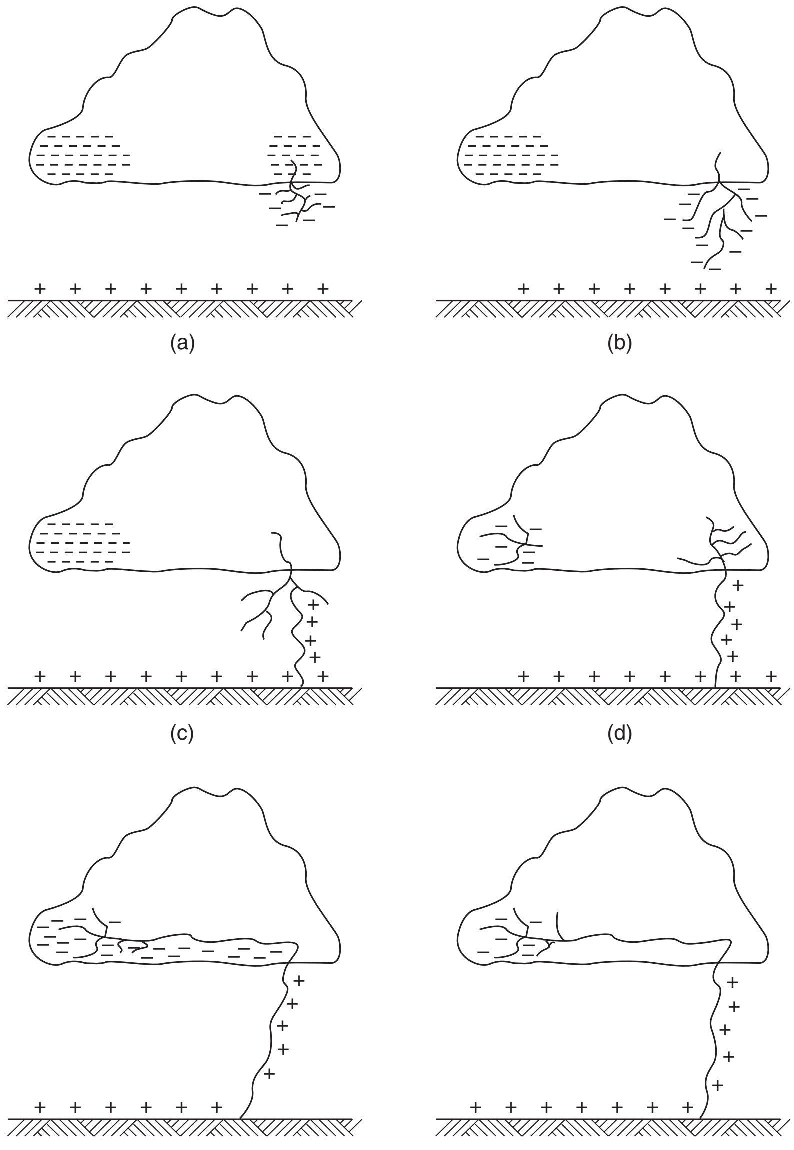

Fig. 12.29 Lightning mechanism

The discharge current in the return streamer is relatively very large but as it lasts only for a few microseconds the energy contained in the streamer is small and hence this streamer is known as cold lightning stroke whereas the dart leader is known as hot lightning stroke because even though the current in this leader is relatively smaller but it lasts for some milliseconds and therefore the energy contained in this leader is relatively larger.

It is found that each thunder cloud may contain as many as 40 charged cells and a heavy lightning stroke may occur. This is known as multiple stroke.

12.9 LINE DESIGN BASED ON LIGHTNING

The severity of switching surges for voltage 400 kV and above is much more than that due to lightning voltages. All the same it is desired to protect the transmission lines against direct lightning strokes. The object of good line design is to reduce the number of outages caused by lightning. To achieve this the following actions are required:

(*i*) The incidence of stroke on to power conductor should be minimised.

(*ii*) The effect of those strokes which are incident on the system should be minimized.

To achieve (*i*) we know that, lightning normally falls on tall objects; thus tall towers are more vulnerable to lightning than the smaller towers. In order to keep smaller tower height for a particular ground clearance, the span lengths will decrease which requires more number of towers and hence the associated accessories like insulators etc. The cost will go up very high. Therefore, a compromise has to be made so that adequate clearance is provided, at the same time keeping longer span and hence lesser number of towers.

With a particular number of towers the chances of incidence of lightning on power conductors can be minimized by placing a ground wire at the top of the tower structure. Refer to article 16.3 for ground wires.

Once the stroke is incident on the ground wire, the lightning current propagates in both the directions along the ground wire. The tower presents a discontinuity to the travelling waves; therefore they suffer reflections and refraction. The system is, then, equivalent to a line bifurcated at the tower point.

We know that, the voltage and current transmitted into the tower will depend upon the surge impedance of the tower and the ground impedance (tower footing resistance) of the tower. If it is low, the wave reflected back up the tower will largely remove the potential existing due to the incident wave. In this way the chance of flash over is eliminated. If, on the other hand, the incident wave encounters a high ground impedance, positive reflection will take place and the potential on the top of the tower structure will be raised rather than lowered. It is, therefore, desired that for good line design high surge impedances in the ground wire circuits, the tower structures and the tower footing should be avoided. Various methods for lowering the tower footing resistances have been discussed in article 16.3.

PROBLEMS

12.1. Given an RL circuit with a sudden 50 Hz sinusoidal voltage applied where R = 20 ohms, L = 0.36 H and voltage V = 220 V.

(*a*) The switch is closed at such a time as to permit maximum transient current. What is the instantaneous value of V upon closing the switch ?

(*b*) What is the maximum value of current in part (*a*) ?

(*c*) Let the switch be closed so as to yield minimum transient current. What instantaneous values of V and α correspond to this instant of closing the switch ?

12.2. Determine the relative attenuation occurring in two cycles in the over voltage surge set up on a 132 kV cable fed through an air blast breaker when the breaker opens on a system short circuit.