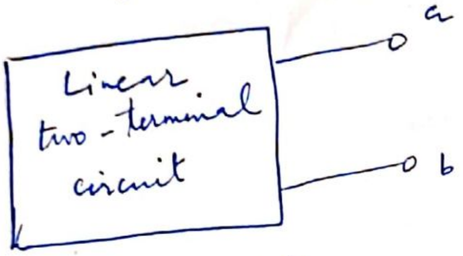
L-9.

Norton's Theorm

Statement: The theorm states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a current source (I_N) in parallel with a resistor (R_N), where I_N is the short circuit current through the terminals and R_N is the input or equivalent resistance at the terminals when the independent sources are turned off.



(a) original ckt

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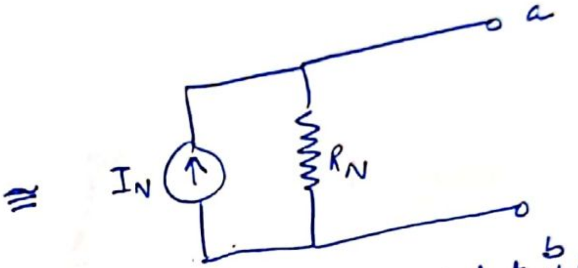


(b) Norton's equivalent ckt.

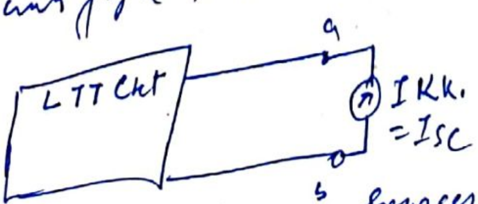
How to get I_N & R_N ?

R_N is found in the same way as R_{th}. They are infact equal.

$\therefore \boxed{R_N = R_{th}}$

To find I_N; we short ckt the two terminals and find the current through this short ckted element flowing from a to b. The short ckt current from a to b for both circuit fig.(a) & (b) is I_N.

$\therefore \boxed{I_{sc} = I_N}$



However dependent ~~XI~~'s sources are to be kept intact as were treated a. sypna.

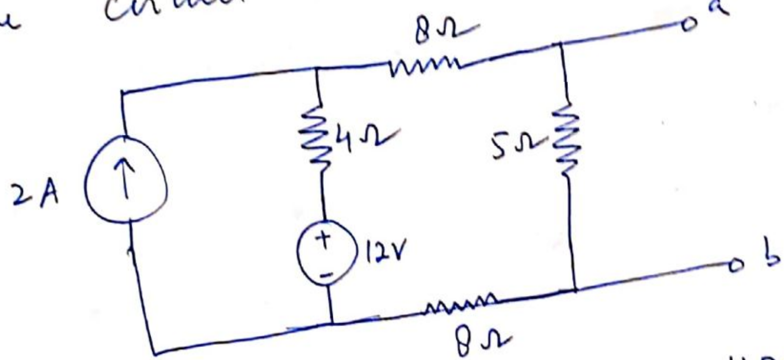
Also while observing close relationship b/w Thevenin's & Norton theorems, we have,

$$R_N = R_{th};$$

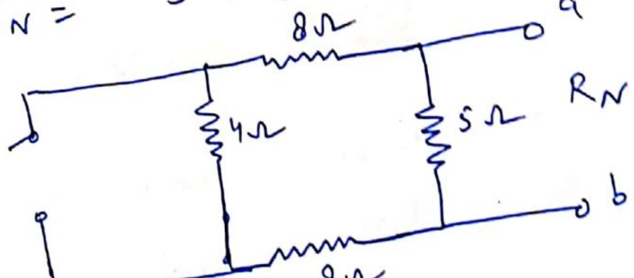
$$\boxed{I_N = \frac{V_{th}}{R_{th}}}$$

This is essentially the source transformation. This source transformation is also called Thevenin-Norton Transformation

Example:- Find the Norton equivalent circuit of the circuit shown below at terminals a-b.



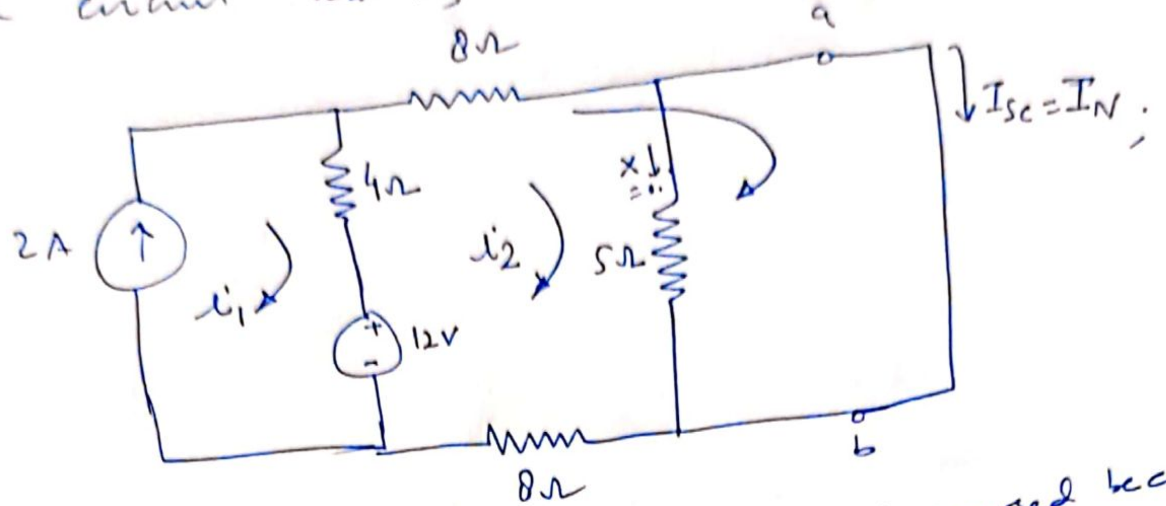
Sol./ $R_N = 5 \parallel (4+8+8) = 5 \parallel 20 = \frac{5\times 20}{25} = 4\Omega;$



$R_N = 4\,\Omega.$

Next is to find short ckt current I_N (I_{sc}).

The circuit becomes;



$\boxed{i_1 = 2A}$; The resistor 5Ω is ignored because its two terminals are shortcircuited, There no current flows through it.

Mesh 2.

$$-4 i_1 + (8 + 8 + 4) i_2 = 12$$

$$20 i_2 - 4 i_1 = 12$$

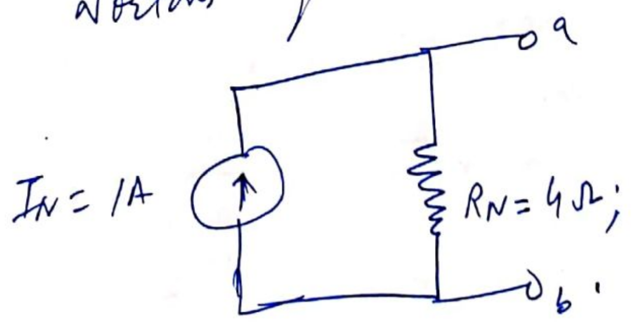
$$20 i_2 - 4 \times 2 = 12$$

$$20 i_2 = 20$$

$$\boxed{i_2 = 1A}$$

Thus short ckt current $I_N = I_{SC} = i_2 = 1A$;

Thus Nortons equivalent ckt will look like as.



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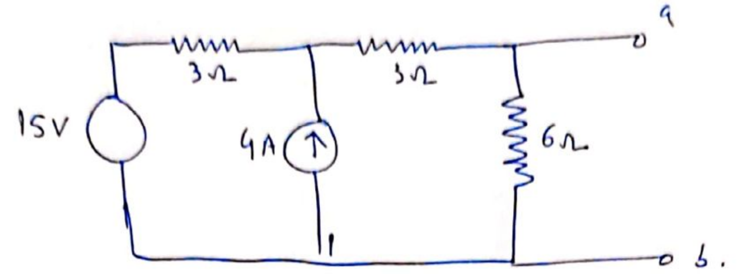
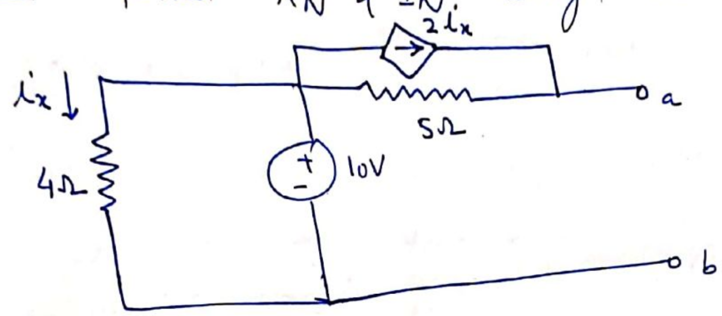
Exp Find the Norton equivalent circuit for the figure shown below across a & b terminals.



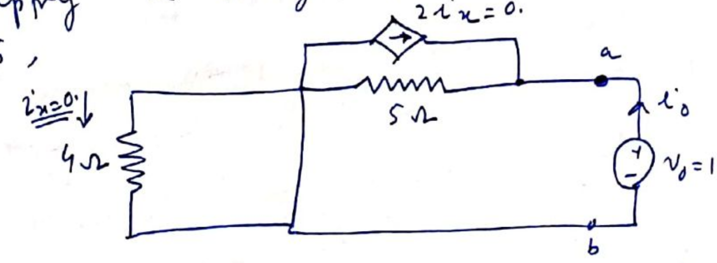
$\boxed{R_N = 3\,\Omega \quad I_N = 4.5\,A}$

Expl Find R_N & I_N using Nortons theorem.



R_N:

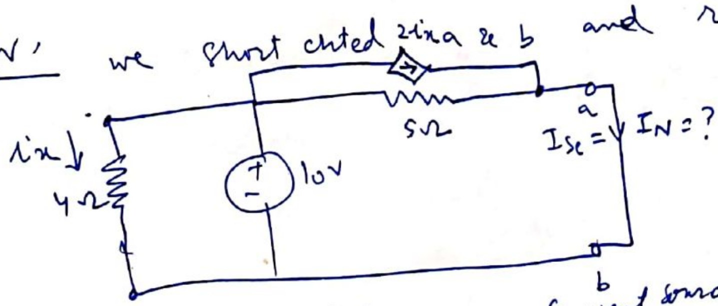
Sol. To find R_N we short ckt 10V source and let us apply a voltage V_0 across ab (=1V), the ckt as we get is,



4Ω is short ckted, thus ignore it; $i_0 = \frac{1}{5} = 0.2\,A$,

$\therefore R_N = \frac{V_0}{i_0} = \frac{1}{0.2} = 5\,\Omega.$

I_N: we short ckted a & b and redraw the ckt as;

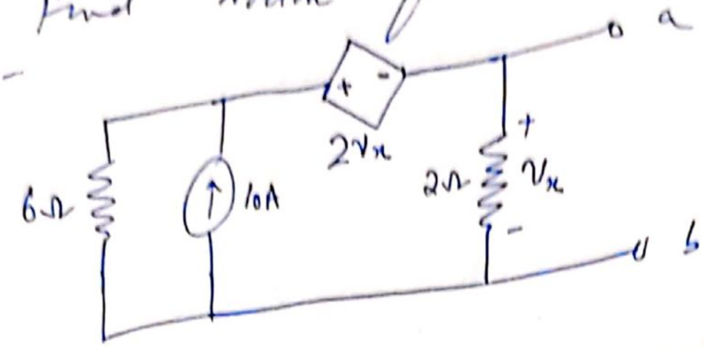


$i_x = \frac{10}{4} = 2.5\,A$; $\therefore$ Current source current $= 2i_x = 2\times2.5 = 5A$.

KCL at a; $5A + \frac{10}{5} - I_N = 0$;

$I_N = 2+5 = 7A.$

Example :- Find Norton equivalent ckt of the following circuit :-



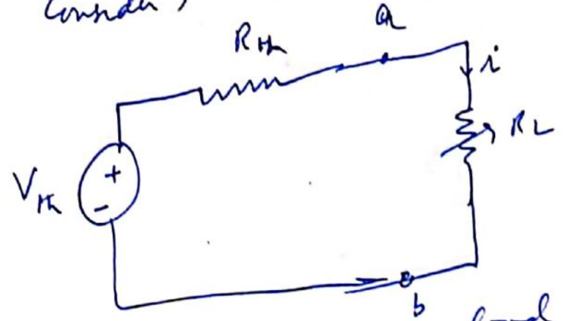
$R_N = 1\Omega$
$I_N = 10A$.

Maximum Power Transfer Theorem

We know, a circuit is designed to provide power to a load. We have applications where internal resistance comes on the way of transferring max. power to load. These internal losses are very significant in ~~deter~~ delivering power to a load.

The Thevenin's equivalent is useful in finding the maximum power a linear circuit can deliver to a load. We assume that load resistance R_L can be adjusted;

Consider, Thevenin's equivalent circuit.



The power delivered to load $P_L = i^2 R_L$;

where $i = \left(\frac{V_{Th}}{R_{Th}+R_L}\right)$;

$\therefore$ load power $P_L = \dfrac{V_{th}^2}{(R_{th}+R_L)^2} \cdot R_L$

$$P_L = \frac{V_{th}^2 R_L}{(R_{th}+R_L)^2}$$

Thus the load power varies as R_L. In order to find max. power, we diff. P_L w.r.t. R_L and equal result $=0$;

$$\frac{dP_L}{dR_L} = V_{th}^2 \left[\frac{(R_{th}+R_L)^2 \cdot 1 - 2(R_{th}+R_L) \cdot R_L}{(R_{th}+R_L)^2} \right] = 0;$$

$$\frac{V_{th}^2}{(R_{th}+R_L)^2}\left[R_{th}^2 + R_L^2 + \cancel{2R_{th}R_L} - \cancel{2R_L R_{th}} - 2R_L^2\right] = 0.$$

we get $R_{th}^2 - R_L^2 = 0. \Rightarrow \boxed{R_{th} = R_L}$

Thus the value of load resistance R_L which gives maximum power across R_L is that $\boxed{R_L = R_{th}}$.

now let us find max. power;

$$P_{max} = \frac{(V_{th})^2 \cdot R_{th}}{(R_{th}+R_{th})^2} = \frac{V_{th}^2 \cdot R_{th}}{4R_{th}^2}$$

$$\boxed{P_{max} = \frac{V_{th}^2}{4R_{th}}} \quad \text{or} \quad \boxed{P_{max} = \frac{V_{th}^2}{4R_L}}$$

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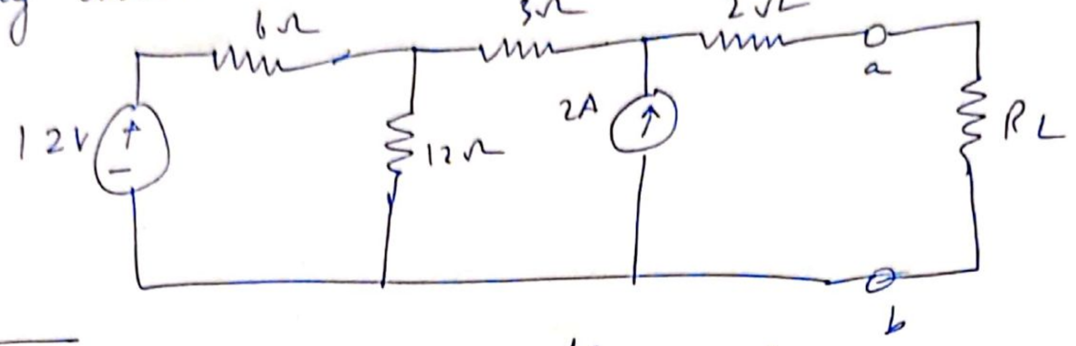
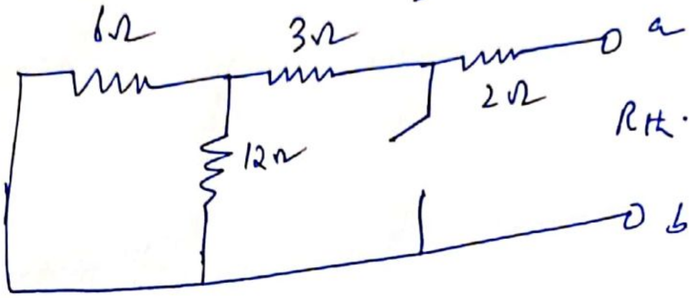
Exple Find the value of R_L for maximum power in the following circuit. Find also maximum power.



R_{th}



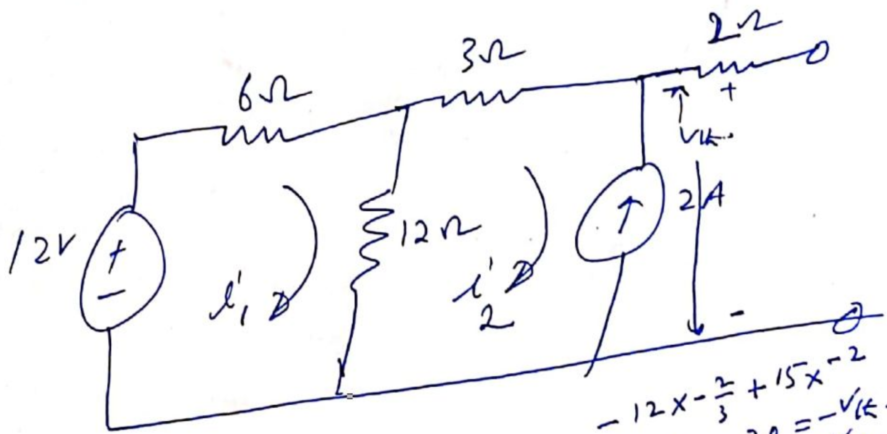
$R_{th} = 2+3+6 \| 12$

$= 5 + \frac{6 \times 12}{18}$

$= 5+4$

$\boxed{R_{th} = 9\Omega}$

V_{th}



$-12 \times -\frac{2}{3} + 15 \times -2 = -V_{th}$.

$8 - 30 = -V_{th}$.

$-22 = -V_{th}$.

$i'_2 = -2A$;

Mesh 1

$18 i'_1 - 12 i'_2 = 12$

$18 i'_1 - 12(-2) = 12$

$18 i'_1 + 24 = 12$

$18 i'_1 = -12$

$i'_1 = \frac{-12}{18} = -\frac{2}{3} A$.

Mesh 2

$-12 i'_1 + 15 i'_2 = -V_{th}$;

$12 - 6 i'_1 - 3 i'_2 - V_{th} = 0$.

$V_{th} = 12 - 6\left(\frac{-2}{3}\right) - 3(-2)$

$= 12 + 4 + 6$

$\boxed{V_{th} = 22V}$;

for max power $R_L = R_{th} = 9\Omega$;

Max. power $P_{max} = \frac{V_{th}^2}{4R_L} = \frac{(22)^2}{4 \times 9} = 13.44W$.