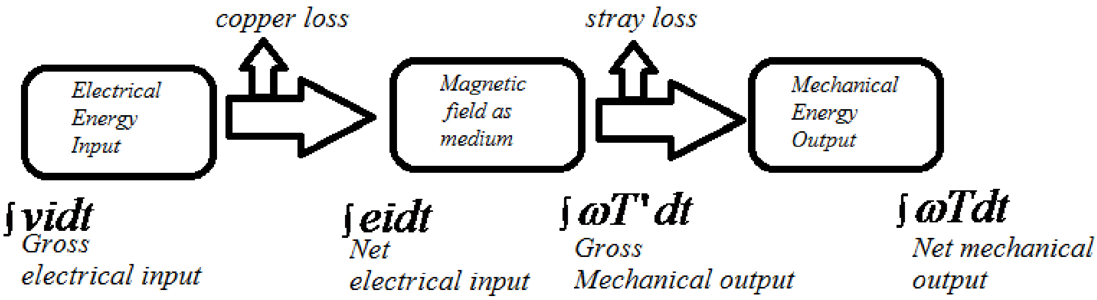
Principles of electro mechanical energy conversion :

Energy neither be created nor destroyed but can be converted from one form to the other form.

Here in the rotating electrical machines the energy conversion is done from either ***mechanical to electrical or electrical to mechanical***

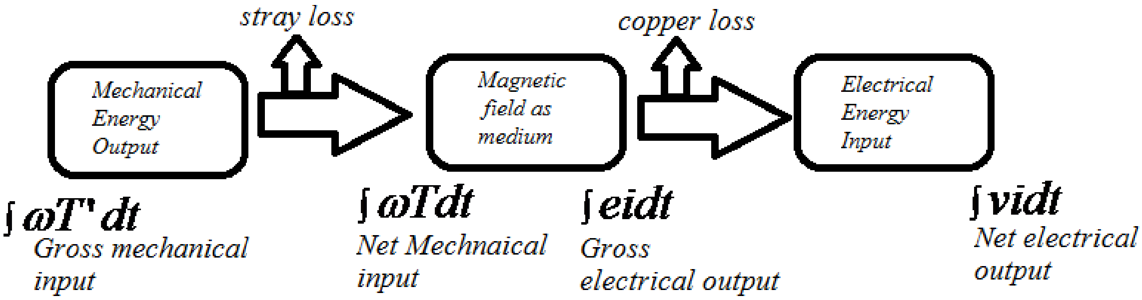
For this energy conversion process the medium is taken as ***magnetic field medium*** since it stores the much more amount of energy when compared to electric field

Energy flow diagram in dc motor:



$dWe = dWm + dWf$ …… motor

Energy flow diagram in DC Generator:



$dWm = dWe + dWf$ …… Generator

Singly excited system:

Consider a singly excited linear actuator as shown below.

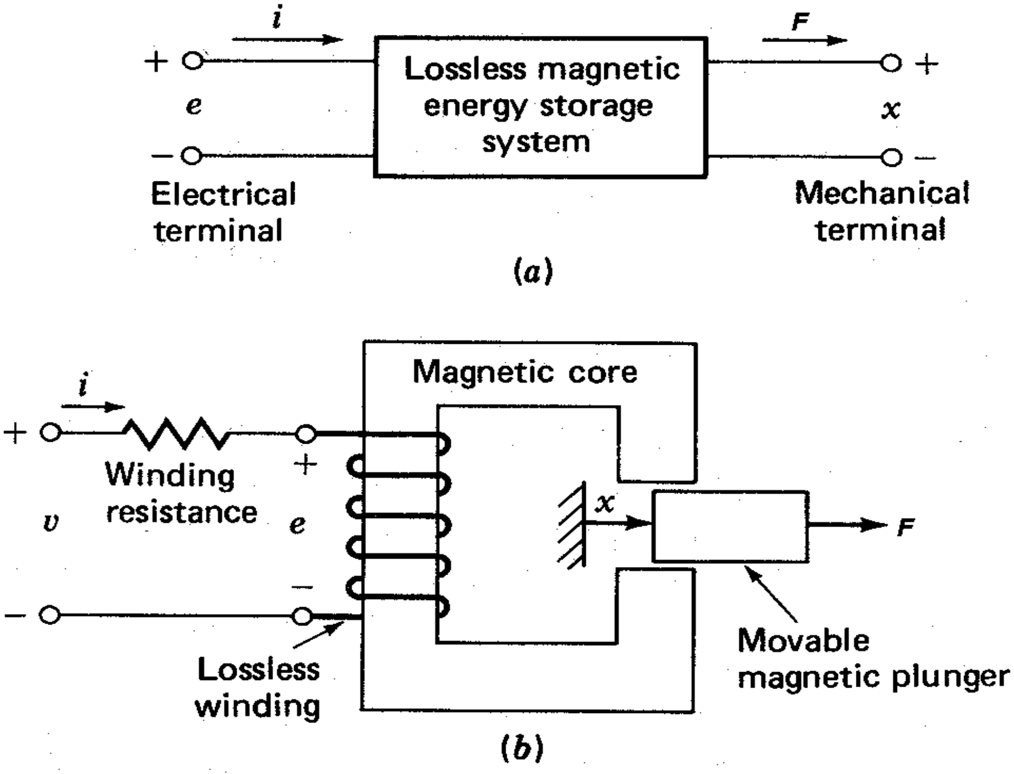
Let,

1. The winding resistance is R.
2. At a certain time instant *t*, we record that the terminal voltage applied to the excitation winding is *v* volt.
3. The excitation winding current is *i Ampere*
4. The position of the movable plunger is *x*
5. The force acting on the plunger is **F** with the reference direction chosen in the positive direction of the *x* axis, as shown in the diagram.

After a time interval *dt*, we notice that the plunger has moved for a distance *dx* under the action of the force **F**.

The mechanical work done by the force acting on the plunger during this time interval is

$$dW_m = Fdx$$



A singly excited linear actuator

The amount of electrical energy that has been transferred into the magnetic field and converted into the mechanical work during this time interval can be calculated by subtracting the power loss dissipated in the winding resistance from the total power fed into the excitation winding as

$dWe = dWm + dWf$ …… motor

As per KVL

$e = v - ir$,

Also from fundamentals induced emf in the coil is $e = N\frac{d\varphi}{dt}$

$$edt = Nd\varphi \Rightarrow eidt = Nid\varphi \Rightarrow (v - ir)idt = Nid\varphi$$

$$\Rightarrow (v - ir)idt = Nid\varphi \Rightarrow (vi - i^2 r)dt = Nid\varphi = Fd\varphi$$

Thus,

$$dWe = eidt = Fd\varphi = id(\varphi N) = id(\Psi)$$

Total electrical energy input is transformed to the magnetic field for energy storage as $i\psi$ hence, $\int dWe = \int id(\Psi) \Rightarrow We = i\Psi$

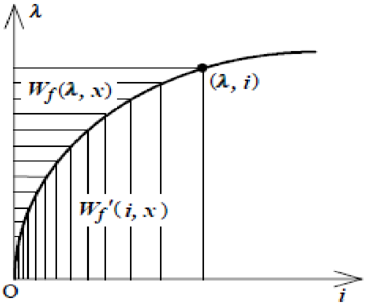
The energy to the magnetic field medium is shared as field energy W_f and co energy W_f'

From the knowledge of the magnetic systems the field energy and co energy is given by

From the linear relations the

The field energy is $W_f(\Psi, x) = \frac{1}{2} i(\Psi, x)\Psi$

The co-energy is $W_f'(i, x) = \frac{1}{2} i\Psi(i, x)$



Energy and coenergy

The mechanical work done is given by the product of the force (F_f)on the plunger and displacement(dx) of the plunger and is equal to dW_m.

Thus,

$$dWm = F_f dx = dWe - dW_f = id\Psi - dW_f$$

Finally

$$F_f dx = id\Psi - dW_f$$

In the above expression, the systems are of two types namely, Voltage excited system and current excited systems.

Case-1: Current excited system

As the mechanical work done by the plunger is $F_f dx = id\Psi - dW_f$

Here,

The flux linkages ψ is a function of (i, x) that is $\psi(i, x)$

Thus $d\Psi(i, x) = \frac{d\Psi(i, x)}{\partial i}\partial i + \frac{d\Psi(i, x)}{\partial x}\partial x$

and $dW_f(i, x) = \frac{dW_f(i, x)}{\partial i}\partial i + \frac{dW_f(i, x)}{\partial x}\partial x$

Substituting the above two equations in the mechanical work done formula it becomes

$$F_f\, dx = i\left(\frac{d\Psi(i,x)}{\partial i}\partial i + \frac{d\Psi(i,x)}{\partial x}\partial x\right) - \left(\frac{dW_f(i,x)}{\partial i}\partial i + \frac{dW_f(i,x)}{\partial x}\partial x\right)$$

$$F_f\, dx = \left(i\frac{d\Psi(i,x)}{\partial x} - \frac{dW_f(i,x)}{\partial x}\right)\partial x + \left(\frac{d\Psi(i,x)}{\partial i} - \frac{dW_f(i,x)}{\partial i}\right)\partial i$$

As *di* part is zero, there is no coefficient hence F_f is given by he coefficient of *dx*

$$F_f = \left(i\frac{d\Psi(i,x)}{\partial x} - \frac{dW_f(i,x)}{\partial x}\right)$$

$$F_f = \frac{\partial}{\partial x}\left(i\Psi(i,x) - W_f(i,x)\right)$$

$$F_f = \frac{\partial W_f^{'}(i,x)}{\partial x}$$ …… Expression for force with current excited system

Case-2: Voltage excited system

As the mechanical work done by the plunger is $F_f\, dx = id\Psi - dW_f$

Here, the current (i) changes as flux linkages (ψ) and disploacement (x) changes.
Therefore i (*ψ, x*) and W$_f$ *(ψ, x)*

Thus $dW_f(\Psi,x) = \frac{dW_f(\Psi,x)}{\partial\Psi}\partial\Psi + \frac{dW_f(\Psi,x)}{\partial x}\partial x$

Substituting the above equation in the mechanical work done formula it is

$$F_f\, dx = id\Psi - \left(\frac{dW_f(\Psi,x)}{\partial\Psi}\partial\Psi + \frac{dW_f(\Psi,x)}{\partial x}\partial x\right)$$

$$F_f\, dx = \left(i - \frac{dW_f(\Psi,x)}{\partial\Psi}\right)\partial\Psi - \frac{dW_f(\Psi,x)}{\partial x}\partial x$$

As *dψ* part is zero, since there is no coefficient hence F_f is given by the coefficient of *dx* as

$$F_f = (-)\frac{dW_f(\Psi,x)}{\partial x}$$ …… Expression for force with voltage excited system

Summary of singly excited system

1. Electrical energy input: dW$_e$ = eidt = F dΦ = idψ
2. Field energy = $W_f = \frac{1}{2}\frac{\Psi^2}{L(x)}$
3. Co-energy = $W_f^{'} = \frac{1}{2}L(x)i^2$
4. Mechanical Force $F_f = (-)\frac{dW_f(\Psi,x)}{\partial x}$
5. Mechanical Force $F_f = \frac{\partial W_f^{'}(i,x)}{\partial x}$

Doubly excited system:

- Consider the doubly excited system as a synchronous motor having the coils on both stator and rotor as shown in the below figure
- Let $v_1, e_1, i_1\ \psi_1$ and v_2, e_2, i_2, ψ_2 are the parameters related to coil1 and coil2 respectively.
- θ_r be the angle between the stator and rotor magnetic axis.
- ω be the angular velocity of the rotor initially without rotation of the rotor.
- T_e is the torque developed on the rotor when it rotates.
- L_1, L_2 are the self inductances of coil 1 and coil2, $M_{12}=M_{21}$ are the mutual inductances between the coils 1 and 2.
- These inductances are constant when the rotor is stationary or else variables when the rotor is rotating
- Also the field energy W_f is same irrespective to the rotor condition whether stationary or rotating

Case-1:

Let the rotor is static and hence there is no mechanical work done. Therefore, $dW_e = dW_f$

From case-1 find the value of the field energy, which can be used in case-2 with rotor rotating.

Electrical energy input dW_e is

$$dW_e = e_1 i_1 dt + e_2 i_2 dt = i_1 d\Psi_1 + i_2 d\Psi_2$$

$$dW_e = i_1 d(L_1 i_1 + M_{12} i_2) + i_2 d(L_2 i_2 + M_{12} i_1)$$

$$dW_e = i_1 L_1 di_1 + M_{12} i_1 di_2 + i_2 L_2 di_2 + M_{12} i_2 di_1$$

$$dW_e = i_1 L_1 di_1 + i_2 L_2 di_2 + M_{12}(i_1 di_2 + i_2 di_1)$$

$$dW_e = i_1 L_1 di_1 + i_2 L_2 di_2 + M_{12}(di_1 i_2) = dW_f$$

For getting the field energy W_f integrate the dW_f equation on both sides

$$\int dW_f = \int_0^{i_1} i_1 L_1 di_1 + \int_0^{i_2} i_2 L_2 di_2 + \int_0^{i_1 i_2} M_{12}(di_1 i_2)$$

$$W_f = \frac{1}{2} i_1^2 L_1 + \frac{1}{2} i_2^2 L_2 + M_{12} i_1 i_2 \qquad \text{...........Eq.1}$$

Case -2:

Let the rotor rotates with ω rad/sec due to a torque of T_e and moves from initial by $d\theta_r$ degrees.

Then , the mechanical work done is

$$dW_m = P_m dt = \omega t_e dt = \frac{d\theta_r}{dt} T_e dt = d\theta_r T_e$$

As the energy balance equation is $dW_e = dW_m + dW_f$

Here, electrical energy input with rotation is given by

$$dW_e = e_1 i_1 dt + e_2 i_2 dt = i_1 d\Psi_1 + i_2 d\Psi_2$$

$$dW_e = i_1 d(L_1 i_1 + M_{12} i_2) + i_2 d(L_2 i_2 + M_{12} i_1)$$

$$dW_e = i_1 L_1 di_1 + M_{12} i_1 di_2 + i_2 L_2 di_2 + M_{12} i_2 di_1 + i_1^2 dL_1 + i_2 i_1 dM_{12} + i_2^2 dL_2 + i_2 i_1 dM_{12} \quad \text{..Eq(2)}$$

Also, from Eq.(1) the dW_f is the differentiation to the W_f

$$dW_f = i_1 L_1 di_1 + \frac{1}{2} i_1^2 dL_1 + \frac{1}{2} i_2^2 dL_2 + i_2 L_2 di_2 + M_{12} i_1 di_2 + M_{12} i_2 di_1 + i_1 i_2 dM_{12} \quad \text{..Eq(3)}$$

Therefore, $dW_m = dW_e - dW_f$ i.e Eq(2)-Eq(3)

$$dW_m = \frac{1}{2} i_1^2 dL_1 + \frac{1}{2} i_2^2 dL_2 + i_1 i_2 dM_{12}$$

$$T_e d\theta_r = \frac{1}{2} i_1^2 dL_1 + \frac{1}{2} i_2^2 dL_2 + i_1 i_2 dM_{12}$$

$$T_e = \frac{1}{2} i_1^2 \frac{dL_1}{d\theta_r} + \frac{1}{2} i_2^2 \frac{dL_2}{d\theta_r} + i_1 i_2 \frac{dM_{12}}{d\theta_r} \quad \text{……..Eq(4)}$$

WORKING PRINCIPLE OF DC GENERATOR:

1. DC Generator is an electro mechanical energy conversion device used to ***convert mechanical energy to electrical energy.***
2. It works as per ***Faradays laws of electromagnetic induction*** which states that

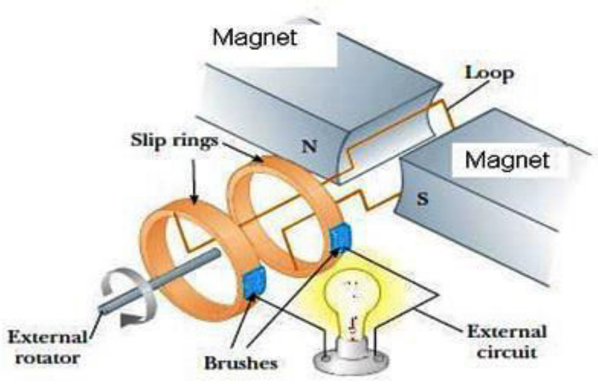
> I Law: *"Whenever the conductor cuts the magnetic flux a dynamical emf is induced in the conductor"*
>
> II Law: *"The magnitude of the emf induced is directly proportional to the rate of change of flux linkages"*

3. ***Fleming's right hand rule*** is used to obtain the direction of the current in the coil of the DC Generator.

Simple loop dc generator:

The simple loop dc generator is assumed to have the following parts as shown in Fig (a)

- Two permanent magnets (North pole and South pole)
- A single turn rectangular coil(placed on the shaft)
- Two Slip rings
- Two brushes
- External load



(a)

The rectangular coil is assumed to be rotated in clockwise with an angular velocity of ω rad/sec.

The working operation of the simple loop generator is explained over one complete rotation of the coil for 360^0 is shown in the below figure at different positions of the coil.

At 0 degrees Position (A):

1. This position is also known as the "Neutral Plane";
2. In this position the loop is parallel to the magnetic lines of flux
3. In this position there is maximum flux passing through the coil.
4. No EMF is induced in the coil because of no "Change in flux through the loop".

At 90 degrees Position (B):

1. After the loop has been rotated 90 degrees clockwise through the magnetic field the flux linkage through it now becomes zero.
2. But the rate of change of flux through it was maximum,
3. This results in an induced EMF which climbs from zero to its peak value.

At 180 degrees Position (C):

1. Once again the coil is rotated 90 degrees clockwise resulting in the completion of a 180 degrees cycle.
2. Here the loop is perpendicular to the magnetic lines of force
3. This means that there is maximum flux density through it resulting the EMF to falls back to zero.

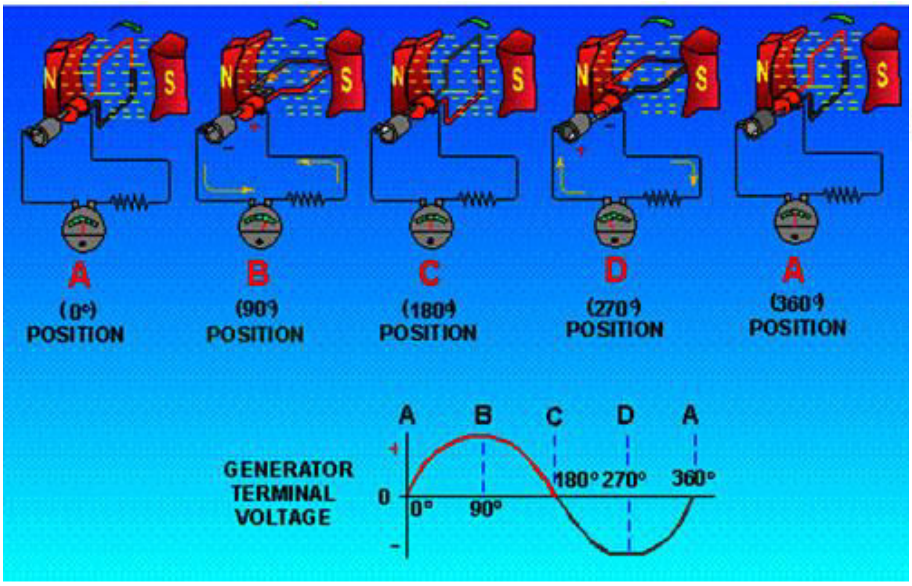
At 270 degrees Position (D):

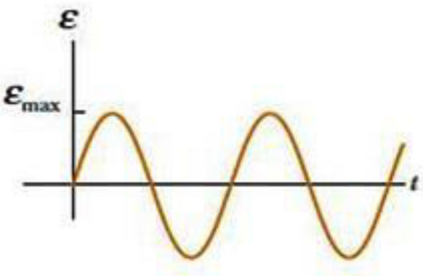
1. At 270 degrees the flux linkage through the loop is once again zero,
2. but the rate of change of flux is maximum.

3. In this position, the EMF induced goes up to its peak value, but this time it's in the reverse direction.

At 360 degrees Position(A):

1. The loop is rotated through another 90 degrees such that it has completed a rotation of 360 degrees.
2. The flux linkage through it is maximum and the voltage decreases back to zero.

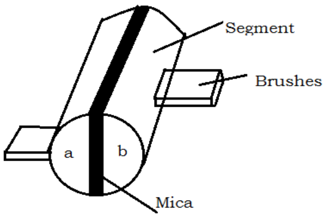




- Hence, it is observed that the nature of the emf induced in the armature coil is alternating quantity (i.e, positive voltage during first half cycle and negative voltage during second half cycle)
- Thus, to convert the induced alternating ac to dc nature the Commutator (or) split rings are used in the place of the slip rings of a simple loop generator.

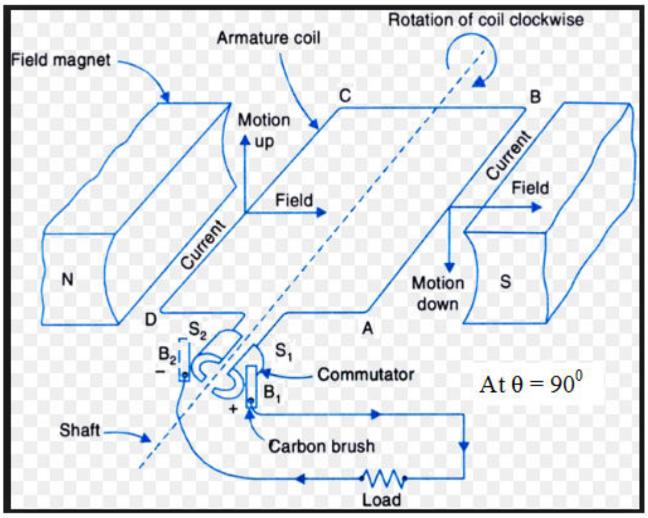
Action of Commutator:

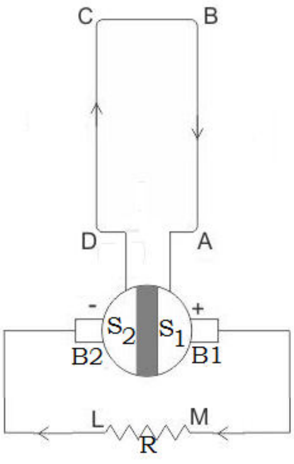
- The Commutator is a mechanical rectifier used to convert AC to DC
- Here, the split rings or Commutator segments (s_1 and s_2) are placed instead of slip rings
- The split rings or commutator are made out of conducting cylinder, which is cut into two halves or segments insulated from each other by a thin sheet of mica.
- Brushes B_1 and B_2 are mounted on two Commutator segments having + and - polarities



At $\theta = 90^0$position :

- Conductor AB is under South Pole and Conductor CD are under North Pole, with coil rotating in clock wise direction.
- Using Flemings right hand rule, the current in the conductor AB is from B-A and in conductor CD is from D-C
- Therefore the current flow is in the path of A – S_1 - B_1 – **M – L** - B_2 - S_2 – D – C – B - A

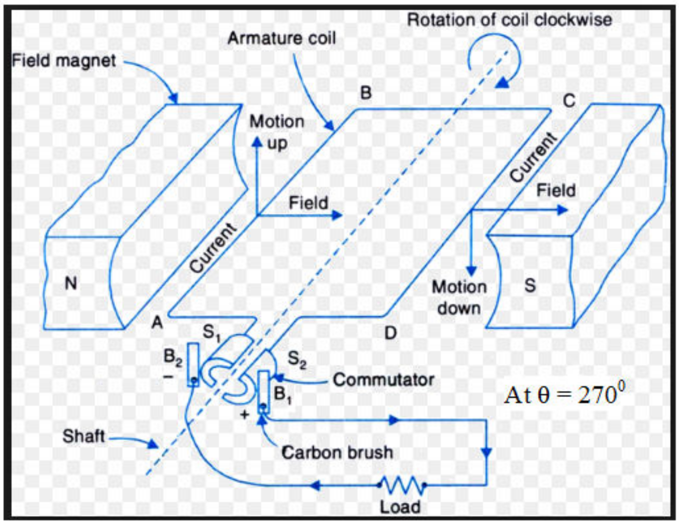


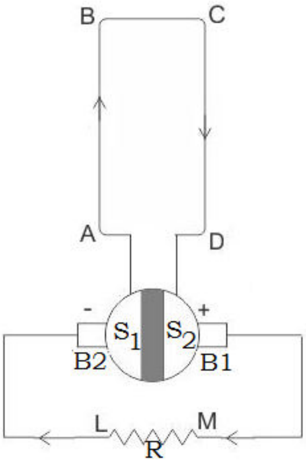


At θ = 90^0

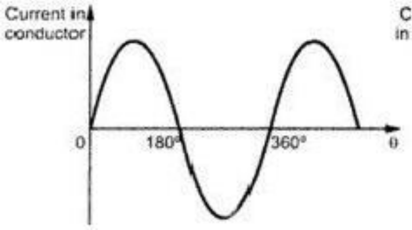
At θ = 270^0position :

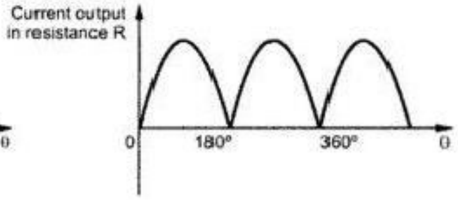
- Conductor AB is under North Pole and Conductor CD are under South Pole, with coil rotating in clock wise direction.
- Using Flemings right hand rule, the current in the conductor AB is from A-B and in conductor CD is from C-D
- Therefore the current flow is in the path of A – B – C - D- S_2- B_1 – **M – L** - B_2 – S_1- A





At θ = 270^0



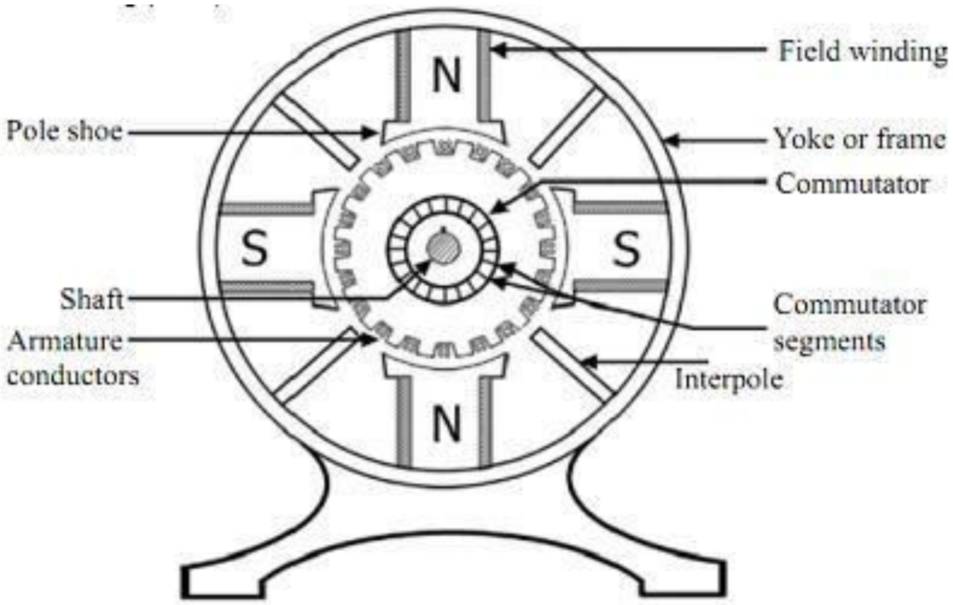


- Thus using the Commutator, the current in the load is unidirectional from M to L at all positions i.e current in coil is alternating and current in Resistance R is unidirectional (pulsating DC)

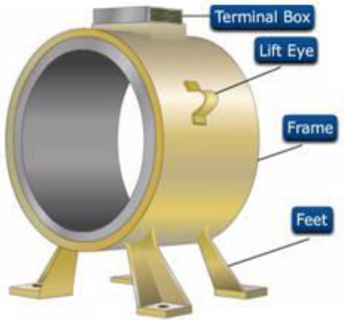
CONSTRUCTION OF DC GENERATOR

A DC generator has the following parts

1. Yoke (or) Magnetic frame
2. Pole core and pole shoe
3. Field winding (or) Pole coils
4. Armature Core
5. Armature winding
6. Commutator
7. Brushes
8. Bearing

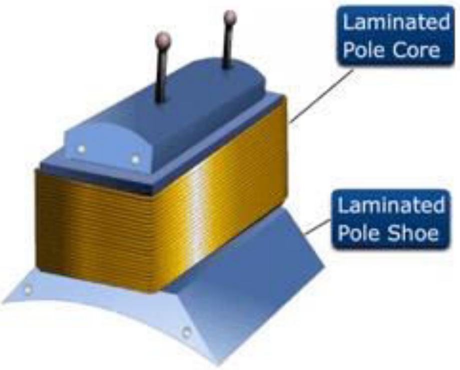


Yoke:



- Yoke or the outer frame of DC generator serves two purposes,
  1. It holds the magnetic pole cores of the generator and acts as cover of the generator.
  2. It carries the magnetic field flux.
- Yoke is made of cast iron for small rating generators, due to the cheaper in cost but heavier than steel.
- Yoke is made of lighter cast steel or rolled steel for larger rating generators , where weight of the machine is concerned.

Pole core and pole shoe



- The field magnets consist of pole cores and pole shoes.
- The pole core is fixed to the inner periphery of the yoke by means of bolts through the yoke and into the pole body.
- The pole core carries the field winding and there are two types of construction

  One: Solid pole core, where it is made of a single piece of cast iron or cast steel.

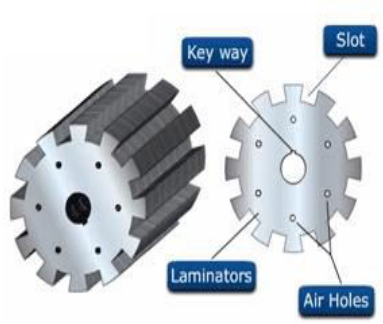
  Two: Laminated pole core, where it made of numbers of thin, limitations of annealed steel which are riveted together.
- The thickness of the lamination is in the range of 0.04" to 0.01".

- The ***pole shoes*** serve two purposes:
    1. They spread out the flux in the air gap and also, being of larger cross-section, reduce the reluctance of the magnetic path
    2. They support the exciting coils (or field coils)

Field winding (or) Pole coils

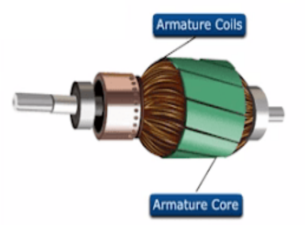
- The function of the field system is to produce uniform magnetic field within which the armature rotates.
- Field coils are mounted on the poles and carry the dc exciting current.
- The field coils are connected in such a way that adjacent poles have opposite polarity.
- The m.m.f. developed by the field coils produces a magnetic flux that passes through the pole pieces, the air gap, the armature and the frame.
- Practical d.c. machines have air gaps ranging from 0.5 mm to 1.5 mm.
- By reducing the length of air gap, we can reduce the size of field coils

Armature Core



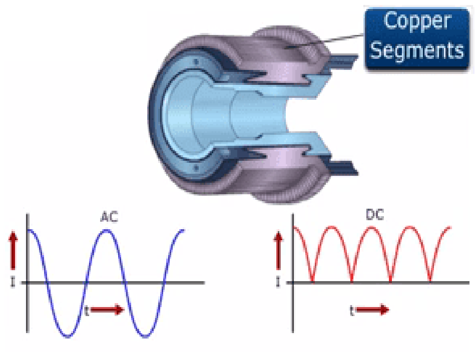
- The armature core consists of slotted soft-iron laminations (about 0.4 to 0.6 mm thick) that are stacked to form a cylindrical core as shown in adjacent figure.
- The purpose of laminating the core is to reduce the eddy current loss.
- Thinner the lamination, greater is the resistance offered to the induced e.m.f., smaller the current and hence lesser the I^2R loss in the core.
- The laminations are slotted to accommodate and provide mechanical security to the armature winding and to give shorter air gap for the flux to cross between the pole face and the armature "teeth".

Armature winding



- The slots of the armature core hold insulated conductors that are connected in a suitable manner. This is known as armature winding.
- This is the winding in which "working" emf is induced. The armature conductors are connected in series-parallel; the conductors being connected in series so as to increase the voltage and in parallel paths so as to increase the current.
- The armature winding of a D.C. machine is a closed-circuit winding; the conductors being connected in a symmetrical manner forming a closed loop or series of closed loops.
- There are two types of armature winding based on the connection to the Commutator they are (a) Lap winding and (b) Wave winding

Commutator

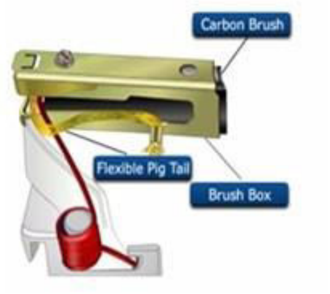


- A Commutator is a mechanical rectifier which converts the alternating voltage generated in the armature winding into direct voltage across the brushes
- The Commutator is made of copper segments insulated from each other by mica sheets and mounted on the shaft of the machine

- The armature conductors are soldered to the Commutator segments in a suitable manner to give rise to the armature winding

Brushes

- The function of the brushes in DC generator is to collect current from Commutator segments.
- The brushes are made of carbon and rest on the Commutator.
- The brush pressure is adjusted by means of adjustable springs.



Bearing of DC Generator

- For small machine, ball bearing is used and for heavy duty DC generator, roller bearing is used.
- The bearing must always be lubricated properly for smooth operation and long life of generator.

EMF EQUATION OF DC GENERATOR:

Let,
E = Average emf induced in volts
Z = No. of armature conductors
N = Speed of the rotor in RPM
P = No. of the poles
A = No. of parallel paths
Φ = Flux per pole in Weber's

As per Faradays second Law,

- The magnitude of the emf induced is directly proportional to the rate of change of flux linkages

  $e \alpha \frac{d\Psi}{dt} = e \alpha N \frac{d\Phi}{dt} = e = k\, N \frac{d\Phi}{dt}$, In SI unit system $k = 1$, $\therefore e = N \frac{d\Phi}{dt}$

- Emf per conductor is $e = \frac{d\Phi}{dt}$

Where,

- $d\Phi$ = total flux in the airgap that cuts the conductor for one revolution.

  As (P) No. of poles and each pole produces the Φ flux, then $d\Phi = P\Phi$

- dt = time taken by the conductor to cut the flux of $d\Phi$

  i.e The time taken for the armature coil to complete one rotation $dt = \frac{60}{N}$ sec

Thus,

- $e = \frac{P\phi}{\left(\frac{60}{N}\right)} = \frac{P\phi N}{60}$
- As there are (A) No. of parallel paths with 'Z' No. of conductors, then the emf per parallel path is given by

$$e = \frac{P\phi N}{60} * \left(\frac{Z}{A}\right) = \frac{\phi ZN}{60} * \frac{P}{A}$$

- Therefore, average value of the emf induced is $E = \frac{\phi ZN}{60} * \frac{P}{A}$
- The No. of parallel paths in the armature winding depends on the type of the armature windings

For Wave connected Armature (A=2)

$$E = \frac{\phi ZN}{60} * \frac{P}{2}$$

For Lap connected Armature (A=P)

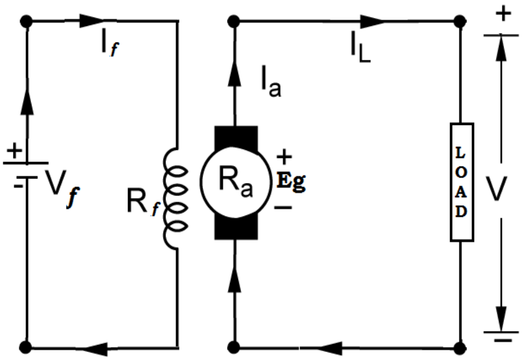
$$E = \frac{\phi ZN}{60} * \frac{P}{P}$$

TYPES OF DC GENERATORS :

- Based on the excitation given to the field winding, the dc generators are classified in to two types
  - a. Separately excited dc generator
  - b. Self excited dc generator

SEPARATELY EXCITED DC GENERATOR:

1. In a separately excited generator field winding is energized from a separate voltage source in order to produce flux in the machine and is shown in the below figure.



2. The flux produced will be proportional to the field current in unsaturated condition of the poles.
3. The armature conductors when rotated in this field will cuts the magnetic flux and generates the emf (Eg).
4. The emf will circulate the current against the armature resistance (Ra), brushes and to the load.
5. Applying KVL to the armature loop the Eg is

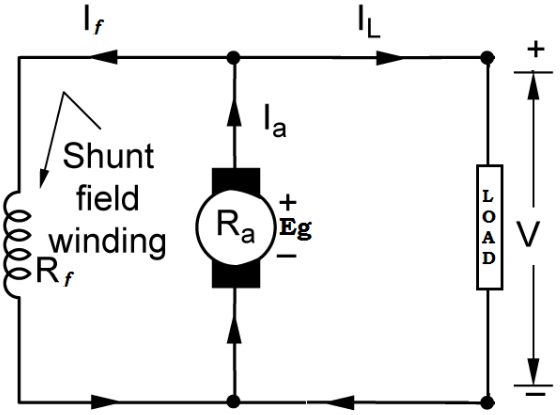
$$E_g = V + I_a R_a + V_{brush}$$

$I_a = I_L$ and $I_L = \frac{P_L}{V}$ and $I_f = \frac{V_f}{R_f}$

SELF EXCITED DC GENERATOR:

1. In self excited generator field winding is energized from the armature induced emf and there is an electrical connection in between this armature and field winding.
2. There are three possibilities of connecting the field winding to the armature they are
   a. Shunt generator
   b. Series generator
   c. Compound generator
      i. Long shunt compound generator
      ii. Short shunt compound generator

DC SHUNT GENERATOR

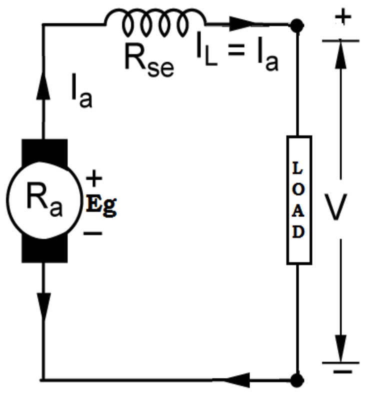


1. In the dc shunt generator the field winding circuit is connected in parallel to the armature circuit and as well as to the load.
2. The armature current is divided in to the field and the load as I_f and I_L.
3. The shunt field winding has ***more number of turns with thin wire***, so that resistance of the field will be in the range of hundreds and was designed to withstand for the rated voltage.
4. Applying KVL to the armature loop the E_g is

$$E_g = V + I_a R_a + V_{brush}$$

$$I_a = I_L + I_f \quad \text{and} \quad I_L = \frac{P_L}{V} \quad \text{and} \quad I_f = \frac{V}{R_f}$$

DC SERIES GENERATOR



1. In the dc series generator the field winding circuit is connected in series to the armature circuit and as well as to the load.
2. Here the armature current is equal to the series field current and also equal to the load.
5. The series field winding has ***less number of turns with thick wire***, so that resistance of the field will be in the smaller values and was designed to carry the rated current.
6. Applying KVL to the armature loop the E_g is

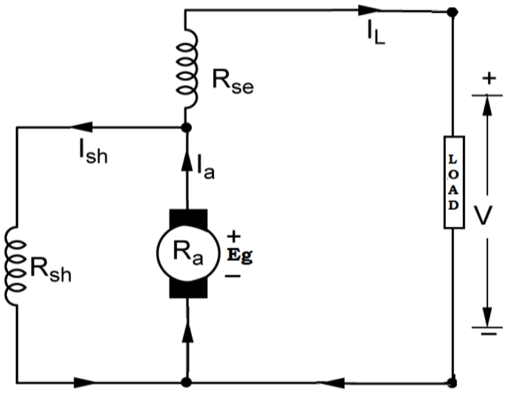
$$E_g = V + I_a(R_a + R_{se}) + V_{brush}$$

$$I_a = I_L = I_{se} \quad \text{and} \quad I_L = \frac{P_L}{V}$$

DC COMPOUND GENERATORS

1. A compound generator has two field coils wound over the field poles.
2. The coil having large number of turns and thinner cross sectional area is called the shunt field coil and the other coil having few numbers of turns and large cross sectional area is called the series field coil.
3. Based on the series field winding connected to the armature the compound generators are classified as long shunt generator and short shunt generator

SHORT SHUNT GENERATOR

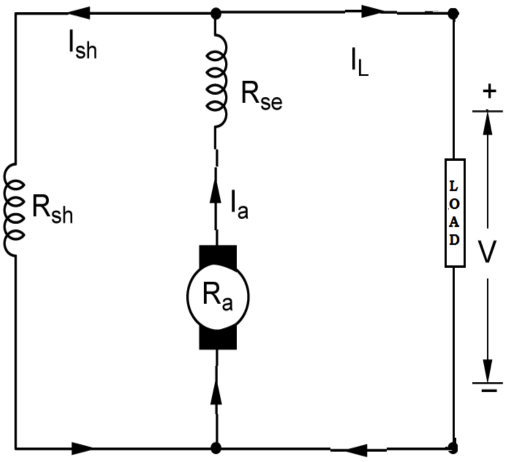


1. In a short shunt dc compound generator, the series field is connected in series to the load and shunt field winding is connected in parallel to the armature and the series combination of the load and series winding.
2. Thus, the series field current will depend on the load variations which will effect in further the shunt field current.
3. Applying KVL to the armature loop the E_g is

$$E_g = V + I_a R_a + I_L R_{se} + V_{brush}$$

$$I_a = I_L + I_{se} \quad \text{and} \quad I_L = \frac{P_L}{V} \quad \text{and} \quad I_f = \frac{V + I_L R_{se}}{R_f}$$

LONG SHUNT GENERATOR



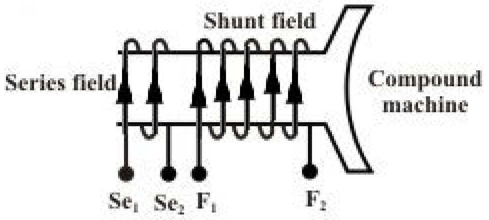
1. In a long shunt dc compound generator, the series field is connected in series to the armature and shunt field winding is connected in parallel to the armature and to the load.
2. Applying KVL to the armature loop the E_g is

$$E_g = V + I_a (R_a + R_{se}) + V_{brush}$$

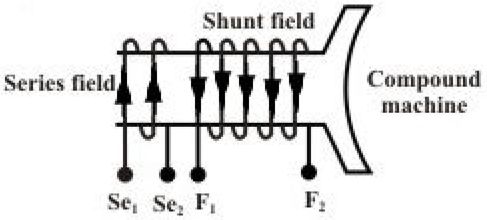
$$I_a = I_L + I_{se} \quad \text{and} \quad I_L = \frac{P_L}{V} \quad \text{and}$$

$$I_f = \frac{V}{R_f}$$

- Also, the dc compound generators are further classified into two types based on the compounding of the series flux to the shunt flux. They are cumulatively compounded and differentially compounded generators
- In the cumulatively compound generator, the series flux aids to the shunt field flux and the net flux increases, whereas in the differentially compounded generators the series flux opposes the shunt field flux and the net resultant flux decreases.
- The below figure shows the arrangement of the series and shunt field coils iin the pole core in both cumulative and differentially compounded generators.



Cummulatively compounded



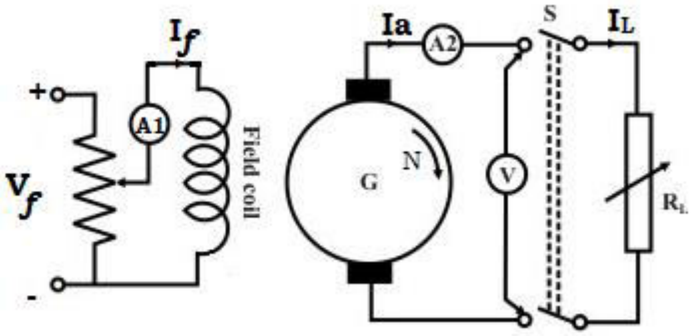
differentially compounded

Characteristics of dc generators :

There are three characteristics to be analyzed for any type of the dc generator, they are

1. Open circuit characteristics (or) No-Load characteristics (or) Magnetization characteristics (E_0 *Vs* I_f)
2. Internal characteristics (E_g *Vs* I_a)
3. External or Load characteristics (V *Vs* I_L)

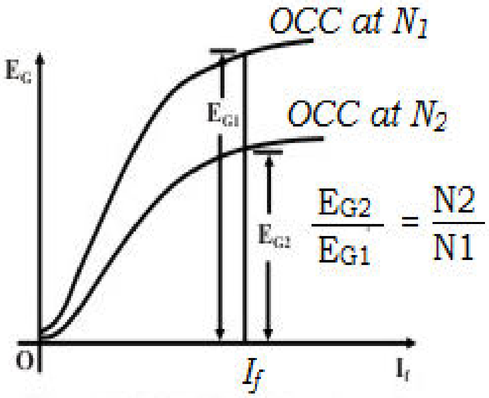
OCC or No-Load Characteristics of Separately excited DC Generator :



1. OCC is the characteristics drawn between open circuit voltage (E_0) for various field currents (I_f) at constant speed.
2. In this generator field winding is excited from a separate source V_f as shown in above circuit, hence field current is independent of armature terminal voltage
3. The generator is driven by a prime mover at rated speed, say constant speed *N* rpm.
4. With switch *S* in opened condition, field coil is excited via a *potential divider* connection from a separate d.c source and field current is gradually increased by moving the wiper from minimum position gradually.
5. The field current will establish the flux per pole Φ.
6. The voltmeter V connected across the armature terminals of the machine will record the generated emf $\left(E=\frac{PZ}{60A}*\phi N=k*\phi N\right)$. Where *k* is a constant of the machine.
7. As field current is increased, E_0 will increase.
8. E_0 versus I_f plot at constant speed *N rpm* is shown in below figure.
9. It may be noted that even when there is no field current, a small voltage (OD) is generated due to *residual flux* and the small voltage is called *residual voltage.*
10. If field current is increased, φ increases linearly initially and O.C.C follows a straight line.
11. However, when saturation sets in, φ practically becomes constant and hence E*g* too becomes constant.
12. In other words, O.C.C follows the B-H characteristic, hence this characteristic is sometimes also called the magnetization characteristic of the machine.

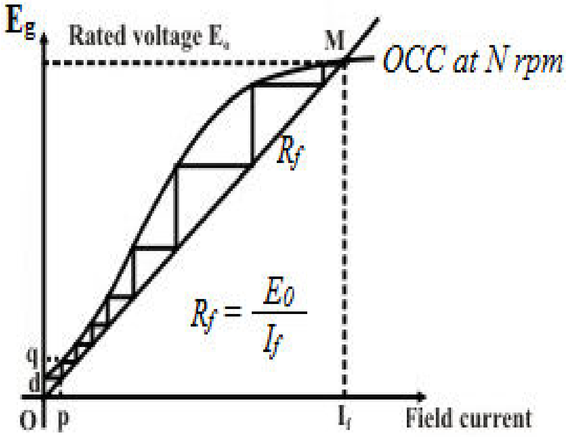
Procedure to draw OCC at different speeds

1. It is important to note that if O.C.C is known at a certain speed N_1, O.C.C at another speed N_2 can easily be predicted from the emf equation $E = k * \phi N$
2. Emf at speed N_1 rpm for a field current of I_f, producing the flux Φ is E_1 and is given by $E_1 = k * \phi N_1$
3. Emf at speed N_2 rpm for the same field current of I_f, producing the flux Φ is E_2 and is given by
$$E_2 = k * \phi N_2$$
4. Therefore, the emf E_2 at speed N_2 is
$$\frac{E_2}{E_1} = \frac{k * \phi N_2}{k * \phi N_1} \Rightarrow \frac{E_2}{E_1} = \frac{N_2}{N_1} \Rightarrow E_2 = E_1 \times \frac{N_2}{N_1}$$



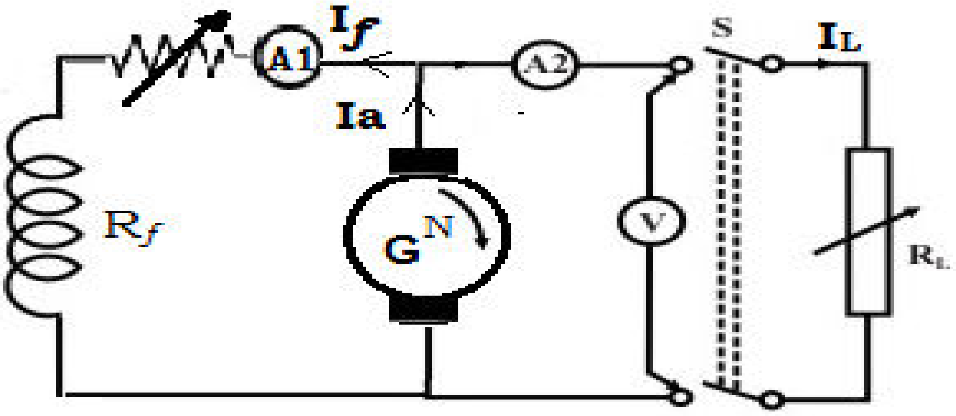
EMF BULID P PROCESS IN A SELF EXCITED DC GENERATOR:

1. For the buildup of emf in the self excited dc generator, the poles or magnets ***must have residual flux*** in them.
2. Therefore, if the generator is driven at rated speed of N rpm, then a small voltage ($k\varphi_{res}N$) will be induced across the armature.
3. This small voltage will be directly applied across the field circuit since it is connected in parallel with the armature.
4. Hence a small field current flows producing additional flux.
5. If it so happens that this additional flux aids the already existing residual flux, total flux now becomes more and generating more voltage.
6. This more voltage will drive more field current generating more voltage.
7. Both field current and armature generated voltage grow *cumulatively*.
8. This process will be explained clearly from the plot shown above
9. Initially voltage induced due to residual flux is observed from O.C.C as Od.
10. The field current thus produced can be obtained from field circuit resistance line and given by Op. With this Op filed current the flux is increased and correspondingly the induced voltage

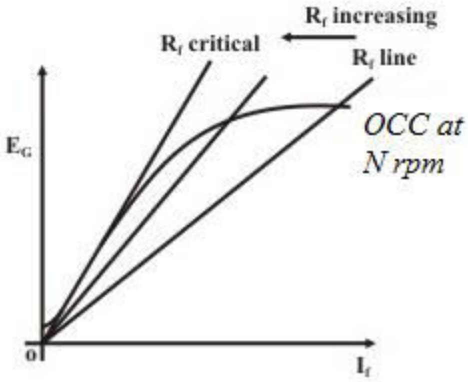


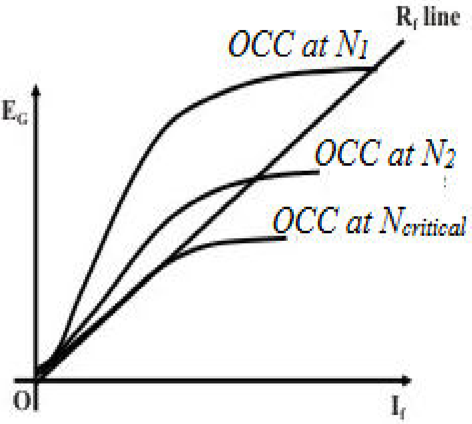
also increases from Od to Oq and so on. In this way voltage build up process continues along the stair case.

OCC or No-Load Characteristics of self excited DC shunt Generator :



1. The OCC of the shunt generator is obtained in a similar way to the dc separately excited generator by disconnecting its field winding from the armature and connecting it to a separate dc source
2. Therefore, the OCC curve at rated speed N rpm is shown in the above figure, with Od as residual voltage and increases gradually.
3. Later, the R_f line is drawn which is a straight line passing through the origin having a slope of its value R_f
4. This R_f line intersects the OCC at point M and gives the rated voltage of the generator.
5. If the R_f value is increased then its slope increases and the voltage generated by the generator reduces and if the value of the R_f is such that it becomes the tangential to the given OCC, then the field resistance is called critical field resistance($R_{f\,critical}$).
6. At this critical field resistance, the emf or voltage of the generator will be very small and it doesn't generates any voltage if the R_f selected is greater than the R_{fc}
7. Thus, ***R_f must be always less than the R_{fc}***
8. Similarly, for the $R_f < R_{fc}$, if the speed decreases then also the voltage generated by the generator reduces.
9. Thus the generator doesn't generates any voltage at a speed called critical speed for which the





given R_f line will become the tangent for the OCC drawn at N_c and is shown in the fig.

10. If the speed of the generator is made to run less than its critical speed then no emf will be induced, so the ***speed must be always greater than the critical speed.***

Conditions to build up the emf in the generator:

1. The magnets in the machine must have the ***residual flux.***
2. Field winding connection should be such that the residual flux is strengthened by the field current in the coil. If due to this, no voltage is being built up, reverse the field terminal connection.
3. Total field circuit resistance ***must be less than the critical field resistance.***
4. Speed of the generator ***must be greater than the critical speed.***

Unit -2

Working principle of dc motor

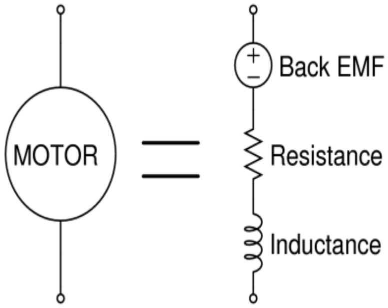
A dc motor is a electro mechanical energy conversion device that converts *electrical energy into mechanical energy.*

Its operation is based on the principle that ***"when a current carrying conductor is placed in a magnetic field, the conductor experiences a mechanical force"***.

The direction of the force is given by Fleming's left hand rule which states that " Stretch the first three fingers of left hand mutually perpendicular to each other in such a way that central finger indicates the direction of the current in the conductor, fore finger in the direction of the magnetic field, then the thumb indicates the direction of the force developed on the conductor

The magnitude of the force developed on the conductor is $F = BIL \sin\theta$

Back EMF:



When the armature of a d.c. motor rotates under the influence of the driving torque, the armature conductors move through the magnetic field and hence an e.m.f. is induced in them as per Faradays laws of electromagnetic induction.

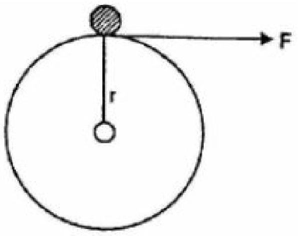
This induced e.m.f. acts in opposite direction to the applied voltage V (Lenz's law) and is known as back or counter e.m.f. E_b .

Significance of Back E.M.F

The presence of back e.m.f. makes the d.c. motor a self-regulating machine i.e., it makes the motor to draw as much armature current as is just sufficient to develop the torque required by the load. Back e.m.f. in a d.c. motor regulates the flow of armature current i.e., it automatically changes the armature current to meet the load requirement.

Armature Torque of a DC Motor

Torque is the turning and twisting moment of a force about an axis and is measured by the product of force (F) and radius (r) at right angle to which the force acts i.e $T = F*r$



Let

T = Torque developed on the rotor of the motor in Nm

Φ = Flux per pole in weber

Z = No. of the armature conductors

I_a= Armature current in A

P = No. of poles

A = No. of Parallel paths

r = radius of the pulley in mts

Work done by the pulley, W= Force * distance = F * 2πr

$$\text{Power} = \frac{\text{work done}}{\text{time}} \qquad P = \frac{F \times 2\pi r}{60/N} = \frac{F \times 2\pi r \times N}{60} = \frac{2\pi N}{60} \times (F * r) = \frac{2\pi NT}{60} = \omega T \Rightarrow P = \omega T$$

As, power developed in the armature is the gross mechanical power and is given by

$P = E_g I_a$, therefore $E_g I_a = \omega T$

$$\frac{\phi ZN}{60}\left(\frac{P}{A}\right)I_a = \frac{2\pi NT}{60} \quad \Theta\, \mathrm{E_g} = \frac{\phi ZN}{60}\left(\frac{P}{A}\right)$$

$$\frac{\phi Z}{2\pi}\left(\frac{P}{A}\right)I_a = T$$

$$\therefore T = \frac{\phi Z}{2\pi}\left(\frac{P}{A}\right)I_a$$

$$T = \frac{1}{2\pi}\phi Z I_a \frac{P}{A}$$

Also, from the fundamentals, the gross torque or armature torque is

$$P = \omega T \Rightarrow E_b I_a = \omega T$$

$$T = \frac{E_b I_a}{\omega} = \frac{E_b I_a * 60}{2\pi N} = \left(\frac{60}{2\pi}\right) * \frac{E_b I_a}{N} = 9.55\frac{E_b I_a}{N} = 9.55\frac{P_m}{N}$$

Also, the shaft torque or useful torque is

$$P_{sh} = \omega T_{sh}$$

$$T_{sh} = \frac{P_{sh}}{\omega} = \frac{P_m - Mechloss}{\omega} = \left(\frac{60}{2\pi}\right) * \frac{P_{sh}}{N} = 9.55\frac{P_{sh}}{N}$$

Therefore,

$$T \,\alpha\, \phi I_a$$

Torque relations in a dc motor

$$\frac{T_2}{T_2} = \frac{\phi_2 I_{a2}}{\phi_1 I_{a1}}$$

Speed of a DC Motor

$$\mathrm{E_b = V - I_a R_a}$$

But $$\mathrm{E_b = \frac{P\phi ZN}{60\ A}}$$

$$\therefore \quad \mathrm{\frac{P\phi ZN}{60\ A} = V - I_a R_a}$$

or $$\mathrm{N = \frac{(V - I_a R_a)}{\phi}\frac{60\ A}{PZ}}$$

or $$\mathrm{N = K\frac{(V - I_a R_a)}{\phi}} \quad \text{where} \quad \mathrm{K = \frac{60A}{PZ}}$$

But $$\mathrm{V - I_a R_a = E_a}$$

$$\therefore \quad \mathrm{N = K\frac{E_b}{\phi}}$$

or $$\boxed{\mathrm{N \propto \frac{E_b}{\phi}}}$$

Therefore,

In a dc motor speed is directly proportional to back emf, E_b and inversely proportional to flux, φ.

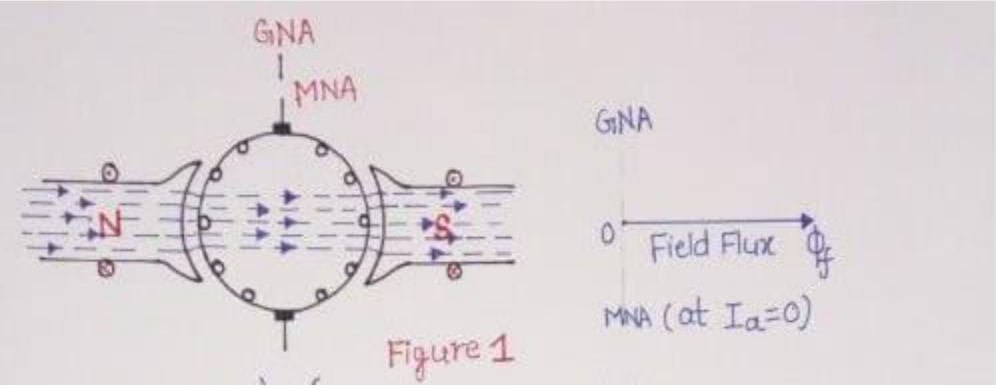
Armature reaction:

- The effect of armature MMF or flux on the main field MMF or flux is called as **armature reaction**.
- Actually, the armature flux so produced has two undesirable effects on main field flux.
    - ➢ Demagnetization effect leads to Net reduction in the main field flux
    - ➢ Cross magnetizing effect results in Distortion of the main field flux
- Reduction in main field flux per pole due to demagnetization effect reduces the generated voltage ($Eg=P\Phi ZN/60A$) in dc generator and reduces the torque ($Ta=K\Phi Ia$) developed in armature of dc motor as flux appears in expressions of both E_g and T_a
- The distortion of main flux due to cross magnetizing effect leads to sparking at the brushes.

Detailed explanation of armature reaction

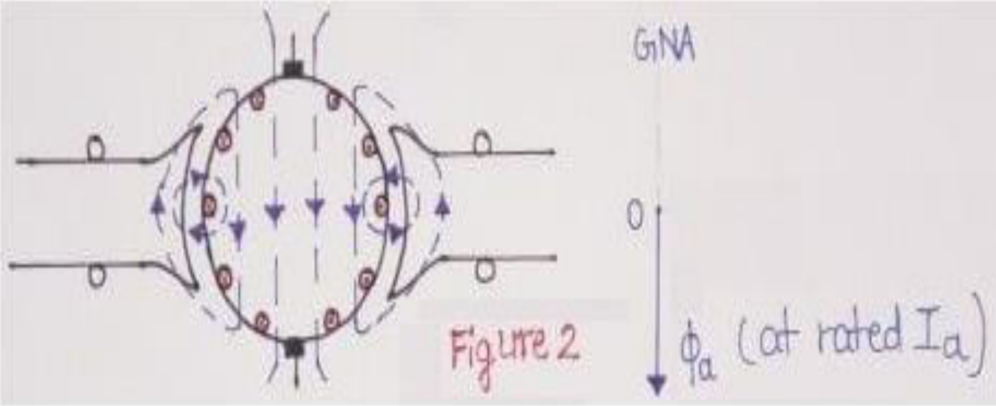
Case:1 (No load with poles excited)

- Consider, no current is flowing in the armature conductors and only the field winding is energized (as shown in the figure 1).
- In this case, magnetic flux lines of the field poles are uniform and symmetrical to the polar axis.
- The 'Magnetic Neutral Axis' (M.N.A.) coincides with the 'Geometric Neutral Axis' (G.N.A.).
    - MNA (Magnetic Neutral Axis) may be defined as the axis along which no emf is generated in the armature conductors as they move parallel to the flux lines.
    - Brushes are always placed along the MNA because reversal of current in the armature conductors takes place along this axis.
    - GNA (Geometrical Neutral Axis) may be defined as the axis which is perpendicular to the stator field axis.
- Thus, the main field mmf Φ_f directs towards right side and is perpendicular to GNA or MNA or Brush axis.



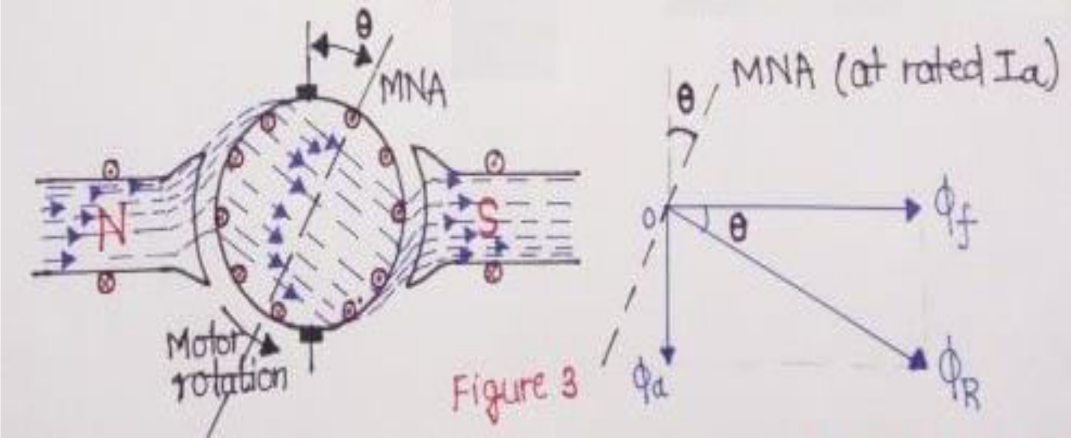
Case:2 (on load with poles un - excited)

- Figure 2 shows armature flux lines due to the armature current on loading of the machine. With field poles are de-energized.
- Here, the armature field mmf Φ_a directs towards down and lies on GNA or MNA or Brush axis.



Case:3 (Machine on load and poles excited)

- Now, when a DC machine is running on load and poles excited, both the fluxes (flux due to the armature conductors and flux due to the field winding) will present at a time.
- The armature flux superimposes with the main field flux and, hence, disturbs the main field flux (as shown in figure 3).
- This effect is called as armature reaction in DC machines which redirects the main field flux such a way that the flux was crowded at the trailing pole tips but weakened or thinned out at the leading pole tips (the pole tip which is first met during rotation by armature conductors is known as the leading pole tip and the other as trailing pole tip).

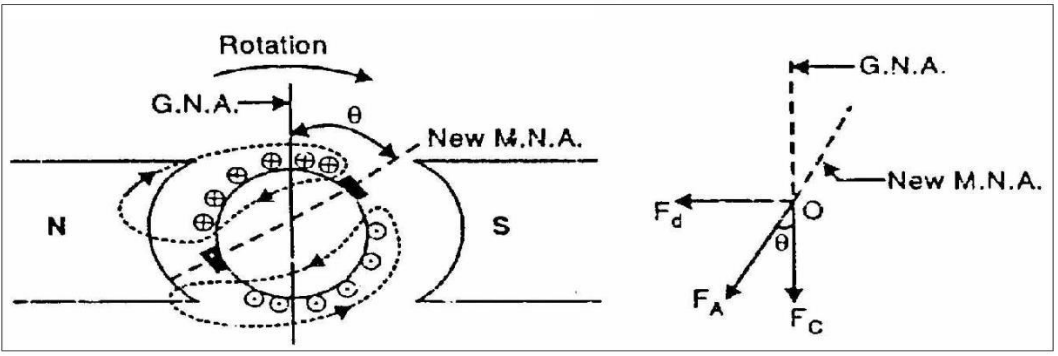


- Therefore, this tends to form a resultant flux line (Φ_R) which makes an angle of θ from the main field mmf (Φ_F)
- This makes MNA to shift from GNA by an angle θ

- The direction of the shift of brush axis or MNA is ***same*** *as that of the rotation of the armature in the case of* ***generator*** and is ***opposite*** *to the rotation of the armature in the case of* ***motor***
- The magnitude of the brush shift θ is directly proportional to the amount of the load on the machine.

<u>Effect of brush shift or MNA shift:</u>

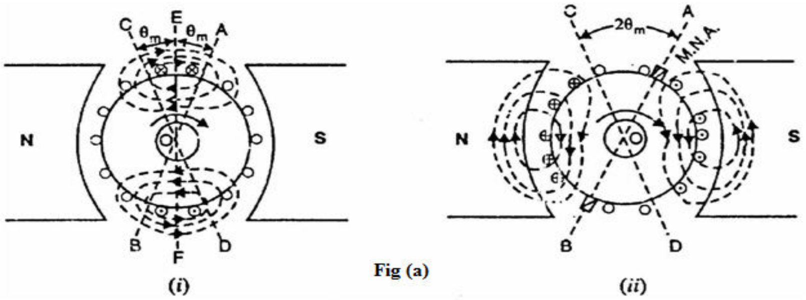
- Due to brush shift, the m.m.f. FA of the armature is also rotated through the same angle q.
- It is because some of the conductors which were earlier under N-pole now come under S-pole and vice-versa.
- The result is that armature m.m.f. FA will no longer be vertically downward but will be rotated in the direction of rotation through an angle q as shown in below Fig.
- Now FA can be resolved into rectangular components Fc and Fd.



  - The component F_d is in direct opposition to the main m.m.f. OF_m and is called the demagnetizing or weakening component of armature reaction, which reduces the main field flux.
  - The component F_c is at right angles to the main m.m.f. OF_m and is called the cross magnetizing or distorting component of armature reaction, which distorts the main field.

<u>Demagnetizing and Cross-Magnetizing Conductors</u>

- Consider a 2-pole generator with brushes shifted (lead) qm mechanical degrees from G.N.A.
- The armature conductors qm degrees on either side of G.N.A. produce flux in direct opposition to main flux as shown in Fig(a)(i).
- Thus the conductors lying within angles AOC = BOD = 2qm at the top and bottom of the armature produce demagnetizing effect.
- These are called demagnetizing armature conductors and constitute the demagnetizing ampere-turns of armature reaction



(i) Fig (a) (ii)

- The armature conductors lying between angles AOD and COB is at right angles to the main flux as shown Fig(a)(ii).
- As these conductors produce the cross-magnetizing (or) distortion effect they are called cross-magnetizing conductors and constitute the cross-magnetizing ampere-turns of armature reaction.
- **Calculation of Demagnetizing Ampere-Turns Per Pole (AT_d/Pole)**

  Let

  Z = total number of armature conductors
  I = current in each armature conductor
  = Ia/2 ... for simplex wave winding
  = Ia/P ... for simplex lap winding
  θ_m = forward lead in mechanical degrees

  Total demagnetizing armature conductors

  $$= \text{Conductors in angles AOC and BOD} = \frac{4\theta_m}{360} \times Z$$

  Since two conductors constitute one turn,

  $$\therefore \quad \text{Total demagnetizing ampere-turns} = \frac{1}{2}\left[\frac{4\theta_m}{360} \times Z\right] \times I = \frac{2\theta_m}{360} \times ZI$$

  These demagnetizing ampere-turns are due to a pair of poles.

  $$\therefore \quad \text{Demagnetizing ampere-turns/pole} = \frac{\theta_m}{360} \times ZI$$

  i.e.,

  $$AT_d / \text{pole} = \frac{\theta_m}{360} \times ZI$$

  As mentioned above, the demagnetizing ampere-turns of armature reaction can be neutralized by putting extra turns on each pole of the generator.

  $$\therefore \quad \text{No. of extra turns/pole} = \frac{AT_d}{I_{sh}} \qquad \text{for a shunt generator}$$

  $$= \frac{AT_d}{I_a} \qquad \text{for a series generator}$$

- **Calculation of cross magnetizing Ampere-Turns Per Pole (AT_c/Pole)**

  Total armature reaction ampere-turns per pole $= \frac{Z/2}{P} \times I = \frac{Z}{2P} \times I$

  As Demagnetizing ampere-turns per pole is given by $AT_d / \text{pole} = \frac{\theta_m}{360} \times ZI$

  Cross-magnetizing ampere-turns/pole are

  $$AT_d / \text{pole} = \frac{Z}{2P} \times I - \frac{\theta_m}{360} \times ZI = ZI\left(\frac{1}{2P} - \frac{\theta_m}{360}\right) \quad \therefore \quad AT_d / \text{pole} = ZI\left(\frac{1}{2P} - \frac{\theta_m}{360}\right)$$

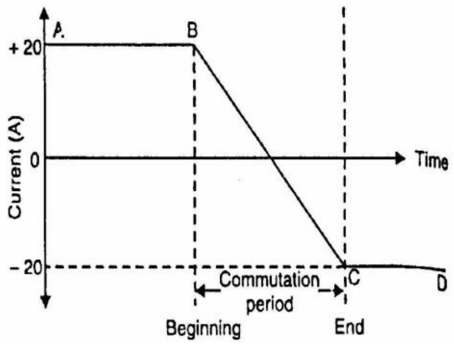
Commutation:

> *"The process by which current in the short circuited coil is reversed while it crosses the MNA is called* ***commutation".***

- The brief period during which coil remains short circuited is known as ***commutation period*** (T_c)
- If the current reversal is completed by the end of commutation period, it is called ***ideal commutation.***
- If the current reversal is not completed by the end of commutation period, it is called ***poor or bad commutation*** and results the sparking between the brush and the commutator segments.

Detailed process of commutation

- In Fig. (B) (i), the brush is in contact with segment 1 of the commutator. The commutator segment 1 conducts a current of 40 A to the brush; 20 A from coil A and 20 A from the adjacent coil as shown. The coil A has yet to undergo commutation.
- As the armature rotates, the brush will make contact with segment 2 and thus short-circuits the coil A as shown in Fig. (B) (ii). There are now two parallel paths into the brush as long as the short-circuit of coil A exists. Fig. (B) (ii) shows the instant when the brush is one-fourth on segment 2 and three-fourth on segment 1. For this condition, the resistance of the path through segment 2 is three times the resistance of the path through segment 1. The brush again conducts a current of 40 A; 30 A through segment 1 and 10 A through segment 2. Note that current in coil A (the coil undergoing commutation) is reduced from 20 A to 10 A.
- Fig. (B) (iii) shows the instant when the brush is one-half on segment 2 and one-half on segment 1. The brush again conducts 40 A; 20 A through segment 1 and 20 A through segment 2 (Q now the resistances of the two parallel paths are equal). Note that now. current in coil A is zero.
- Fig. (B) (iv) shows the instant when the brush is three-fourth on segment 2 and one-fourth on segment 1. The brush conducts a current of 40 A; 30 A through segment 2 and 10 A through segment 1. Note that current in coil A is 10 A but in the reverse direction to that before the start of commutation.
- Fig. (2.7) (v) shows the instant when the brush is in contact only with segment 2. The brush again conducts 40 A; 20 A from coil A and 20 A from the adjacent coil to coil A. Note that now current in coil A is 20 A but in the reverse direction. Thus the coil A has undergone commutation.
- Each coil undergoes commutation in this way as it passes the brush axis. Note that during commutation, the coil under consideration remains short circuited by the brush.
  - Adjacent Fig shows the current-time graph of the coil A undergoing commutation.
  - +20A is the current in coil A before commutation (right to left : AB line in graph)
  - -20A is the current in coil A after commutation (left to right : CD line in graph)



- B to C is the reversal of current in coil A happened completely during commutation period and is known as ideal commutation.
- For good commutation there is no sparking between the brush and the commutator

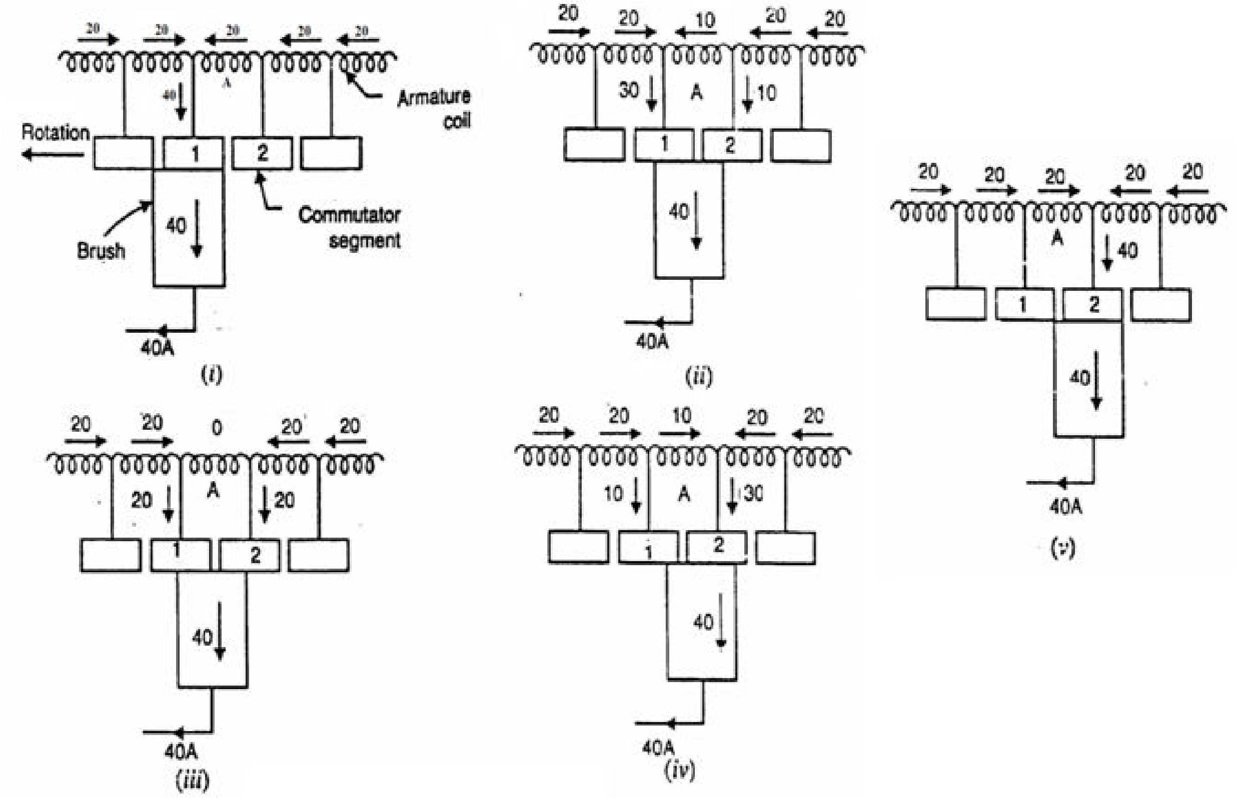
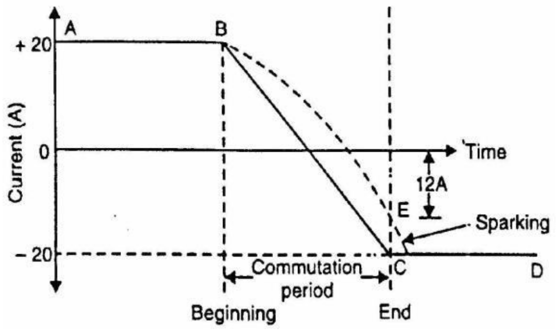
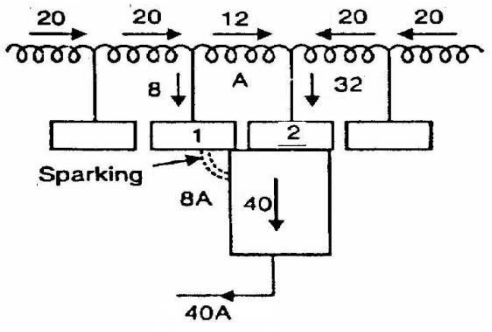


Fig B detail process of ideal commutation

Poor commutation

- As the armature coils have appreciable inductance. the current in the coil when undergoing commutation changes, self-induced e.m.f. is produced in the coil and is generally called reactance voltage.
- This reactance voltage opposes the change of current in the coil undergoing commutation.
- The result is that the change of current in the coil undergoing commutation occurs more slowly than it would be under ideal commutation.
- The straight line BC represents the ideal commutation whereas the curve BE represents the change in current when self-inductance of the coil is taken into account. Note that current CE (= 8A in Fig. below) is flowing from the commutator segment 1 to the brush and this results in sparking.
- The sparking results in overheating of commutator brush contact and causing damage to both.
- Below Fig illustrates how sparking takes place between the commutator segment and the brush. At the end of commutation or short-circuit period, the current in coil A is reversed to a value of 12 A (instead of 20 A) due to inductance of the coil.
- When the brush breaks contact with segment 1, the remaining 8 A current jumps from segment 1 to the brush through air causing sparking between segment 1 and the brush.





The reactance voltage in the short circuited coil under commutation is given by

$$E_R = L \times \frac{2I}{T_c}$$

Where T_c is the time of commutation and is given by

$$\text{Commutation period, } T_c = \frac{W_b - W_m}{v} \quad \text{seconds}$$

Here, Wb = brush width in cm;

Wm = mica thickness in cm;

v = peripheral speed of commutator in cm/s

Methods of Improving Commutation

- Improving commutation means to make current reversal in the short-circuited coil as sparkless as possible.
- The following are the two principal methods of improving commutation:

(i) Resistance commutation and (ii) E.M.F. commutation

Resistance Commutation:

- In this method the contact resistance between the brush and the commutator is made large, and then current would divide in the inverse ratio of contact resistances (as for any two resistances in parallel).
- This Resistance Commutation is achieved by using carbon brushes (instead of Cu brushes) which have high contact resistance.
- It may be noted that the main cause of sparking during commutation is the production of reactance voltage and carbon brushes cannot prevent it.
- Nevertheless, the carbon brushes do help in improving commutation.

The other minor advantages of carbon brushes are:

(i) The carbon lubricates and polishes the commutator.

(ii) If sparking occurs, it damages the commutator less than with copper brushes and the damage to the brush itself is of little importance.

E.M.F. commutation

- In this method, an arrangement is made to neutralize the reactance voltage by producing a reversing voltage in the coil undergoing commutation.
- If the reversing voltage is equal to the reactance voltage, the effect of the latter is completely wiped out and can get sparkless commutation.
- The reversing voltage may be produced in the following two ways:
  (i) By brush shifting
  (ii) By using interpoles or compoles

By brush shifting:

- In this method, the brushes are given sufficient forward lead (for a generator) to bring the short-circuited coil under the influence of the next pole of opposite polarity.
- Since the short-circuited coil is now in the reversing field, the reversing voltage produced cancels the reactance voltage.

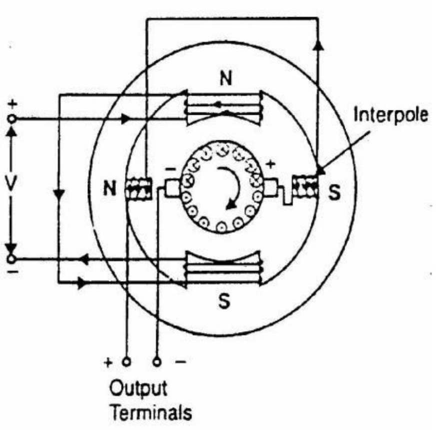
This method suffers from the following drawbacks:

(a) The reactance voltage depends upon armature current. Therefore, the brush shift will depend on the magnitude of armature current which keeps on changing. This necessitates frequent shifting of brushes.

(b) The greater the armature current, the greater must be the forward lead for a generator. This increases the demagnetizing effect of armature reaction and further weakens the main field.

By using interpoles or compoles

- The best way to produce reversing voltage to neutralize the reactance voltage is by using interpoles or compoles.
- These are small poles fixed to the yoke and spaced mid-way between the main poles
- They are wound with comparatively few turns and connected in series with the armature so that they carry armature current.
- Their polarity is the same as the next main pole ahead in the direction of rotation for a generator



Functions of Interpoles: The interpoles perform the following two functions:

I function of the interpole:

- As their polarity is the same as the main pole ahead (for a generator), they induce an e.m.f. in the coil (undergoing commutation) which opposes reactance voltage. This leads to sparkless commutation.
- The e.m.f. induced by compoles is known as commutating or reversing e.m.f.
- Since the interpoles carry the armature current and the reactance voltage is also proportional to armature current, the neutralization of reactance voltage is automatic.

II function of the interpole:

- The m.m.f. of the compoles neutralizes the cross-magnetizing effect of armature reaction in small region in the space between the main poles.
- It is because the two m.m.f.s oppose each other in this region.

Compensating Windings

- The cross-magnetizing effect of armature reaction may cause trouble in d.c. machines subjected to large fluctuations in load.
- In order to neutralize the crossmagnetizing effect of armature reaction, a compensating winding is used.
- A compensating winding is an auxiliary winding embedded in slots in the pole faces as shown in Fig.
- It is connected in series with armature in a manner so that the direction of current through the compensating conductors in any one pole face will be opposite to the direction of the current through the adjacent armature conductors

Calculation of number of compensating conductors/ pole face.

- Note that the current in the compensating conductors placed on the pole faces is the armature current Ia whereas the current in armature conductors is Ia/A

Let Zc = No. of compensating conductors/pole face
Za = No. of active armature conductors
Ia = Total armature current
Ia/A = Current in each armature conductor

$$\therefore \quad Z_c I_a = Z_a \times \frac{I_a}{A}$$

$$Z_c = \frac{Z_a}{A}$$

Calculation of AT/Pole for Compensating Winding

- Only the cross-magnetizing ampere-turns produced by conductors under the pole face are effective in producing the distortion in the pole cores.
- If Z is the total number of armature conductors and P is the number of poles, then,

$$\text{No. of armature conductors/pole} = \frac{Z}{P}$$

$$\text{No. of armature turns/pole} = \frac{Z}{2P}$$

$$\text{No. of armature turns under pole face} = \frac{Z}{2P} \times \frac{\text{Pole arc}}{\text{Pole pitch}}$$

If I is the current through each armature conductor, then,

$$\text{AT/pole required for compensating winding} = \frac{ZI}{2P} \times \frac{\text{Pole arc}}{\text{Pole pitch}}$$

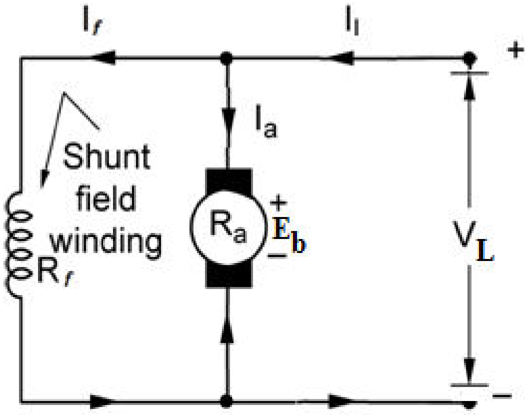
$$= \text{Armature AT/pole} \times \frac{\text{Pole arc}}{\text{Pole pitch}}$$

Types of D.C. Motors

Based on the field winding excited from the armature the dc motors are of three types

1. DC Shunt motor
2. DC Series motor
3. DC compound motor
    a. Long Shunt Compound motor
    b. Short Shunt Compound motor

DC SHUNT MOTOR



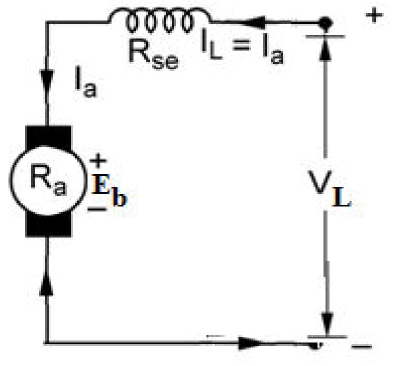
1. In the dc shunt motor the field winding circuit is connected in parallel to the armature circuit and as well as to the line.
2. The line current I_L is divided in to the field and the armature as I_f and I_a.
3. The shunt field winding has ***more number of turns with thin wire***, so that resistance of the field will be in the range of hundreds and was designed to withstand for the rated voltage.
4. Applying KVL to the armature loop the E_g is

$$E_b = V_L - I_a R_a - V_{brush}$$

$$I_a = I_L - I_f \quad \text{and} \quad I_L = \frac{P_L}{V} \quad \text{and} \quad I_f = \frac{V}{R_f}$$

DC SERIES MOTOR

1. In the dc series motor the field winding circuit is connected in series to the armature circuit and as well as to the line.
2. Here the armature current is equal to the series field current and also equal to the line.
4. The series field winding has ***less number of turns with thick wire***, so that resistance of the field will be in the smaller values and was designed to carry the rated current.



5. Applying KVL to the armature loop the E_g is

$$E_b = V_L - I_a(R_a + R_{se}) - V_{brush}$$

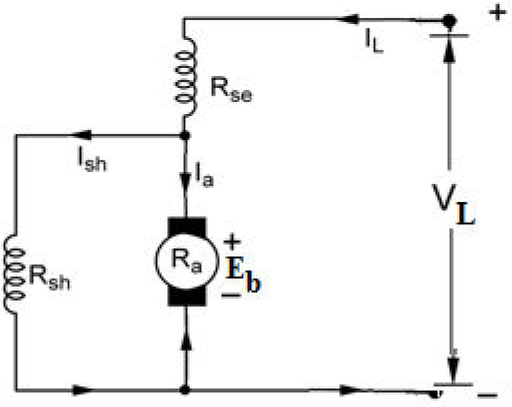
$$I_a = I_L = I_{se} \quad \text{and} \quad I_L = \frac{P_L}{V}$$

DC COMPOUND MOTORS

1. A compound motor has two field coils wound over the field poles.
2. The coil having large number of turns and thinner cross sectional area is called the shunt field coil and the other coil having few numbers of turns and large cross sectional area is called the series field coil.

3. Based on the series field winding connected to the armature the compound motors are classified as long shunt motor and short shunt motor

- **SHORT SHUNT MOTOR**



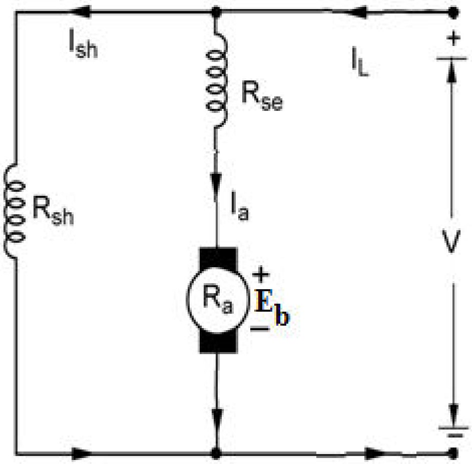
1. In a short shunt dc compound motor, the series field is connected in series to the line and shunt field winding is connected in parallel to the armature and the series combination of the line and series winding.
2. Thus, the series field current will depend on the line variations which will effect in further the shunt field current.
3. Applying KVL to the armature loop the E_g is

$$E_b = V_L - I_a R_a - I_L R_{se} - V_{brush}$$

$$I_a = I_L - I_f \quad \text{and} \quad I_L = \frac{P_L}{V} \quad \text{and}$$

$$I_f = \frac{V_L - I_L R_{se}}{R_f}$$

- **LONG SHUNT MOTOR**

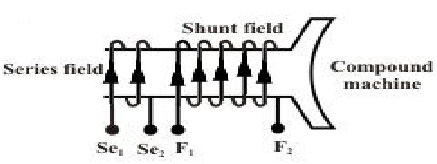


1. In a long shunt dc compound motor, the series field is connected in series to the armature and shunt field winding is connected in parallel to the armature and to the line.
2. Applying KVL to the armature loop the E_g is

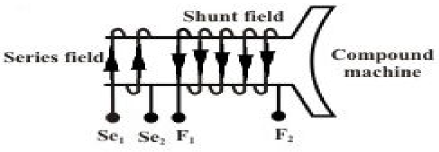
$$E_b = V_L - I_a (R_a + R_{se}) - V_{brush}$$

$$I_a = I_L - I_f \quad \text{and} \quad I_L = \frac{P_L}{V} \quad \text{and} \quad I_f = \frac{V}{R_f}$$

- Also, the dc compound motors are further classified into two types based on the compounding of the series flux to the shunt flux. They are cumulatively compounded and differentially compounded motors
- In the cumulatively compound motor, the series flux aids to the shunt field flux and the net flux increases, whereas in the differentially compounded motors the series flux opposes the shunt field flux and the net resultant flux decreases.
- The below figure shows the arrangement of the series and shunt field coils in the pole core in both cumulative and differentially compounded motors.

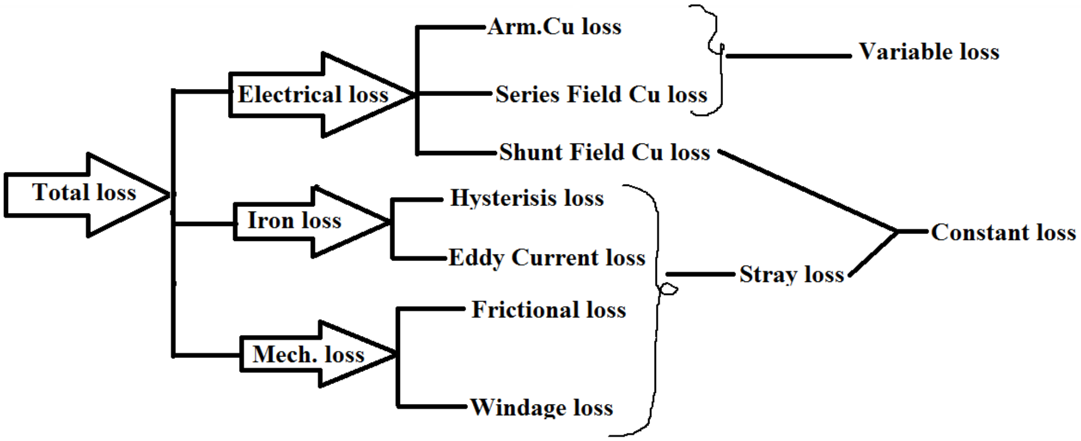


Cummulatively compounded



differentially compounded

Losses in dc machine



Power Stages in DC Generator:

The power stages in a d.c. generator are represented diagrammatically in below Fig.

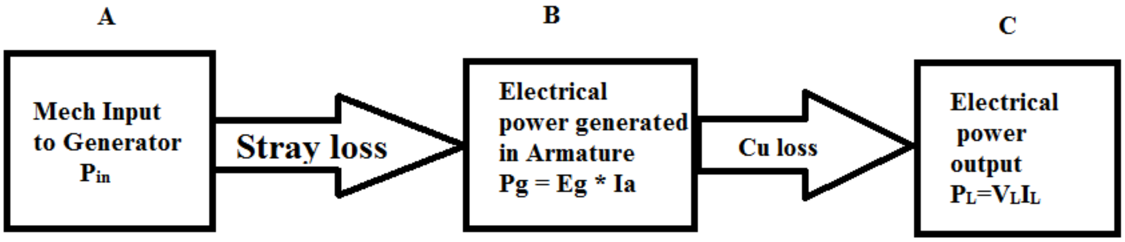
A - B = Iron and friction losses

B - C = Copper losses

Overall efficiency, η_c = C/A

Electrical efficiency, η_e = C/B

Mechanical efficiency, η_m = B/A

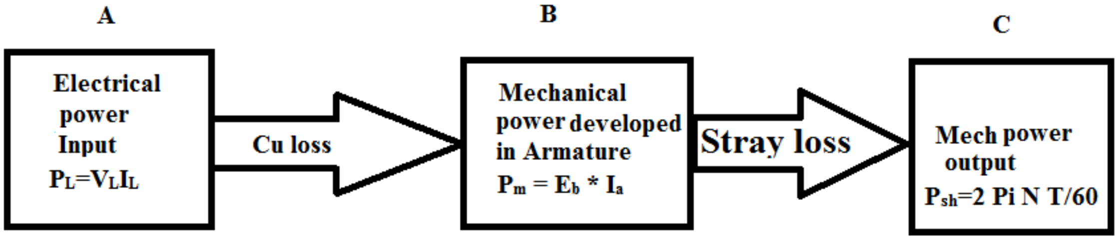


Power Stages in DC Motor:

The power stages in a d.c. motor are represented diagrammatically in below Fig.

A - B = Copper losses

B - C = Iron and friction losses



Overall efficiency, η_c = C/A

Electrical efficiency, η_e = B/A

Mechanical efficiency, η_m = C/B

Condition for maximum efficiency for dc motor:

We assume that field current I_f remains constant during change of loading. Let,

$$
\begin{aligned}
P_{rot} &= \text{constant rotational loss} \\
V\,I_f &= \text{constant field copper loss} \\
\text{Constant loss } P_{const} &= P_{rot} + V\,I_f \\
\text{Now, input power drawn from supply} &= V\,I_L \\
\text{Power loss in the armature,} &= I_a^2 r_a \\
\text{Net mechanical output power} &= VI_L - I_a^2 r_a - (VI_f + P_{rot}) \\
&= VI_L - I_a^2 r_a - P_{const} \\
\text{so, efficiency at this load current } \eta_m &= \frac{VI_L - I_a^2 r_a - P_{const}}{VI_L}
\end{aligned}
$$

Now the armature copper loss $I_a^2 r_a$ can be approximated to $I_L^2 r_a$ as $I_a \approx I_L$. This is because the order of field current may be 3 to 5% of the rated current. Except for very lightly loaded motor, this assumption is reasonably fair. Therefore replacing I_a by I_f in the above expression for efficiency η_m, we get,

$$
\begin{aligned}
\eta_m &= \frac{VI_L - I_L^2 r_a - P_{const}}{VI_L} \\
&= 1 - \frac{I_L r_a}{V} - \frac{P_{const}}{VI_L}
\end{aligned}
$$

Thus, we get a simplified expression for motor efficiency η_m in terms of the variable current (which depends on degree of loading) I_L, current drawn from the supply. So to find out the condition for maximum efficiency, we have to differentiate η_m with respect to I_L and set it to zero as shown below.

$$
\begin{aligned}
\frac{d\eta_m}{dI_L} &= 0 \\
\text{or, } \frac{d}{dI_L}\left(\frac{I_L r_a}{V} - \frac{P_{const}}{VI_L}\right) &= 0 \\
\text{or, } -\frac{r_a}{V} + \frac{P_{const}}{VI_L^2} &
\end{aligned}
$$

$$\therefore \text{Condition for maximum efficiency is } I_L^2 r_a \approx I_a^2 r_a = P_{const}$$

$$\text{So, the armature current at which efficiency becomes maximum is } I_a = \sqrt{P_{const}/r_a}$$

Characteristics of dc motors:

(i) Torque and Armature current characteristic ($T_a V_s I_a$)

It is the curve between armature torque Ta and armature current I_a of a d.c. motor. It is also known as electrical characteristic of the motor.

(ii) Speed and armature current characteristic ($N\ V_s I_a$)

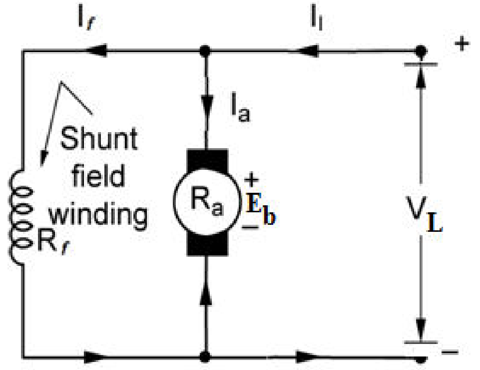
It is the curve between speed N and armature current Ia of a d.c. motor. It is very important characteristic as it is often the deciding factor in the selection of the motor for a particular application.

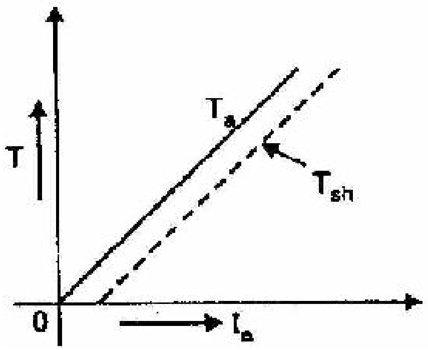
(iii) Speed and torque characteristic ($N\ V_s$ Ta)

It is the curve between speed N and armature torque Ta of a d.c. motor. It is also known as mechanical characteristic.

Characteristics of Shunt Motors

Fig. below shows the connections of a d.c. shunt motor. The field current I_f is constant since the field winding is directly connected to the supply voltage V which is assumed to be constant. Hence, the flux in a shunt motor is approximately constant



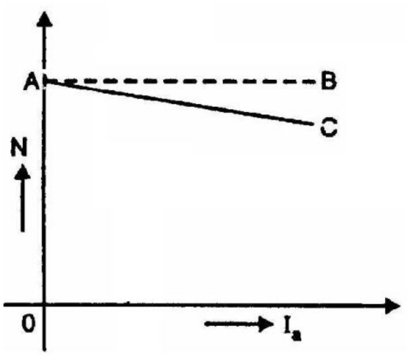


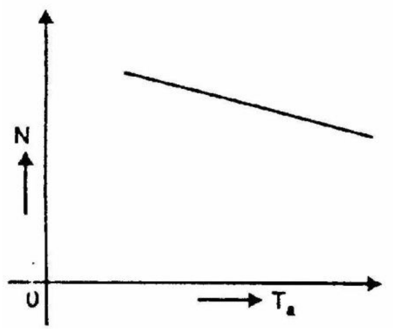
(i) $T_a V_s I_a$ Characteristic.

- We know that in a d.c. motor, $T_a \alpha I_a$ Since the motor is operating from a constant supply voltage, flux Φ is constant $T_a \alpha \Phi I_a$
- T_a/I_a characteristic is a straight line passing through the origin as shown in Fig. The shaft torque (T_{sh}) is less than T_a and is shown by a dotted line.
- It is clear from the curve that a very large current is required to start a heavy load. Therefore, a shunt motor should not be started on heavy load.

(ii) $N\ V_s I_a$ Characteristic.

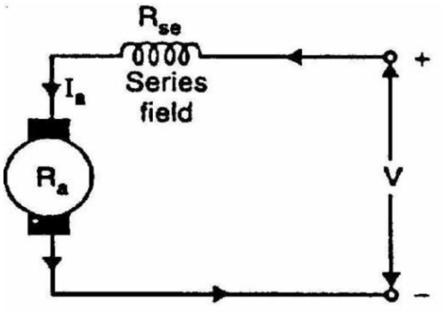
- The speed N of a. d.c. motor is given by $N \alpha \frac{Eb}{\varphi}$
- The flux Φ and back e.m.f. E_b in a shunt motor are almost constant under normal conditions.
- Therefore, speed of a shunt motor will remain constant as the armature current varies
- Strictly speaking, when load is increased, ($E_b = V_L - I_a R_a$) and E_b decrease due to the armature resistance drop and armature reaction respectively.
- However, E_b decreases slightly so that the speed of the motor decreases slightly with load (line AC).

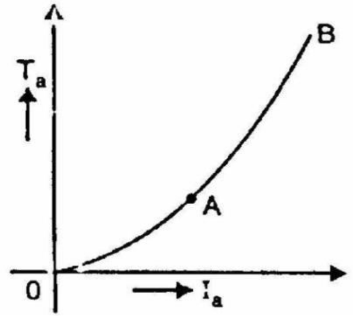




Characteristics of Series Motors:

- Fig. shows the connections of a series motor. Note that current passing through the field winding is the same as that in the armature.
- If the mechanical load on the motor increases, the armature current also increases.
- Hence, the flux in a series motor increases with the increase in armature current and vice-versa.





Ta/Ia Characteristic of DC Series motor:

- We know that Ta αΦIa
- Upto magnetic saturation, Φ α Ia so that Ta α I_a^2
- After magnetic saturation, Φ is constant so that Ta α Ia
- Thus upto magnetic saturation, the armature torque is directly proportional to the square of armature current. If Ia is doubled, Ta is almost quadrupled.
- Therefore, Ta/Ia curve upto magnetic saturation is a parabola (portion OA in Fig above.). However, after magnetic saturation, torque is directly proportional to the armature current. Therefore, Ta/Ia curve after magnetic saturation is a straight line (portion AB of the curve).
- It may be seen that in the initial portion of the curve (i.e. upto magnetic saturation), Ta α I_a^2
- This means that starting torque of a d.c. series motor will be very high as compared to a shunt motor (where that Ta α Ia).

(ii) **N/I_a Characteristic.** The speed N of a series motor is given by;

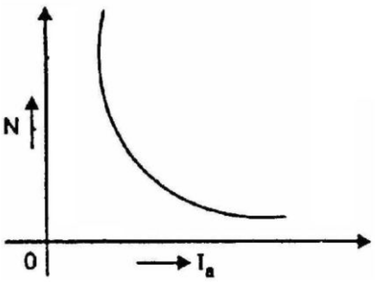
$$N \propto \frac{E_b}{\phi} \qquad \text{where} \qquad E_b = V - I_a(R_a + R_{se})$$

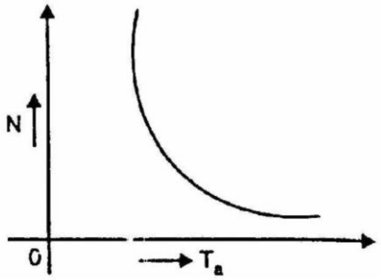
When the armature current increases, the back e.m.f. E_d decreases due to $I_a(R_a + R_{se})$ drop while the flux ϕ increases. However, $I_a(R_a + R_{se})$ drop is quite small under normal conditions and may be neglected.

$$\therefore \quad N \propto \frac{1}{\phi}$$

$$\propto \frac{1}{I_a} \text{ upto magnetic saturation}$$

- Thus, upto magnetic saturation, the N/Ia curve follows the hyperbolic path as shown in Fig.
- After saturation, the flux becomes constant and so does the speed.





(iii) N/Ta Characteristic of DC Series motor:

- The N/Ta characteristic of a series motor is shown in above Fig.
- It is clear that series motor develops high torque at low speed and vice-versa.
- It is because an increase in torque requires an increase in armature current, which is also the field current.
- The result is that flux is strengthened and hence the speed drops (N α 1/Φ). Reverse happens should the torque be low.

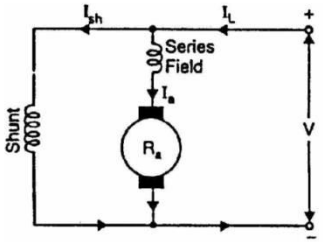
Compound Motors:

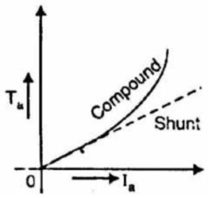
A compound motor has both series field and shunt field. The shunt field is always stronger than the series field.

Compound motors are of two types:

(i) *Cumulative-compound motors* in which series field aids the shunt field.

(ii) *Differential-compound motors* in which series field opposes the shunt field.





Differential compound motors are rarely used due to their poor torque characteristics at heavy loads.

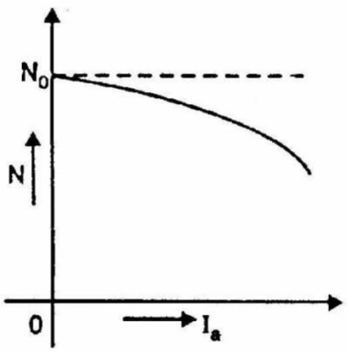
Ta/Ia Characteristics of DC Compound motor:

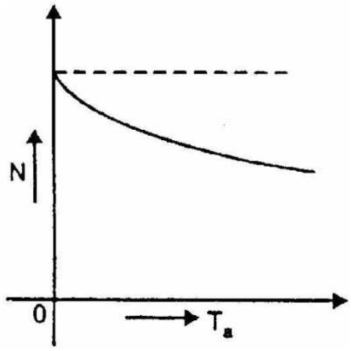
- As the load increases, the series field increases but shunt field strength remains constant. Consequently, total flux is increased and hence the armature torque.

- It may be noted that torque of a cumulative-compound motor is greater than that of shunt motor for a given armature current due to series field

(ii) N/Ia Characteristics of DC compound motors.

- As explained above, as the load increases, the flux per pole also increases. Consequently, the speed ($N\ \alpha 1/\Phi$) of the motor tails as the load increases.
- It may be noted that as the load is added, the increased amount of flux causes the speed to decrease more than does the speed of a shunt motor.
- Thus the speed regulation of a cumulative compound motor is poorer than that of a shunt motor.

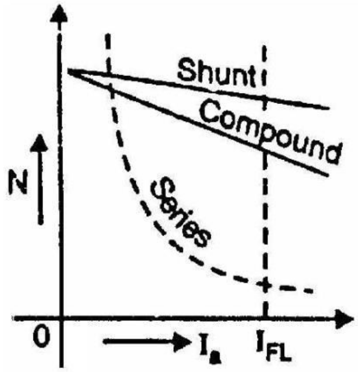


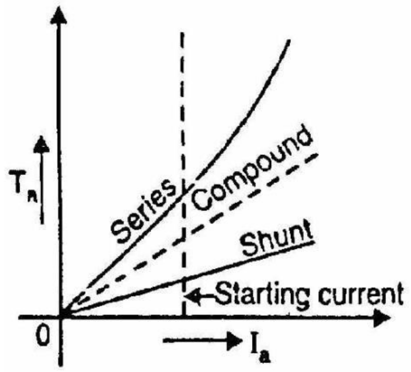


N/Ta Characteristics of DC compound motor:

Fig. above shows N/Ta characteristic of a cumulative compound motor. For a given armature current, the torque of a cumulative compound motor is more than that of a shunt motor but less than that of a series motor.

Comparison of Three Types of Motors





(i) The speed regulation of a shunt motor is better than that of a series motor.

However, speed regulation of a cumulative compound motor lies between shunt and series motors.

(ii) For a given armature current, the starting torque of a series motor is more than that of a shunt motor. However, the starting torque of a cumulative compound motor lies between series and shunt motors

(iii) Both shunt and cumulative compound motors have definite no-load speed. However, a series motor has dangerously high speed at no-load.

Applications of D.C. Motors

1. Shunt motors

The characteristics of a shunt motor is an approximately constant speed motor. It is, used

(i) where the speed is required to remain almost constant from no-load to full-load

***Industrial use*: Lathes, drills, boring mills, shapers, spinning and weaving machines etc.**

2. Series motors

It is a variable speed motor i.e., speed is low at high torque and vice-versa. It is used

(i) where large starting torque is required e.g., in elevators and electric Traction

(ii) where the load is subjected to heavy fluctuations and the speed is automatically required to reduce at high torques and vice-versa

***Industrial use*: Electric traction, cranes, elevators, air compressors, vacuum cleaners, hair drier, sewing machines etc.**

3. Compound motors

Differential-compound motors are rarely used because of their poor torque characteristics. However, cumulative-compound motors are used where a fairly constant speed is required with irregular loads or suddenly applied heavy loads.

***Industrial use*: Presses, shears, reciprocating machines etc.**

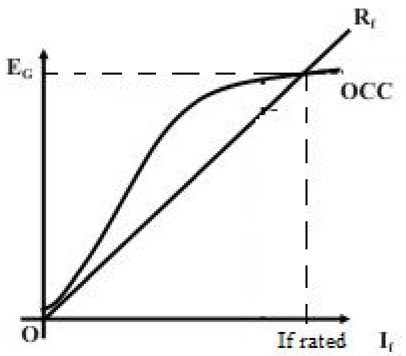
Characteristics of DC Generator:

There are three characteristics for a dc generator, they are

1. Open circuit characteristics - OCC (or) Magnetization Characteristics (or) No load characteristics (E_0 vs I_f)
2. Internal Characteristics (E_g vs I_a)
3. External Characteristics (V_L vs I_L)

DC Shunt Generator:

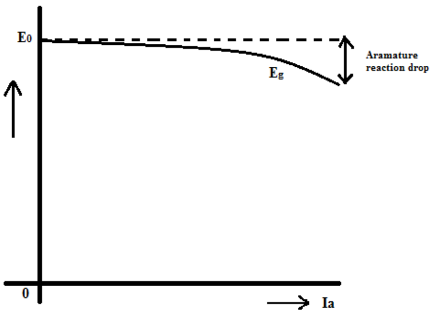
OCC of DC Shunt Generator (E_0 vs I_f):



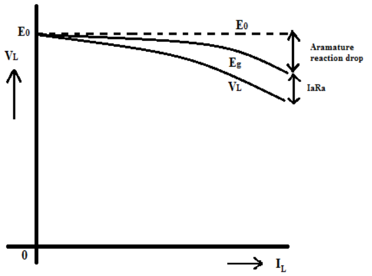
- The OCC is drawn at constant speed and the intersection of R_f line with drawn OCC gives the open circuit voltage E_0
- R_{fc} is the critical field resistance which is the tangent line for the OCC drawn at a given speed.
- N_c is the critical speed for which the R_f of the shunt generator will becomes as R_{fc}

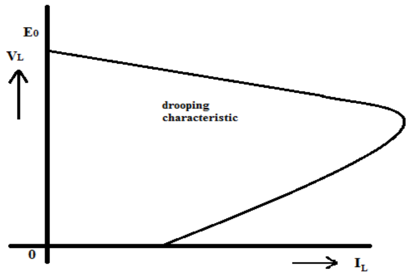
Internal Characteristics of shunt generator (E_g vs I_a):

- At no-load the voltage is E_g=E_0
- Speed is constant for all the operating conditions
- As the load increases the armature reaction effect increases there by the armature reaction drop is observed in the E_0 and the reduced voltage is called E_g
- This drop is small and can be considered as almost constant voltage.



External Characteristics of shunt generator (V_L vs I_L):





- At no-load the voltage is $E_g=E_0$
- Speed is constant for all the operating conditions
- As the load increases the armature reaction effect increases there by the armature reaction drop is observed in the E_0 and the reduced voltage is called E_g
- In addition to this the I_aR_a drop increases with increase in the load and hence the voltage drops, therefore the shunt generator characteristics are ***drooping characteristics.***
- This drop is small and can be considered as almost constant voltage with field regulators.

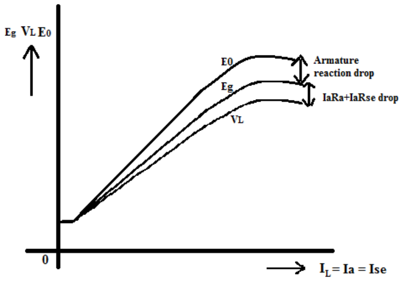
DC Series Generator:

The flux in the series generator is directly proportional to the armature current i.e., load

At no-load Ia=0 Eg is small voltage due to the residual flux.

As the load increases the voltage increases in proportion to the field and the curves of OCC, Internal and external are shown in the figure.

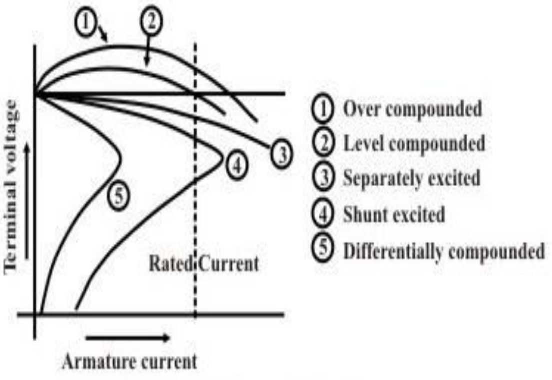
Therefore, the series generator characteristics are ***raising characteristics.***



DC Compound Generator:

The dc compound generators are three types

1. ***Over compound :***
   In over compound at full load operating condition the $V_L > V_{Lrated}$
2. ***Under compound:***
   In under compound at full load operating condition the $V_L < V_{Lrated}$
3. ***Level or Flat compound:***
   In level compound at full load operating condition the $V_L = V_{Lrated}$



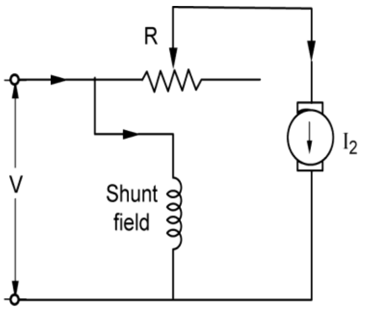
Load characteristics of d.c generators.

- The adjacent figure shows the external characteristics of all the generators taking the reference as the full load voltage and full load current respectively.

Unit -3 Electrical machines – I

Necessity of starter:

- The function of the starter is to limit the starting current in the motor.
- The current drawn by the motor armature is given by $I_a = \frac{V - E_b}{R_a}$ where V is the supply voltage, E_b is the back emf and R_a is the armature resistance of the motor.
- At starting, when motor is at rest there is no back emf in the armature (since $E_b \propto N$)
- Now the total supply voltage is applied across the stationary armature and it will draw a very large current because of small armature resistance.



- Consider the case of 440 V, 5 HP (3.73 KW) motor having a cold armature resistance of 0.25 Ω and full load current of 50A.
- If this motor is started from the line directly, it will draw a starting current of **Fehler!** = 1760 A which is **Fehler!** = 35.2 times its full-load current.
- This excessive current will blow out the fuses and damages the commutator and brushes. To avoid this, a resistance is placed in series to the armature for the time duration until the motor pickups the speed.
- Once the motor pickups the speed, the back emf is developed and the current was limited by the small voltage (V_L-E_b) applied to the armature against the small resistance.
- Thus, the starter is used to limit this starting current by inserting the resistance only at the starting time.
- There are three types of starters used namely
  a) 3 point starter b) Four point starter c) Two point starter

Three Point Starter

- The 3 terminals of the three point starter are marked A, B and C.
- First terminal A is connected to the handle arm (L) through the overload release (OLR) from the supply terminals
- Second terminal B is connected to the field winding of the motor through the Hold ON coil from the stud 1 of the external resistance placed in series to the armature.
- Third terminal C is connected to the armature by inserting the external resistance.
- The handle initially is at OFF position and when the supply is given, to start the motor the handle is dragged towards the stud 1.
- This position of the handle divides the line current into two paths one path to the armature through the current limiting resistance and second path to the field winding.
- Thus the current is limited by this resistance placed in series with the armature. Also as the speed picksup, the handle was dragged over the studs from off position to ON position.
- At this ON position all the external resistance is removed from the armature and the spring on the other side of the handle develops the restraining torque with the spring placed.
- The soft iron piece (S) on the handle is attracted by the hold on coil in normal running conditions
- The resistance that was removed from the armature circuit will be added to the field circuit.
- Thus the field current is reduced, to overcome the drawback of weakening of the flux the field winding terminal is connected from the brass arc placed below the studs and is shown in the figure

Hold ON coil (or) No Volt Release (NVR)

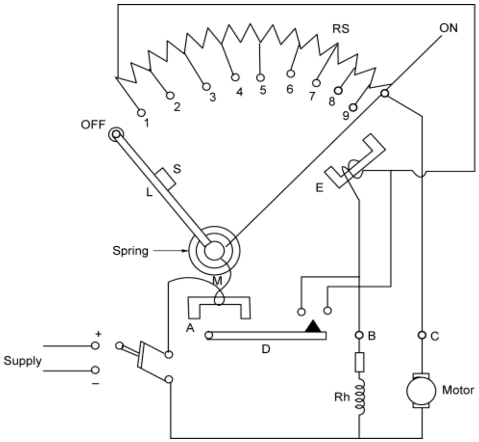
- The Normal function of the HOLD-ON coil is to hold on the arm in then full running position when the motor is in normal operation.
- When the supply failure (or) disconnection, it is de-energised, so that handle is released from the hold on coil and pulled back by the spring to the OFF position.
- The Hold ON coil protects the motor from dangerous speed when field circuit opens.

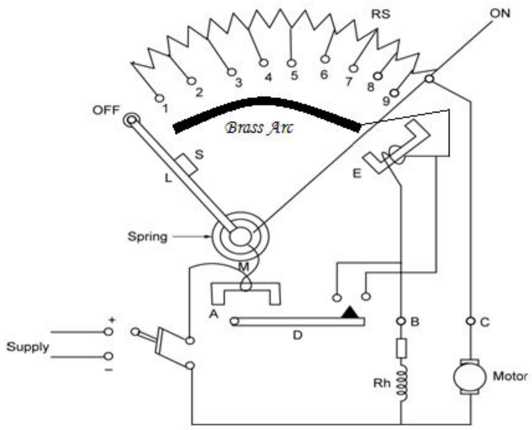
Over Load Release (OLR)

- It consists of an electro-magnet connected in the supply line.
- If motor becomes over loaded, then D is lifted and short circuits the electro-magnet. Hence arm is released and returns to OFF position.

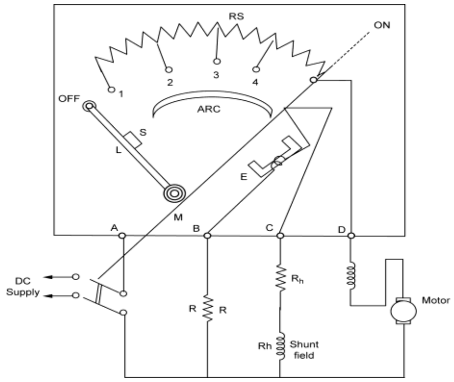
Disadvantage of three point starter:

- To control the speed of motor, a field rheostat is connected in the field circuit.The motor speed is increased by decreasing the flux ($N \propto I/\phi$). There is a difficulty that if too much resistance is added by the field rheostat, then field current is reduced very much so that the current in the hold on coil is unable to create enough Electromagnetic pull to overcome the spring tension. Hence arm is pulled back to OFF position.
- Therefore the shunt motor with this three point starter is not suitable for adjustable speed drive applications.





Four Point Starter:



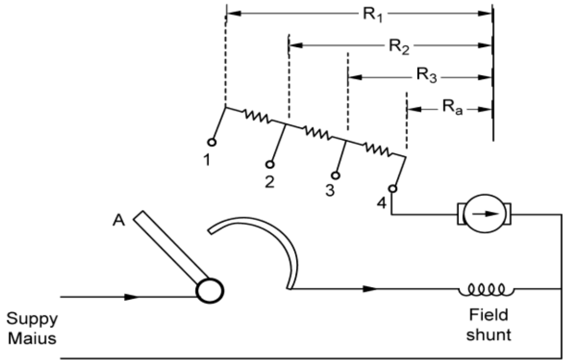
It is connected to a long shunt compound motor then compared to the three-point starter, one change is the HOLD-ON coil has been connected across the line through a protecting resistance.

When the aim touches stud No. 1, then the line current will divide into three parts.

1. One part passes through starting resistance R_S, series field and motor armature which limits the starting current.
2. The second part passes through the shunt field and its field rheostat R_h and
3. The third part passes through the HOLD-ON coil and current-protecting resistance R. It should be particularly noted that with this arrangement any change of current in the shunt field circuit does not affect the current passing through the HOLD-ON coil because the two circuits are independent of each other. It means the electro-

magnetic pull exterted by the HOLD-ON coil will always be sufficient and will prevent the spring from restoring the starting aim to OFF position no matter how the field Rheostat (or) regulator is adjusted.

Designing formulas for the 3 point starter:



$$\frac{I_1}{I_2}=\frac{R_1}{R_2}=\frac{R_2}{R_3}=\frac{R_3}{R_4}=\frac{R_4}{R_5}.....\frac{R_{n-1}}{R_a}=K$$

$$R_1=K^{n-1}R_a$$

$$K^{n-1}=\frac{V}{I_{\max}R_a} \text{ and } K^n=\frac{V}{I_{\min}R_a}$$

$$n=1+\frac{\log(V/R_aI_{\max})}{\log K}$$

Ex: ***A 220 V shunt motor has an armature resistance of 0.5 Ω. The armature current at starting must not exceed 40A. If the number of sections is 6, calculate the values of the resistor steps to be used in this starter.***

Since the number of starter sections is specified, we will use the relation.

Fehler! $= K^{\textbf{Fehler!}}$ (or) $R_1 = R_a K^{n-1}$

$\Rightarrow R_1 =$ **Fehler!** $= 5.5\ \Omega$, $R_a = 0.4\ \Omega$, $n - 1 = 6 \Rightarrow n = 7$

$6 \log_{10} K = \log_{10} 13.75 = 1.1383$, $K = 1.548$

$\Rightarrow R_1 = R_a K^{n-1} \Rightarrow 5.5 = 0.4 K^6$ (or) $K^6 = 13.75$

$\Rightarrow$ $R_2 =$ **Fehler!** $=$ **Fehler!** $= 3.553\ \Omega$

$\Rightarrow$ $R_3 =$ **Fehler!** $=$ **Fehler!** $= 2.295\ \Omega$

$R_4 =$ **Fehler!** $= 1.482\ \Omega$

$R_5 =$ **Fehler!** $= 0.958\ \Omega$

$R_6 =$ **Fehler!** $= 0.619\ \Omega$

Resistance of 1st section $= R_1 - R_2 = 5.5 - 3.553 = 1.947\ \Omega$

2nd section $= R_2 - R_3 = 3.553 - 2.295 = 1.258\ \Omega$

3rd section $= 2.295 - 1.482 = 0.813\ \Omega = R_3 - R_4$

4th section $= R_4 - R_5 = 1.482 - 0.956 = 0.524\ \Omega$

5th section $= R_5 - R_6 = 0.958 - 0.619 = 0.339\ \Omega$

6th section $= R_6 - R_a = 0.619 - 0.4 = 0.219\ \Omega$

Speed control of DC motors:

The speed of a d.c. motor is given by:

$$N\alpha\frac{Eb}{\Phi} \text{ or } N\alpha\frac{V_L-I_aR}{\Phi}$$ where R is Ra for shunt motor and (Ra + Rse). for series motor

From the above expression,

The speed of a d.c. motor is controlled

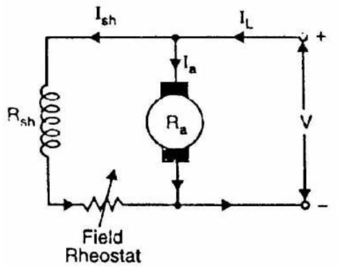
(i) By varying the flux per pole (f) known as flux control method.

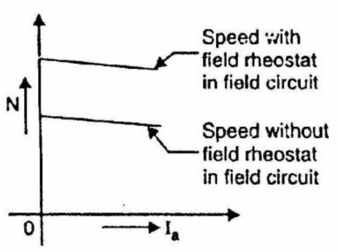
(ii) By varying the R_a and is known as armature control method.

(iii) By varying the applied voltage V and is known as voltage control method.

Speed Control of D.C. Shunt Motor

a) Field control method:





- In this field control method the variable is flux (f)
- The rheostat is placed in series to the field winding, as the field resistance increases the field current decreases and this weakens the flux
- The weakening of the flux increases the speed since speed is inversely proportional to the flux.
- Thus using the field control, above base speeds can be controlled.
- This method is also known as constant power method or variable torque method.

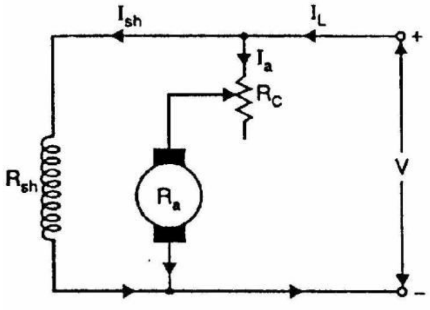
Advantages

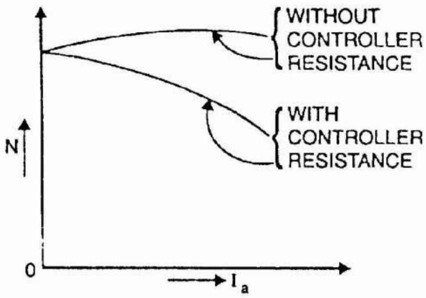
- This is an easy and convenient method.
- It is an inexpensive method since very little power is wasted in the shunt field rheostat due to relatively small value of I_f
- The speed control exercised by this method is independent of load on the machine.

Disadvantages

- Only speeds higher than the normal speed can be obtained.
- There is a limit to the maximum speed obtainable by this method. It is because if the flux is too much weakened, commutation becomes poorer.

b) Armature control method





- In this armature resistance control method the variable is R_a
- The rheostat is placed in series to the armature winding, as the R_a increases the I_aR_a drop increases and this decreases the speed.
- The decreasing of the back emf decreases the speed since speed is directly proportional to E_b.
- Thus using the R_a control method, below base speeds can be controlled.
- This method is also known as constant torque method or variable power method.

Disadvantages

- A large amount of power is wasted in the controller resistance since it carries full armature current Ia.

- The speed varies widely with load since the speed depends upon the voltage drop in the controller resistance and hence on the armature current demanded by the load.
- The output and efficiency of the motor are reduced.
- This method results in poor speed regulation.

c) Voltage control method by Ward-Leonard system

- This method is used to get the wide range of speed control 10:1.
- As the speed of the motor is directly proportional to the applied voltage to the armature, thus by applying the variable voltage the speed is controlled.
- The armature of the shunt motor M (whose speed is to be controlled) is connected directly to a d.c. generator G driven by a constant-speed a.c. motor A.
- The field of the shunt motor is supplied from a constant-voltage exciter E.
- The field of the generator G is also supplied from the exciter E.
- The voltage of the generator G can be varied by means of its field regulator.
- By reversing the field current of generator G by controller FC, the voltage applied to the motor may be reversed.

Advantages

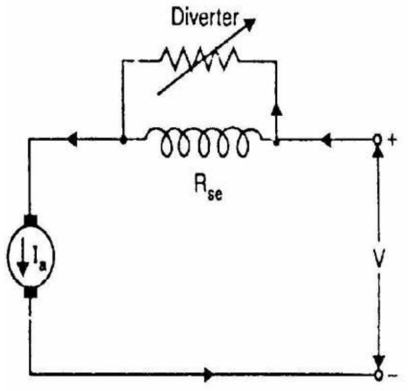
- The speed of the motor can be adjusted through a wide range without resistance losses which results in high efficiency.
- The motor can be brought to a standstill quickly, simply by rapidly reducing the voltage of generator G.
- The disadvantage of the method is that a special motor-generator set is required for each motor and the losses in this set are high if the motor is operating under light loads for long periods.

Speed Control of D.C. Series Motor

a) Flux control method

In this method, the flux produced by the series motor is varied and hence the speed.

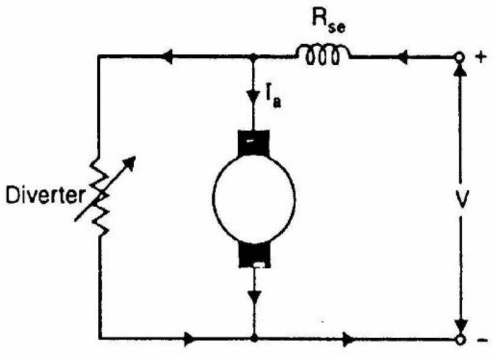
The variation of flux can be achieved in the following ways:



(i) Field diverters.

- In this method, a variable resistance (called field diverter) is connected in parallel with series field winding as shown in Fig.
- Its effect is to shunt some portion of the line current from the series field winding, thus weakening the field and increasing the speed (Nα1/Φ).
- This method can only provide speeds above the normal speed. The series field diverter method is often employed in traction work.

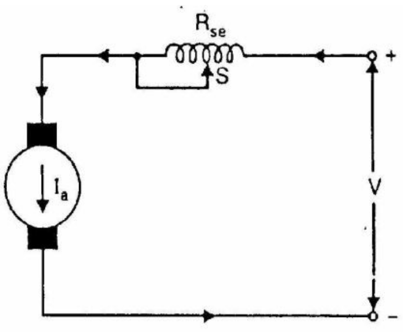
ii) Armature diverter.



- In order to obtain speeds below the normal speed, a variable resistance (called armature diverter) is connected in parallel with the armature as shown in Fig.
- The diverter shunts some of the line current, thus reducing the armature current.
- Now for a given load, if Ia is decreased, the flux Φ must increase ($T\alpha\Phi I_a$).

- Since ($N \alpha 1/\Phi$). The motor speed is decreased.
- By adjusting the armature diverter, any speed lower than the normal speed can be obtained.

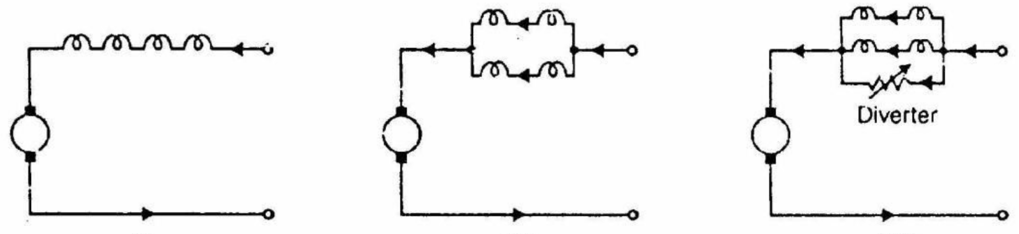
iii) Tapped field control.



- In this method, the flux is reduced by decreasing the number of turns of the series field winding as shown in Fig, and hence speed is increased
- The switch S can short circuit any part of the field winding, thus decreasing the flux and raising the speed.
- With full turns of the field winding, the motor runs at normal speed and as the field turns are cut out; speeds higher than normal speed are achieved.

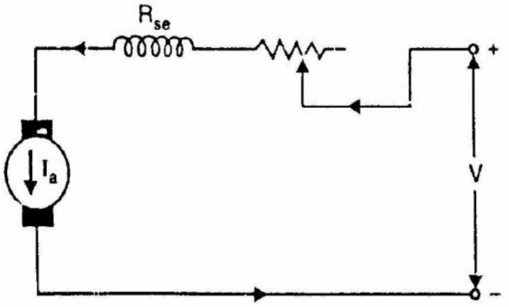
iv) Paralleling field coils.

This method is usually employed in the case of fan motors. By regrouping the field coils as shown in Fig below, several fixed speeds can be obtained.



(*i*) (*ii*) (*iii*)

b) Armature-resistance control:



- In this method, a variable resistance is directly connected in series with the supply to the complete motor as shown in Fig.
- This reduces the voltage available across the armature and hence the speed falls.
- By changing the value of variable resistance, any speed below the normal speed can be obtained.
- This is the most common method employed to control the speed of d.c. series motors.
- Although this method has poor speed regulation, this has no significance for series motors because they are used in varying speed applications.
- The loss of power in the series resistance for many applications of series motors is not too serious since in these applications.

<u>Testing of DC machines:</u>

Testing of DC machines can be broadly classified as

i) Direct method of Testing

ii) Indirect method of testing

Direct method of testing:

In this method, the DC machine is loaded directly by means of a brake applied to water cooled pulley coupled to the shaft of the machine. The input and output are measured and efficiency is determined by $\eta = \frac{output}{input}$

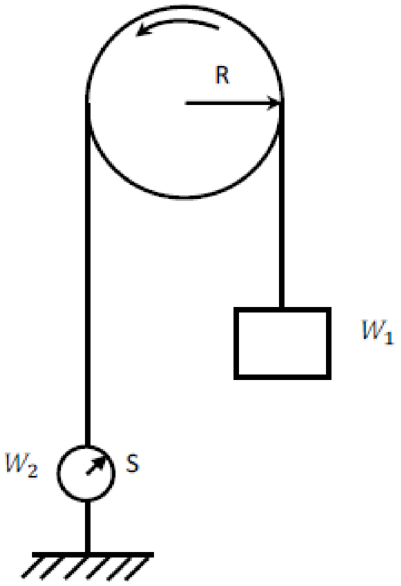
It is not practically possible to arrange loads for machines of large capacity.

Indirect method of testing:

In this method, the losses are determined without actual loading the machine. If the losses are known, then efficiency can be determined. Swinburne's test, Hopkinson's test and retardation tests are commonly used on shunt motors.

(i) **BRAKE TEST**: is a direct method of testing.

In this method of testing motor shaft is coupled to a Water cooled pulley which is loaded by means of weight as shown in figure



W_1= suspended weight in kg

W_2 = Reading in spring balance in kg

R = radius of pulley

N = speed in rpm

V = Supply voltage

I = Full Load Current

Net pull due to friction $= (W_1 - W_2)$ kg

$= 9.81\ (W_1 - W_2)$ Newton 1

Shaft torque $T_{sh} = (W_1 - W_2)R\ kg - mt.$

$= 9.81\ (W_1 - W_2)\ R\ N - mt$ 2

Motor output power = $T_{sh}\ X\ \frac{2\ \pi\ N}{60}$ Watt

Input power = VI watts 3

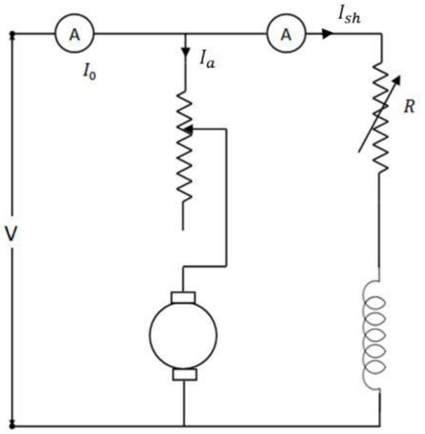
Therefore efficiency $= \frac{output}{input}$

This method of testing can be used for small motors only because for a large motor it is difficult to arrange for dissipation of heat generated at the brake.

(ii)Swinburne's Test:

This test is a no load test and hence cannot be performed on series motor.

The circuit connection is shown in Figure



The machine is run on no load at rated speed which is adjusted by the shunt field resistance.

Advantages

1. Economical, because no load input power is sufficient to perform the test
2. Efficiency can be pre-determined
3. As it is a no load test, it cannot be done on a dc series motor

Disadvantages

1. Change in iron loss from no load to full load is not taken into account. (Because of armature reaction, flux is distorted which increases iron losses).
2. Stray load loss cannot be determined by this test and hence efficiency is over estimated.
3. Temperature rise of the machine cannot be determined.
4. The test does not indicate whether commutation would be satisfactory when the machine is loaded.

I_O = No load current; I_{sh} = shunt field current

I_{ao} = No load armature current = $(I_o - I_{sh})$

V= Supply Voltage

No load input =VI_o watts.

No load power input supplies

(i) Iron losses in the core

(ii) Friction and windings loss and

(iii) Armature copper loss.

Let I = load current at which efficiency is required

$I_a = I - I_{sh}$ if machine is motoring; $I + I_{sh}$ if machine is generating

Efficiency as a motor:

Input = VI; $I_a^2 r_a = (I - I_{sh})^2 r_a$

Constant losses $W_c = VI_o - (I_o - I_{sh})^2 r_a$ …………… 7

Total losses = $(I - I_{sh})^2 r_a + W_c$

Therefore efficiency of motor = $\frac{input-losses}{input}$; = $\frac{VI - ((I-Ish)2ra+Wc)}{VI}$ ……. 8

EFFICIENCY OF A GENERATOR:

Output = VI

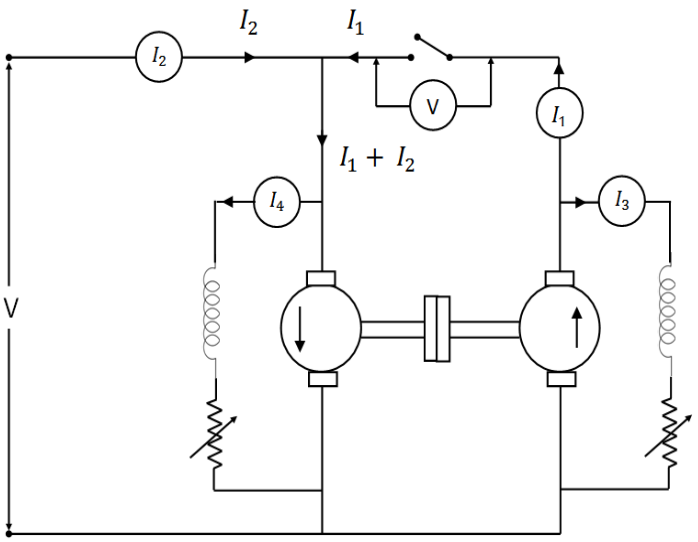
$I_a^2 r_a = (I+I_{sh})^2 r_a$

Total losses = $W_c + (I+I_{sh})^2 r_a$ ………………………

Efficiency of generator = $\frac{output}{output+losses} = \frac{VI}{VI+(I+I_{sh})}$

Hopkinson's Or Regenerative Or Back To Back Test:

- This is a regenerative test in which two identical DC shunt machines are coupled mechanically and tested simultaneously.



- One of the machines is run as a generator, while the other as motor supplied by the generator.
- The set therefore draws only losses in the machines.
- The circuit connection is shown in below Figure.
- The machine is started as motor and its shunt field resistance is varied to run the motor at its rated speed.
- The voltage of the generator is made equal to supply voltage by varying the shunt field resistance of the generator which is indicated by the zero reading of the voltmeter connected across the switch.
- By adjusting the field currents of the machines, the machines can be made to operate at any desired load with in the rated capacity of the machines

Advantages:

- The two machines are tested under loaded conditions so that stray load losses are accounted.
- Power required for the test is small as compared to the full load powers of the two machines. Therefore economical for long duration tests like "Heat run tests".
- Temperature rise and commutation qualities can be observed.
- By merely adjusting the field currents of the two machines the two machines can be loaded easily and the load test can be conducted over the complete load range in a short time.

Disadvantages:

- Availability of two identical machines
- Both machines are not loaded equally and this is crucial in smaller machines.
- There is no way of separating iron losses of the two machines which are different because of different excitations.
- Since field currents are varied widely to get full load, the set speed will be greater than rated values.

The efficiency can be determined as follows:

V= supply voltage

Motor input = $V(I_1+I_2)$

Generator output = VI_1 ---------------- (a)

If we assume both machines have the same efficiency 'η', then,

Output of motor = η x input = η x V (I_1+I_2) = input to generator

Output of generator = η x input = η x ηV $(I_1+I_2) = \eta^2 V(I_1+I_2)$-----(b)

Equating (a) and (b),

$VI_1 = \eta^2 V(I_1+I_2)$

Therefore, $\eta = \sqrt{\frac{I_1}{I_1+I_2}}$ 11

Armature copper loss in motor = $(I_1 + I_2 - I_4)^2 r_a$

Shunt field copper loss in motor = VI_4

Armature copper loss in generator = $(I_1 + I_3)^2 r_a$

Shunt field copper loss in generator = VI_3

Power drawn from supply = VI_2

Therefore stray losses = $VI_2 - [(I_1 + I_2 - I_4)^2 r_a + VI_4 + (I_1 + I_3)^2 r_a + VI_3]$ = W (say) …… 12

Stray losses/motor = $\frac{W}{2}$ 13

Therefore for generator

Total losses = $(I_1 + I_3)^2 r_a + VI_3 + \frac{W}{2} = W_g$ (s

Output = VI_1, therefore $\eta_{generator} = \frac{VI_1}{VI_1+W_g} = \frac{}{ou}$

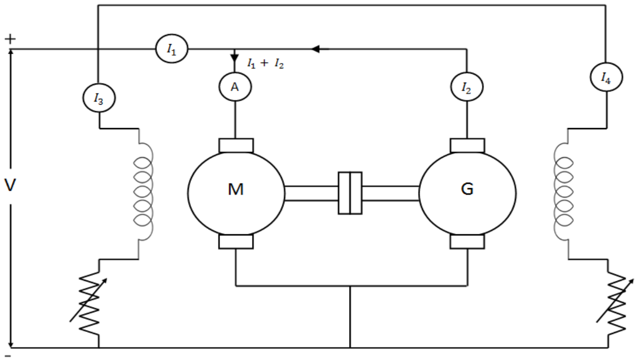
For motor,

Total losses =$(I_1 + I_2 - I_4)^2 r_a + VI_4 + \frac{W}{2} = W$

Input to motor = $V(I_1 + I_2)$

Therefore $\eta_{motor} = \frac{V(I_1+I_2)-W_m}{V(I_1+I_2)}$ 16

ALTERNATIVE CONNECTION:



The Figure shows an alternate circuit connection for this test.
In this connection the shunt field windings are directly connected across the lines. Hence the input current is excluding the field currents.

The efficiency is determined as follows:

Motor armature copper loss $=(I_1+I_2)^2 r_a$

Generator armature copper loss $= {I_2}^2 r_a$

Power drawn from supply $= VI_1$

Stray losses $= VI_1 - \left[(I_1+I_2)^2 r_a - {I_2}^2 r_a\right] = W(\text{say})$ ....

Stray loss/motor $= \frac{W}{2}$ 18

MOTOR EFFICIENCY: motor input = armature input + shun

$= V(I_1+I_2) + VI_3$

Motor loss = Armature copper loss + Shunt copper loss + stray

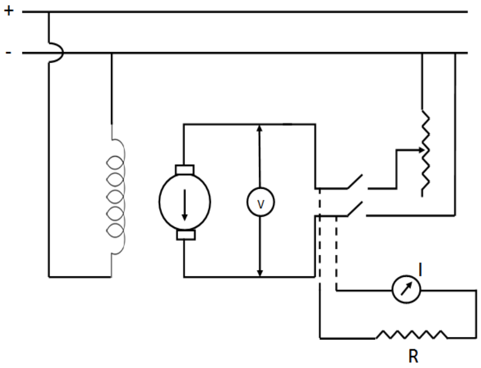
Therefore $\eta_{motor} = \frac{motor\ input - motor\ losses}{motor\ input}$ 20

Generator efficiency 'η': Generator output $= VI_2$

Generator losses $= {I_2}^2 r_a + VI_4 + \frac{W}{2}$ 21

$\eta_{generator} = \frac{VI_2}{VI_2 + Generator\ losses}$ 22

<u>Retardation or running down test:</u>



- This method is applicable to shunt motors and generators and is used for finding the stray losses.
- If armature and shunt copper losses are known for a given load, efficiency can be calculated.
- Machine is speeded up slightly beyond its rated speed and then supply is cut off from the armature while keeping the field excited.
- Armature will slow down and its kinetic energy is needed to meet rotational losses. i.e., friction and windage losses.

Kinetic energy of the armature $= \frac{1}{2} I\omega^2$

I = Moment of inertia of the armature

ω = Angular velocity.

Rotational losses;

N = Rate of loss of K.E.

Rate of loss of Kinetic energy $W = \frac{d}{dt}\left[\frac{1}{2} I\omega^2\right]$

$I \text{ X } W \frac{d\omega}{dt}$................. 30

Two quantities need to be known

(i) Moment of Inertia 'I'

(ii) $\frac{d\omega}{dt}$ or $\frac{dN}{dt}$ (because $\omega \; \alpha \; N$)

(i) **Finding $\frac{d\omega}{dt}$:** The voltmeter 'V' in the circuit shown in Figure 4.11 is used as speed indicator by suitably graduating it because $E \propto N$. when the supply is cut off, the armature speed and hence voltmeter reading falls. Voltage and time at different intervals are noted and a curve is drawn between the time and speed as shown in Figure 4.12.

In the Figure 4.12 AB - tangent drawn at P

There fore $\frac{dN}{dt} = \frac{OB\ (rpm)}{OA\ (sec)}$

$W = I \text{ X } \omega \text{ X } \frac{d\omega}{dt}$

$\omega = \frac{2\pi N}{60}$

$W = I\left(\frac{2\pi}{60}\right)\frac{d}{dt}\left(\frac{2\pi N}{60}\right)$

$W = \left(\frac{2\pi N}{60}\right)^2 I.N.\frac{dN}{dt} = 0.011 \text{ X I X N X } \frac{dN}{dt}$.............. 31

(ii) Finding Moment of Inertial 'I': There are two methods of finding the moment of inertia 'I'

(a) **I is calculated**:

(i) Slowing down curve with armature alone is calculated.

(ii) A fly wheel is keyed to the shaft and the curve is drawn again

(iii) For any given speed, $\frac{dN}{dt}$, and $\frac{dN}{dt_2}$ are determined as before.

Therefore $W = \left(\frac{2\pi}{60}\right)^2 I.N.\frac{dN}{dt_1}$ --- (32) 1st case

$W = \left(\frac{2\pi}{60}\right)^2 (I + I_1)N.\frac{dN}{dt_2}$ --- (33) 2nd case

Equation (32) = Equation (33), losses in both the cases will be almost same.

$$I\frac{dN}{dt_1} = (I + I_1)\frac{dN}{dt_2}. \ \frac{I + I_1}{I}\left(\frac{dN}{dt_2}\right) = \frac{dN}{dt_1}$$

$$\frac{I + I_1}{I} = \frac{dN}{dt_2}$$

$\Rightarrow$ I = I_1 X $\frac{t_2}{t_1 - t_2}$.............. 34

(b) **I is eliminated:** In this method, time taken to slow down is noted with armature alone and then a retarding torque is applied electrically i.e., a non inductive resistance is connected to the armature. The additional loss is $I_a^2(R_a + R)$orVI_a.

Let W^1 be the power then

$W = (\frac{2\pi}{60})^2 IN\frac{dN}{dt_1}$.................. 35

$W + W^1 = (\frac{2\pi}{60})^2 IN\frac{dN}{dt_2}$............ 36; if dN is same.

$\frac{dN}{dt_1}$ =rate of change of speed without electrical load

$\frac{dN}{dt_2}$ =rate of change of speed with electrical load

$\frac{W + W1}{W} = \frac{\frac{dN}{dt_2}}{\frac{dN}{dt_1}}$...... 37 or $\frac{W + W1}{W} = \frac{dt_1}{dt_2}$or $W = W^1 \times \frac{dt_2}{dt_1 - dt_2}$ or $W = W^1 \times \frac{t_2}{t_1 - t_2}$.......... 38

Separation of losses:

At a given excitation, friction losses and hysteresis are proportional to speed. Windage losses and eddy current losses on the other hand are both proportional to square of speed. Hence, for a given excitation (field current) we have,

Friction losses = AN Watts

Windage losses = BN^2 Watts

Hysteresis losses = CN Watts

Eddy current losses = DN^2 Watts

Where, N = speed.

For a motor on no load, power input to the armature is the sum of the armature copper losses and the above losses. In the circuit diagram,

Power input to the armature = V*Ia watts.

Armature copper losses = Ia2*Ra watts

V*Ia – Ia2*Ra = (A + C)N + (B + D)N

W/N = (A+C) + (B+D)N.

The graph between W/N & N is a straight line, from which (A+C) and (B+D) can be found.In order to find A, B, C and D seperately, let the field current be changed to a reduced value IIf and kept constant at that value. If voltage is applied to the armature as before, we now have ,

$W/N = (A+C^1)\,N + (B+D^1)\,N^2$

(at the reduced excitation, function and windage losses are still are AN and BN2, but hysteresis losses become C^1N and eddy current losses become D^1N^2. We can now obtain (A+C) and (B+D) as before.

Now,

C/C^1 = (flux at normal excitation/flux at reduced excitation)

D/D^1 = (flux at normal excitation/flux at reduced excitation)

So, if we determine the ratio (flux at normal excitation/flux at reduced excitation) we can find

A, B, C, D, C^1, & D^1

A+C, A+C', B+D and B+D' values are obtained from the graph and hence A,B,C and D constants can be calculated.

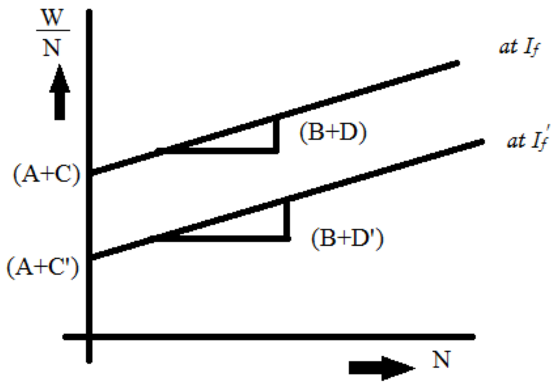
Thus with these constants at any speed the loss are given by

Friction losses = AN Watts

Windage losses = BN^2 Watts

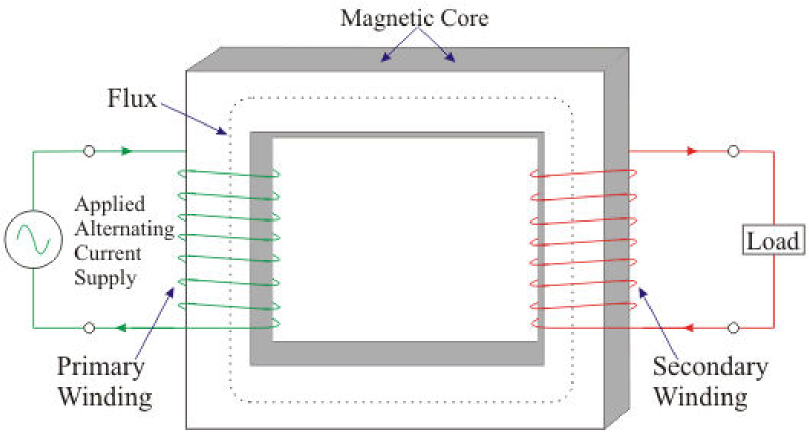
Hysteresis losses = CN Watts

Eddy current losses = DN^2 Watts



Unit – 4

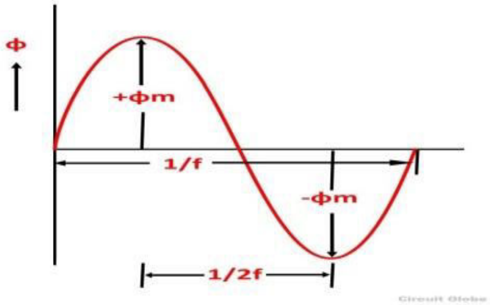
1. Explain the *Working principle* of transformer



Single Phase Transformer

1. The basic working principle of a transformer is mutual induction between two windings linked by common magnetic flux.
2. The primary and secondary coils are electrically separated but magnetically linked to each other.
3. When, primary winding is connected to a source of alternating voltage, alternating magnetic flux is produced around the winding.
4. The core provides magnetic path for the flux, to get linked with the secondary winding. Most of the flux gets linked with the secondary winding which is called as 'useful flux' or main 'flux', and the flux which does not get linked with secondary winding is called as 'leakage flux'.
5. As the flux produced is alternating (the direction of it is continuously changing), EMF gets induced in the secondary winding according to Faraday's law of electromagnetic induction. This induced emf is called 'mutually induced emf', and the frequency of mutually induced emf is same as that of supplied emf. Thus, in a transformer the frequency is same on both sides.
6. If the secondary winding is closed circuit, then mutually induced makes the current flow through it, and hence the electrical energy is transferred from one circuit (primary) to another circuit (secondary).

2. Derive the *EMF Equation* of a Transformer



Let

ϕ_m = Maximum value of flux in Weber

f = Supply frequency in Hz

N_1 = Number of turns in the primary winding

N_2 = Number of turns in the secondary winding

Φ = flux per turn in Weber

As per the faradays laws,

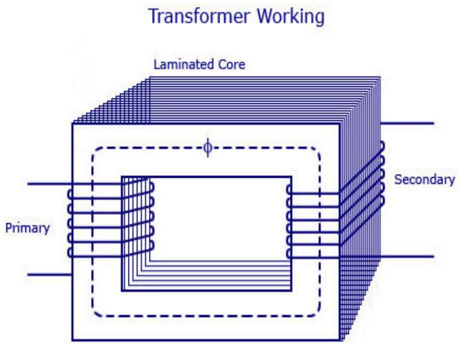
The average value of the emf induced is directly proportional to the rate of change of flux.

- ➢ The flux changes from $+\phi_m$ to $-\phi_m$ in half a cycle of 1/2f seconds.
- ➢ Flux increases from its zero value to maximum value ϕ_m in one quarter of the cycle i.e. in ¼ of the timeperiod.
- ➢ Average rate of change of flux is $\frac{d\phi}{dt} = \frac{\phi_m - 0}{\frac{1}{4f}} = 4\phi_m f$ volts
- ➢ Therefore the average e.m.f per turn is $4\phi_m f$
- ➢ As $\frac{Rmsvalue}{Averagevalue} = Formfactor = 1.11$ for sinusoidal varying quatities

- Hence, RMS value of e.m.f/turn is $1.11 * 4\phi_m f = 4.44\phi_m f$
- RMS value of e.m.f in the primary and secondary winding. =(e.m.f/turn) * No:of turns
- Therefore Emf induced in primary winding having N_1turns is $E_1 = 4.44\phi_m f N_1$
- Emf induced in secondary winding having N_2 turns is $E_2 = 4.44\phi_m f N_2$

3. Explain the *Construction of Transformer*

1. The simple construction of a transformer, need two coils having mutual inductance and a laminated steel core.



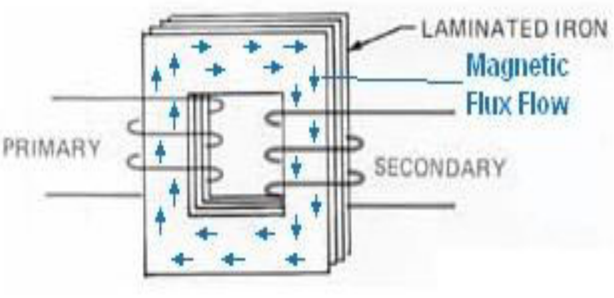
2. The two coils are insulated from each other and from the steel core.
3. The device will also need some suitable container for the assembled core and windings, a medium with which the core and its windings from its container can be insulated.
4. In order to insulate and to bring out the terminals of the winding from the tank, bushings made of porcelain are used.
5. In all transformers, the core is made of transformer sheet steel laminations assembled to provide a continuous magnetic path with minimum of air-gap included.
6. The steel should have high permeability and low hysteresis loss. For this to happen, the steel should be made of high silicon content and must also be heat treated.
7. By effectively laminating the core, the eddy-current losses can be reduced. The lamination can be done with the help of a light coat of core plate varnish or lay an oxide layer on the surface. For a frequency of 50 Hertz, the thickness of the lamination varies from 0.35mm to 0.5mm for a frequency of 25 Hertz.
8. To reduce the leakage fluxes in the transformer the windings of the primary and secondary coils are interleaved in the core type and sandwiched coils in the shell type.
9. To reduce the volume of the cu wire the core used must be the stepped core or cruciform core.

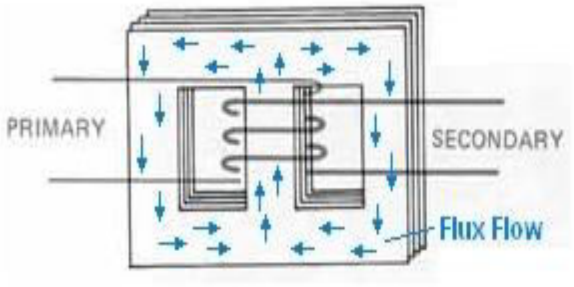
4. Compare and distinguish *the types of transformers*

There are two major types of transformers based on construction. They are

1. Core type and 2. Shell type

S.No	Core type Transformer	Shell type transformer
1	The winding encircles the core	The core encircles the winding
2	The cylindrical type of coils are used	Generally multilayer disc type or sandwiched coils coils are used
3	As windings are distributed, the natural cooling is more effective	As windings are surrounded by the core, the natural cooling does not exists.
4	The coils can be easily removed from the maintenance point of view	For removing any winding for maintenance, a large number of laminations are to be removed. This is difficult.
5	The construction is preferred for low voltage transformers	The construction is used for very high voltage transformers
6	It has a single magnetic core	It has a double magnetic core
7	In a single phase type there are two limbs	In a single phase type the core has three limbs

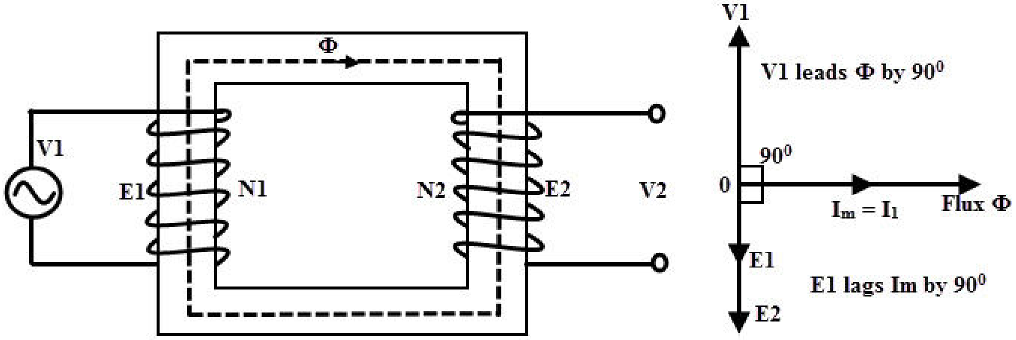




5. Explain the operation of *Transformer on No Load.*

Ideal transformer at No-Load:

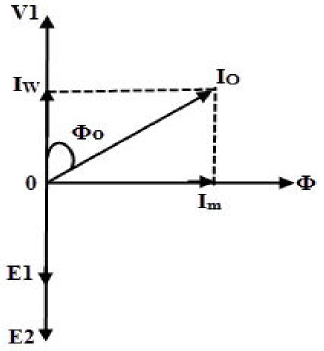
1. The transformer operating at no load, is equivalent to the secondary winding kept open circuited, which means current in the secondary is zero.
2. When primary winding is excited at its rated voltage it draws a current I_m called magnetizing current which is 2 to 10% of the rated current. This generates the magnetic flux in the core by primary mmf N_1I_m
3. As the transformer is ideal, the core loss and cu loss are zero. And the net current taken is to create the mmf or flux of alternating nature.
4. This alternating flux induces the emf's E_1 and E_2 in the coils which lags the flux by 90^0
5. The I_m is inphase to the flux and the applied voltage leads to the I_m by 90^0 being the coil with pure inductive type.
6. Hence, emf's E_1 and E_2 in the coils are inphase to each other and lags the flux by 90^0



Ideal Transformer at No-Load

Transformer at No-Load:

1. The transformer in the practical case draws an additional current I_w to the magnetizing current I_m and total current from the supply mains is I_0 which lags to the applied voltage by an angle Φ_0
2. There are two components of the current in I_0 namely
   i. Active (or) power (or) Watt full component of the current I_w which is in phase to the voltage, and generates the core loss in the transformer
   ii. Reactive (or) Watt less (or) magnetizing component of the current I_m which lags to the voltage by 90^0, and magnetizes the core in the transformer
3. Also, the no-load input power of the transformer is the iron loss (since the cu loss are small at no-load)
4. The no load angle (Φ_0) depends upon the losses in the transformer and is nearly equal to 90^0. So that the power factor is very low and varies from 0.1 to 0.15 lagging.



6.

Working component $I_w = I_0 \cos\varphi_0$

No load current $I_0 = \sqrt{I_w^2 + I_m^2}$

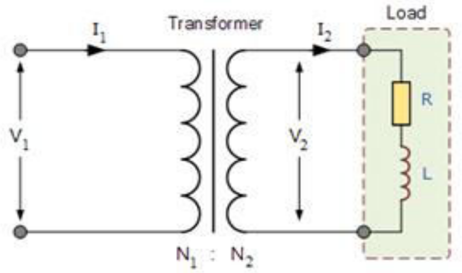
Magnetizing component $I_m = I_0 \sin\varphi_0$

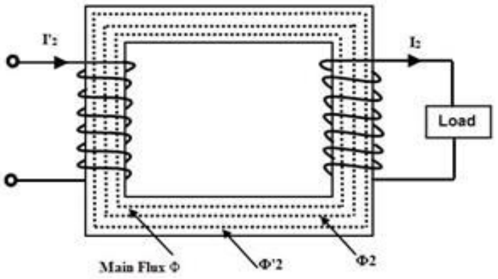
Power factor $\cos\varphi_0 = \frac{I_w}{I_0}$

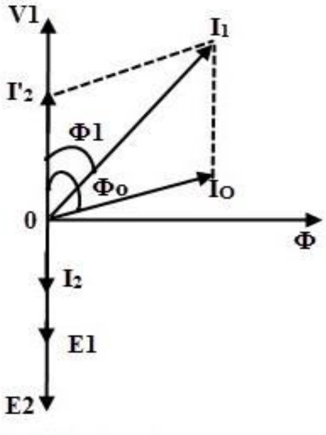
No load power input $P_0 = V_1 I_0 \cos\varphi_0$

6. Explain the operation of Transformer on Load *without leakage impedances* of the coils.

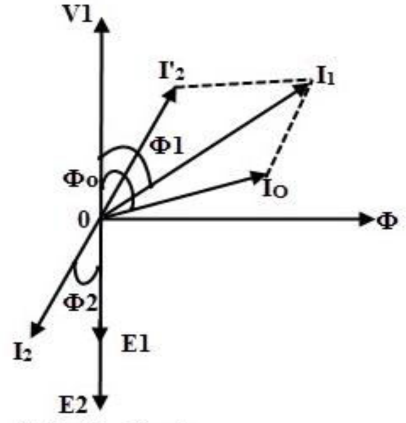
1. When an electrical load is connected to the secondary winding of a transformer a current flows in the secondary winding.
2. This secondary current is due to the induced secondary voltage, set up by the magnetic flux Φ in the core from the primary current(I_0) and the main flux direction is from primary coil to secondary coil (clockwise)



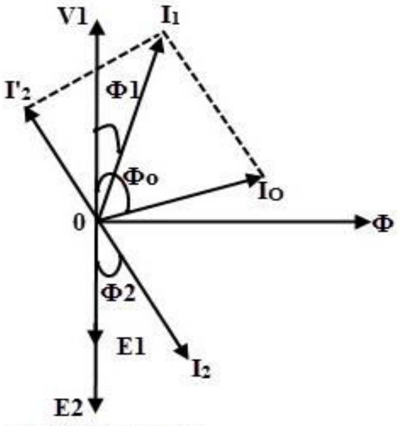




Resistive Load



Inductive Load



Capacitive Load

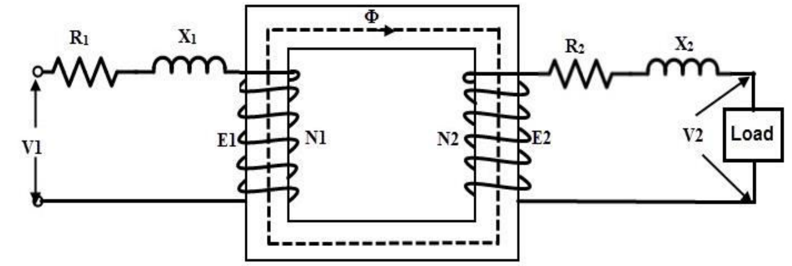
3. The secondary current, I_2 which is determined by the characteristics of the load, creates an secondary or load mmf (N_2I_2) and a secondary magnetic field, Φ_2 is established in the transformer core which flows in the exact opposite direction to the main primary field, Φ_1. i.e Φ_2 is in anti clock wise.

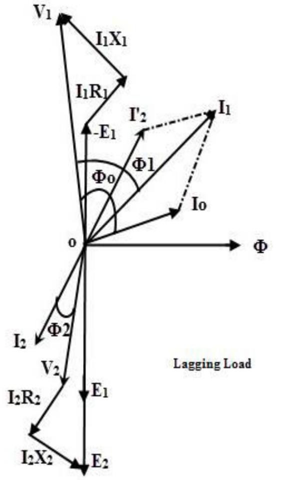
4. These two magnetic fields oppose each other resulting in a combined magnetic field of less magnetic strength than the single field produced by the primary winding alone when the secondary circuit was open circuited.
5. This in turn decreases the primary induced emf and leads to the increase in primary current $I_1=I_0+I_2^1$.
6. This additional I_2^1 current is called load component current in the primary and will be in such a way to balance the load mmf by this mmf on the primary

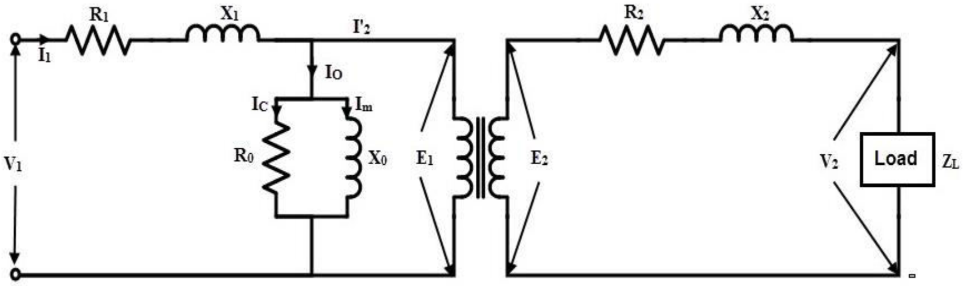
   i.e $N_2I_2=N_1I_2^1$ therefore $I_2^1= I_2K$ where, $K = N_2/N_1$
7. This $N_1I_2^1$ will produce a flux Φ_2^1 equal and opposite to Φ_2. These fluxes will now be cancelled and the net flux in the core will be Φ_1 even under the loading conditions.
8. For lagging load: $I_1^2 = I_0^2 + (I_2^1)^2 + 2I_0I_2^1\cos(\Phi_0\sim \Phi_2)$
9. As the flux remains constant from no-load to load, the iron loss will be same from no-load to load.

7. Explain the operation of transformer with *leakage impedances* of the coils

1. Below figure shows the schematic diagram, equivalent circuit and phasor diagram of the transformer with the leakage impedances of the coils.







Let,

R_1=Resistance of primary coil in Ω *R_2=Resistance of secondary coil in Ω*
X_1=Reactance of primary coil in Ω *X_2=Reactance of secondary coil in Ω*
Z_1=impedance of primary coil in Ω *Z_2=impedance of secondary coil in Ω*
E_1=emf induced in primary coil *E_2=emf induced in secondary coil*
V_1=applied voltage to primary coil *V_2= Load or terminal voltage of transformer*
$I_1Z_1 = I_1(R_1+jX_1)$ = Primary leakage impedance drop
$I_2Z_2 = I_2(R_2+jX_2)$ = Secondary leakage impedance drop

The magnetic core of the transformer is electrically represented with the parallel combination of R_0 and X_0 carrying the currents of I_w and I_m respectively and is placed across the primary coil.

Currents Analysis of the transformer in equivalent circuit

Currents in the transformer at No-load:

$$I_w = \frac{V_1}{R_0} \qquad I_m = \frac{V_1}{X_0} \qquad I_0^2 = I_w^2 + I_m^2 \qquad I_0 = \sqrt{I_w^2 + I_m^2} \qquad \phi_0 = \tan^{-1}\left(\frac{I_m}{I_w}\right)$$

Currents in the transformer with load

$$I_1 = (I_0\angle -\phi_0) + (I_2^1\angle \pm\phi_2) \quad \textit{Where} \;\; I_2^1 = I_2 \times K \quad \textit{and} \quad K = \frac{N_2}{N_1}$$

$$I_1 = (I_0\cos\phi_0 + I_2^1\cos\phi_2) + j(I_0\sin(-\phi_0) + I_2^1\sin(\pm\phi_2)) \qquad \text{- for lag} \qquad \text{and} \;\; \text{+ for lead}$$

Primary phase angle (Φ_1)

$$\phi_1 = \tan^{-1}\left(\frac{I_0\sin(-\phi_0) + I_2^1\sin(\pm\phi_2)}{I_0\cos\phi_0 + I_2^1\cos\phi_2}\right)$$ *and primary power factor is cosΦ_1*

Voltages Analysis of the transformer in equivalent circuit

Primary induced emf

$$\boxed{E_1 = (V_1\angle 0) - (I_1\angle\phi_1 * Z_1)}$$

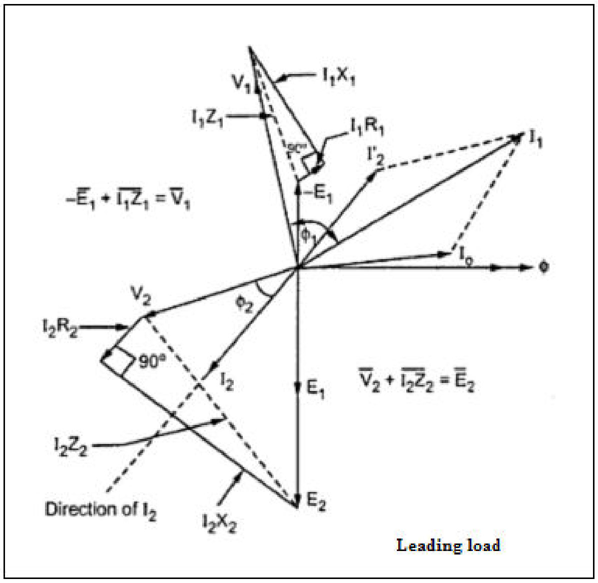
$$E_1 = (V_1 + j0) - \{(I_1\cos\phi_1 + jI_1\sin\phi_1) \times (R_1 + jX_1)\}$$

Using transformation ratio $E_2 = E_1 * K$

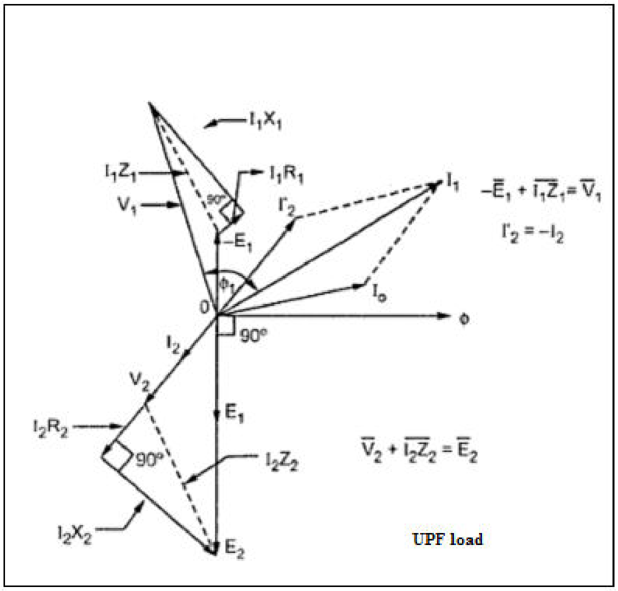
Knowing the E_2 and applying KVL to the secondary loop the load voltage is

$$\boxed{V_2 = E_2 - I_2 Z_2}$$

$$V_2 = E_2\angle\phi' - (I_2\angle\pm\phi_2)Z_2 \qquad V_2 = E_2\angle\phi' - (I_2\angle\pm\phi_2)(R_2 + jX_2)$$



Leading load



UPF load

8. Explain the *equivalent circuits* referred to both primary and secondary of the transformer

The equivalent circuit of the transformer referred to primary is shown in the below figure in which the winding parameters of the secondary are transformed and was referred to primary based on the voltage balancing principle before and after the transformation.

Secondary Resistance referred to primary:

$$R_2^1 = \frac{V_1}{I_1} = \frac{V_1}{I_1} \times \frac{V_2 I_2}{V_2 I_2} = \times \frac{V_1 I_2}{V_2 I_1} \times \frac{V_2}{I_2} = \frac{R_2}{K^2} \qquad \left(\because \frac{V_1}{V_2} = \frac{I_2}{I_1} = \frac{1}{K} \right) also \frac{V_2}{I_2} = R_2$$

$\therefore R_2^1 = \frac{R_2}{K^2}$ Thus, it is the secondary resistance referred to primary

Secondary Reactance referred to primary:

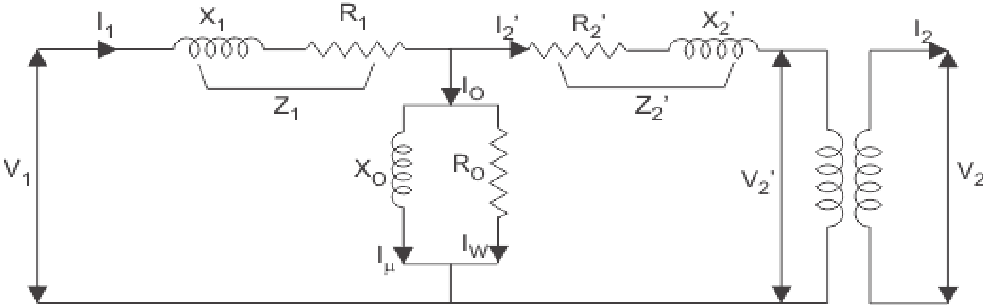
$$X_2^1 = \frac{V_1}{I_1} = \frac{V_1}{I_1} \times \frac{V_2 I_2}{V_2 I_2} = \times \frac{V_1 I_2}{V_2 I_1} \times \frac{V_2}{I_2} = \frac{X_2}{K^2} \qquad \left(\because \frac{V_1}{V_2} = \frac{I_2}{I_1} = \frac{1}{K} \right) also \frac{V_2}{I_2} = X_2$$

$\therefore X_2^1 = \frac{X_2}{K^2}$ Thus, it is the secondary reactance referred to primary

Secondary Impedance referred to primary:

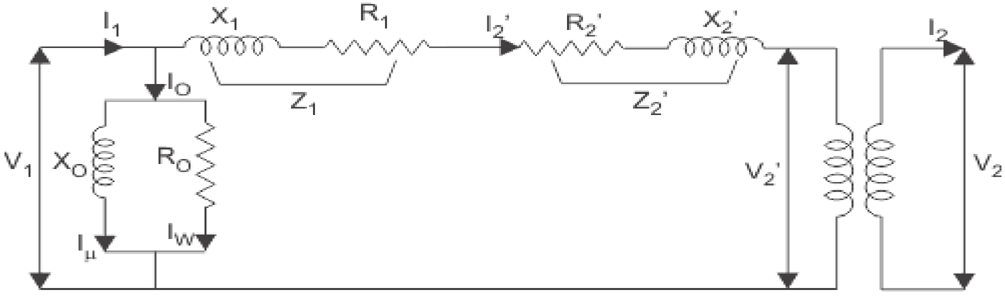
$$Z_2^1 = \frac{V_1}{I_1} = \frac{V_1}{I_1} \times \frac{V_2 I_2}{V_2 I_2} = \times \frac{V_1 I_2}{V_2 I_1} \times \frac{V_2}{I_2} = \frac{Z_2}{K^2} \qquad \left(\because \frac{V_1}{V_2} = \frac{I_2}{I_1} = \frac{1}{K} \right) also \frac{V_2}{I_2} = Z_2$$

$\therefore Z_2^1 = \frac{Z_2}{K^2}$ Thus, it is the secondary impedance referred to primary



Equivalent Circuit of Transformer referred to Primary

To have simplified calculations the equivalent circuit is modified as bringing the core branch towards the supply voltage instead of having in between the primary and secondary parameters



simplified equivalent circuit of transformer reffered to primary

In this simplified circuit the total resistance, reactance and impedances referred to primary are

$$\therefore R_{eq1} = R_1 + R_2^1 = R_1 + \frac{R_2}{K^2} \qquad \therefore X_{eq1} = X_1 + X_2^1 = X_1 + \frac{X_2}{K^2} \qquad \therefore Z_{eq1} = Z_1 + Z_2^1 = Z_1 + \frac{Z_2}{K^2}$$

Similarly, the equivalent circuit referred to secondary of the transformer is shown below with their formulas

Primary Resistance referred to secondary:

$$R_1^1 = \frac{V_2}{I_2} = \frac{V_2}{I_2} \times \frac{V_1 I_1}{V_1 I_1} = \times \frac{V_2 I_1}{V_1 I_2} \times \frac{V_1}{I_1} = K^2 R_1 \qquad \left(\because \frac{V_2}{V_1} = \frac{I_1}{I_2} = K \right) also \frac{V_1}{I_1} = R_1$$

$\therefore R_1^1 = R_1 K^2$ Thus, it is the primary resistance referred to secondary

Primary Reactance referred to secondary:

$$X_1^1 = \frac{V_2}{I_2} = \frac{V_2}{I_2} \times \frac{V_1 I_1}{V_1 I_1} = \times \frac{V_2 I_1}{V_1 I_2} \times \frac{V_1}{I_1} = K^2 X_1 \qquad \left(\because \frac{V_2}{V_1} = \frac{I_1}{I_2} = K \right) also \frac{V_1}{I_1} = X_1$$

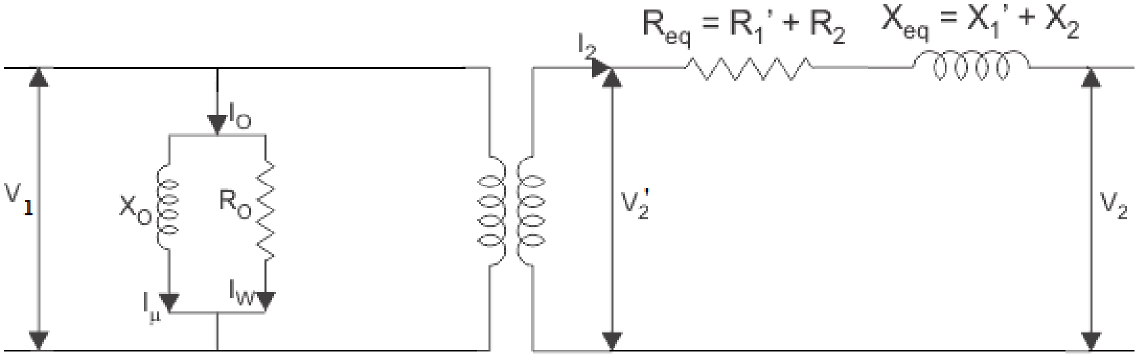
$\therefore X_1^1 = X_1 K^2$ Thus, it is the primary reactance referred to secondary

Primary Impedance referred to secondary:

$$Z_1^1 = \frac{V_2}{I_2} = \frac{V_2}{I_2} \times \frac{V_1 I_1}{V_1 I_1} = \times \frac{V_2 I_1}{V_1 I_2} \times \frac{V_1}{I_1} = K^2 Z_1 \qquad \left(\because \frac{V_2}{V_1} = \frac{I_1}{I_2} = K \right) also \frac{V_1}{I_1} = Z_1$$

$\therefore Z_1^1 = Z_1 K^2$ Thus, it is the primary impedance referred to secondary

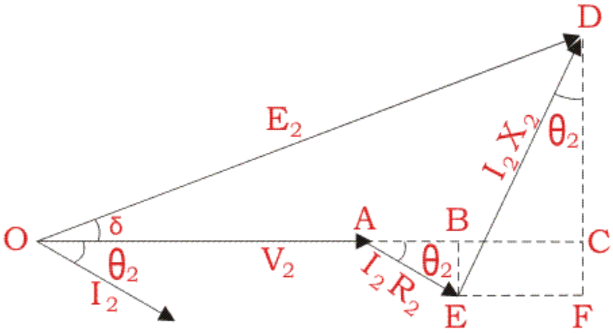
$$\therefore R_{eq2} = R_2 + R_1^1 = R_2 + R_1 K^2 \qquad \therefore X_{eq2} = X_2 + X_1^1 = X_2 + X_1 K^2 \qquad \therefore Z_{eq2} = Z_2 + Z_1^1 = Z_2 + Z_1 K^2$$



Approximate Equivalent Circuit of Transformer referred to Secondary

9. Derive the expression for *voltage regulation and efficiency* of the transformer

Definition of voltage regulation : Voltage regulation is defined as the percentage change in the output voltage from no-load to full-load expressed in full load voltage.



$OC = OA + AB + BC$

$Here,\ OA = V_2$

$Here,\ AB = AE\cos\theta_2 = I_2R_2\cos\theta_2$

$and,\ BC = DE\sin\theta_2 = I_2X_2\sin\theta_2$

Derivation of voltage regulation for the lagging power factor load,

assuming the angle between OC and OD as very small, and neglected it, OD is nearly equal to OC ($E_2 > V_2$)

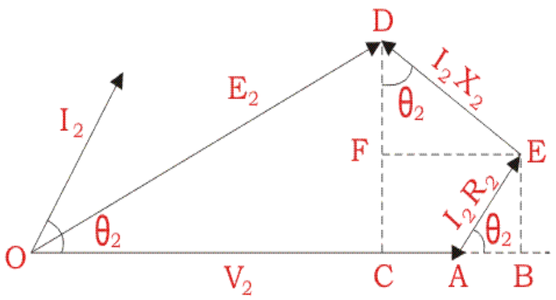
$$E_2 = OC = OA + AB + BC, \quad E_2 = OC = V_2 + I_2 R_{eq2} \cos\phi + I_2 X_{eq2} \sin\phi$$

Thus, the % voltage regulation is

$$\frac{E_2 - V_2}{V_2} * 100 = \frac{V_2 + I_2 R_{eq2} \cos\phi + I_2 X_{eq2} \sin\phi - V_2}{\mathrm{V_2}} \times 100 = \frac{I_2 R_{eq2} \cos\phi + I_2 X_{eq2} \sin\phi}{\mathrm{V_2}} \times 100$$

Derivation of voltage regulation for the leading power factor load,

Similarly, from the phasor diagram of the leading pf load , ($E_2 < V_2$)



Here

$EF = DE \sin\theta = I_2X_2 \sin\theta$

$AB = AE \cos\theta = I_2R_2 \cos\theta$

$OA = V_2$ and $OD = E_2$

assuming the angle between OA and OD as very small, and neglected it, OD is nearly equal to OC ($E_2 < V_2$)

$V_2 - E_2 = OA – OC = CA = CB – AB$, thus $V_2 = E_2 + CB - AB$

Thus, the % voltage regulation is

$$\frac{E_2 - V_2}{V_2} * 100 = \frac{\mathrm{E_2 - E_2 - CB + AB}}{\mathrm{V_2}} \times 100 = \frac{I_2 R_{eq2} \cos\phi - I_2 X_{eq2} \sin\phi}{\mathrm{V_2}} \times 100$$

Therefore,

$$\%regulation = \frac{I_2 R_{eq2} \cos\phi \pm I_2 X_{eq2} \sin\phi}{\mathrm{V_2}} \times 100$$

(+) for lagging pf and (-) for leading pf

- The **Efficiency** of the transformer is defined as the ratio of power output to the input power.

$$\eta = \frac{\text{output power}}{\text{input power}} = \frac{\text{output power}}{\text{output power + losses}}$$

$$\eta = \frac{\text{output power}}{\text{output power + iron losses + copper losses}}$$

$$\eta = \frac{V_2 I_2 \cos\varphi_2}{V_2 I_2 \cos\varphi_2 + P_i + P_c}$$

Where,

V_2 = Secondary terminal voltage

I_2 = Full load secondary current in A

$\cos\phi_2$ = power factor of the load

P_i = Iron losses

= hysteresis losses + eddy current loss

Pc = Full load copper losses = $I_2^2 R_{eq}$

Also, the efficiency at any amount of load(x) is given by

$$\eta = \frac{outputinwatts}{inputinwatts} = \frac{xVA\cos\phi}{xVA\cos\phi + W_i + x^2 W_{FLCu}} \times 100$$

Condition for maximum efficiency in the transformer:

$$\eta = \frac{outputinwatts}{inputinwatts} = \frac{V_2 I_2 \cos\phi}{V_2 I_2 \cos\phi + W_i + I_2^2 r_{e2}} = \frac{1}{1+\frac{W_i}{V_2 I_2 \cos\phi}+\frac{I_2^2 r_{e2}}{V_2 I_2 \cos\phi}} = \frac{1}{1+\frac{W_i}{V_2 I_2 \cos\phi}+\frac{I_2 r_{e2}}{V_2 \cos\phi}}$$

To get the maximum efficiency the denominator must be small, therefore condition to be the denominator minimum is

$$\frac{d\left(1+\frac{W_i}{V_2 I_2 \cos\phi}+\frac{I_2 r_{e2}}{V_2 \cos\phi}\right)}{dI_2} = 0$$

$$\frac{d\left(1+\frac{W_i}{V_2 I_2 \cos\phi}+\frac{I_2 r_{e2}}{V_2 \cos\phi}\right)}{dI_2} = 0 + \left((-)\frac{W_i}{V_2 I_2^2 \cos\phi}\right) + \left(\frac{r_{e2}}{V_2 \cos\phi}\right) = 0$$

$$\frac{r_{e2}}{V_2 \cos\phi} = \frac{W_i}{V_2 I_2^2 \cos\phi} \qquad r_{e2} = \frac{W_i}{I_2^2} \qquad I_2^2 r_{e2} = W_i$$

Therefore the condition for obtaining the maximum efficiency is the variable loss $\left(I_2^2 r_{e2}\right)$ must be equal to the constant loss W_i.

Also, the load current at which the maximum efficiency occurs is $I_{2\max} = \sqrt{\left(\frac{W_i}{r_{e2}}\right)}$

Multiplying both sides with 1000 *V_2

$$1000 * V_2 * I_{2\max} = 1000 * V_2 * \sqrt{\left(\frac{W_i}{r_{e2}}\right)} \qquad \text{Load KVA }_{\max} = 1000 * V_2 * \sqrt{\left(\frac{W_i}{r_{e2}}\right)}$$

$$\text{Load KVA }_{\max} = 1000 * V_2 * \frac{I_{2Fullload}}{I_{2Fullload}} \sqrt{\left(\frac{W_i}{r_{e2}}\right)} \qquad \text{Load KVA }_{\max} = 1000 * V_2 * I_{2Fullload} \sqrt{\left(\frac{W_i}{I_{2Fullload}^2 r_{e2}}\right)}$$

$$\text{Load KVA }_{\max} = \text{Full load KVA} \sqrt{\left(\frac{W_i}{I_{2Fullload}^2 r_{e2}}\right)}$$

$$\text{The Load KVA at which maximum efficiency= Full load KVA} \sqrt{\left(\frac{W_i}{W_{cuFullload}}\right)}$$

$$\boxed{\text{The Load KVA at which maximum efficiency= Full load KVA} \sqrt{\frac{W_i}{W_{cuFullload}}}}$$

10. Explain the *Separation of hysteresis and eddy current loss* in a transformer

The core loss (or) iron loss (or) Magnetic loss in the transformer is of two types namely

a) Hysteresis loss (P_h) and b) Eddy current loss (P_e)

Thus the core loss (P_c) or (W_i) = $P_h + P_e$

The hysteresis loss $P_h = K_h f B_{max}^x$ and the eddy current loss $P_e = K_e f^2 B_{max}^2$

Where

K_h and K_e are proportionality constants of hysteresis and eddy current loss respectively.

f = frequency of the alternating flux

B_{max} = maximum flux density in the core

$W_i = P_h + P_e = K_h f B_{max}^x + K_e f^2 B_{max}^2$

Also, from the emf equation of the transformer

$$E \cong V = 4.44\phi_m fN = 4.44 B_m A_C fN \qquad B_m = \left(\frac{1}{4.44 A_C N}\right)\left(\frac{V}{f}\right)$$

- The hysteresis loss $P_h = K_h f\left(\frac{1}{4.44A_C N}\right)^x \left(\frac{V}{f}\right)^x$ $\quad P_h = K_1 f\left(\frac{V}{f}\right)^x$ where $K_1 = K_h\left(\frac{1}{4.44A_C N}\right)^x$

$P_h = K_1 V^x f^{1-x}$ and the value of $x = 1.6$ $\therefore P_h = K_1 V^{1.6} f^{-0.6}$

Thus, the ***hysteresis loss*** depends on both the ***applied voltage*** and ***frequency.***

- The eddycurrent loss $P_e = K_e f^2\left(\frac{1}{4.44A_C N}\right)^2 \left(\frac{V}{f}\right)^2$ $\quad P_e = K_2 f^2\left(\frac{V}{f}\right)^2$ where $K_2 = K_e\left(\frac{1}{4.44A_C N}\right)^2$

$\therefore P_h = K_2 V^2$ And thus, the ***eddy current loss*** depends on only ***applied voltage.***

The total core loss is $\therefore P_c = K_1 V^{1.6} f^{-0.6} + K_2 V^2$

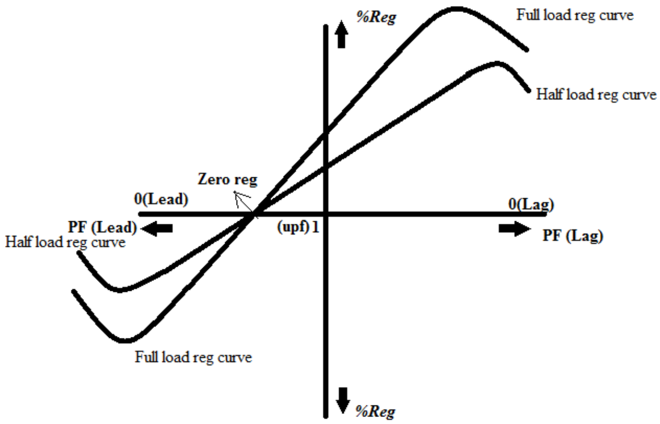
11. Explain in detail about the *all day efficiency* in a single phase transformer

- There are two types of transformers used in power lines namely, Power transformers and Distribution transformers
- The Power transformers are disconnected during the light load periods from the primary, so they are designed to have maximum efficiency at the rated or full load KVA.
- The Distribution transformers are used to supply the utility voltage to the consumer point and these transformers are connected throughout the day to the primary which presents the core loss in the transformer irrespective to the load present on the secondary.

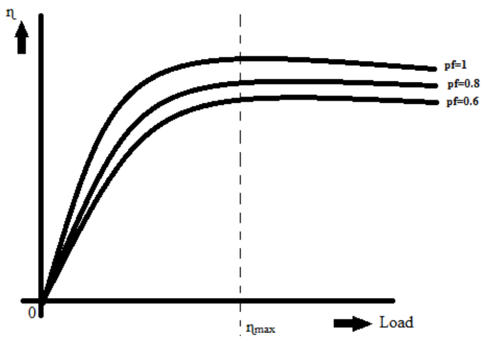
- Thus these distribution transformers are designed to have low value of coreloss, since coreloss is low, the load at which maximum efficiency occurs is also low, i.e the maximum efficiency of the transformer will be at either at half of the load.
- Therefore the performance of the distribution transformers is not judged by the full load efficiency which will be usually less than its maximum efficiency.
- As the distribution transformers are connected to primary continuously throughout the period its performance is therefore determined from the energy efficiency
- The energy efficiency is defined as the ratio of the energy output for a certain period to the total energy input for the same period. When this energy efficiency is calculated for a day of 24hours then this energy efficiency is called All day efficiency.

$$\eta_{allday} = \frac{energyoutput(KWh)}{energyoutput(KWh) + W_i * 24(KWh) + W_{cu} * Load\ present\ period(KWh)}$$

Variation of voltage regulation and efficiency with respect to load and load powerfactors



Regulation curves w.r.t load pf and amount of loads



Varaiation of efficiency with respect to load and load power factor

Emf equation of transformer(Alternate method)

Let the flux be taken as reference varying sinusoidal and given by $\phi = \phi_m \sin wt$

As per faradays second law

$$e/turn = (-)\frac{d\phi}{dt} = (-)\frac{d(\phi_m \sin wt)}{dt} = (-)\phi_m w \cos wt = (-)\phi_m w \sin(90 - wt) = \phi_m w \sin(wt - 90) = \phi_m 2\pi f \sin(wt - 90)$$

$$e/turn = \phi_m 2\pi f \sin(wt - 90) \Rightarrow e = \phi_m 2\pi f N \sin(wt - 90)$$

and is in the form of

$$e = E_{\max} \sin(wt - 90) \therefore E_{\max} = \phi_m 2\pi f N \Rightarrow E_{rms} = (\phi_m 2\pi f N)/\sqrt{2} \Rightarrow E = 4.44\phi_m f N$$

Thus, $E_1 = 4.44\phi_m f N_1$ and $E_2 = 4.44\phi_m f N_2$ $\frac{E_2}{E_1} = \frac{N_2}{N_1} = 4.44\phi_m f = 4.44 B_m A_c f$

Unit – 5

Question no. 1 Explain OC and SC tests on a single phase transformer

Ans: Purpose of conducting OC and SC tests is to find

i) Equivalent circuit parameters ii) Efficiency iii) Regulation

Open Circuit Test:

1. The OC test is performed on LV side at rated voltage and HV side is kept opened.
2. As the test is conducted on LV side the meters selected will be at low range values like smaller voltmeter, smaller ammeter and low pf wattmeter
3. As the no-load current is quite small about 2 to 5% of the rated current, the ammeter required here will be smaller range even after on LV side which are designed for higher current values.
4. The voltmeter, ammeter and the wattmeter readings V_0, I_0 and W_0 respectively are noted by applying rated voltage on LV side.
5. The wattmeter will record the core loss because of noload input power.

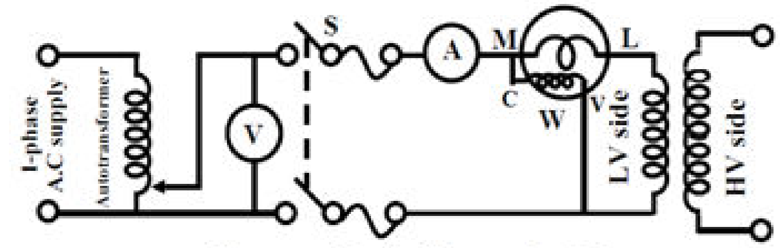


Figure : Circuit diagram for O.C test

Calculations from OC test readings:

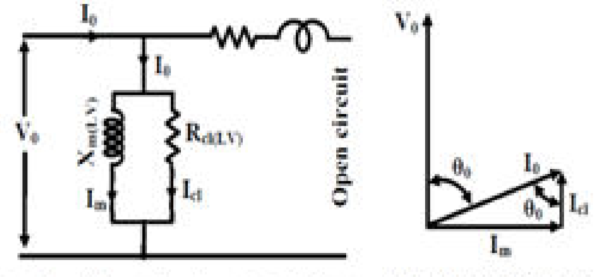
R_0, X_0 and Iron loss are calculated from the OC test results as

Core resistance $R_0 = \dfrac{V_0}{I_w} = \dfrac{V_0}{I_0 \cos\phi_0}$

Magnetizing reactance $X_0 = \dfrac{V_0}{I_m} = \dfrac{V_0}{I_0 \sin\phi_0}$

Where $\cos\phi_0 = \dfrac{P_0}{V_0 I_0}$

and iron loss $W_i = P_0$ (No load input power)



(a) Equivalent circuit under O.C test (b) phasor diagram

Figure 2.2: Equivalent circuit & phasor diagram during O.C test

Short Circuit Test:

1. The SC test is performed on HV side at rated current and LV side is kept Shorted.
2. As the test is conducted on HV side the meters selected will be at low range values like smaller voltmeter, smaller ammeter and unity pf wattmeter

3. As the voltage required to circulate the short circuit rated current is very small about 10 to 15% of the rated HV voltage, so the voltmeter required here will be smaller range even the test is conducted on HV side.
4. The voltmeter, ammeter and the wattmeter readings V_{sc}, I_{sc} and W_{sc} respectively are noted by passing rated current on HV side.
5. The wattmeter will record thecopper loss corresponding to the I_{sc}.

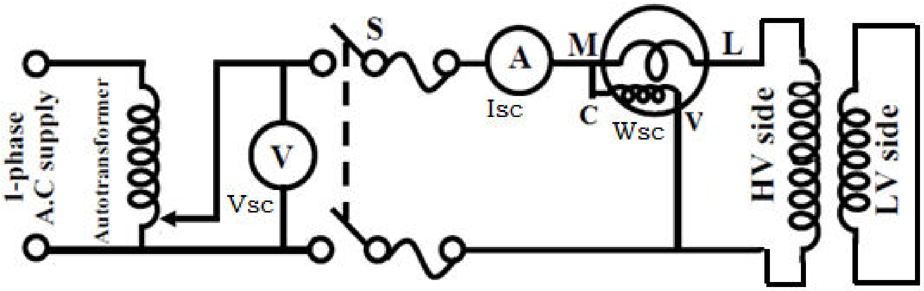
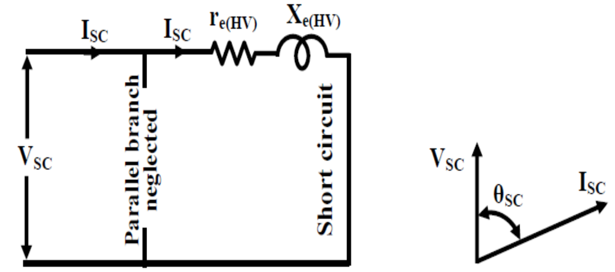


Figure 2.3: Circuit diagram for S.C test

Calculations from SC test readings:



(a) Equivalent circuit under S.C test (b) phasor diagram

Figure 2.4: Equivalent circuit & phasor diagram during S.C test

$r_{e(HV)}$, $x_{e(HV)}$ and cu loss are calculated from the SC test results as

Equivalent resistance referred to HV side is

$$R_{sc} = \frac{P_{sc}}{I_{sc}^{2}} = r_{e(HV)}$$

Equivalent impedance referred to HV side is

$$Z_{sc} = \frac{V_{sc}}{I_{sc}} = z_{e(HV)}$$

Equivalent reactance referred to HV side is $X_{sc} = \sqrt{Z_{sc}^2 - R_{sc}^2} = x_{e(HV)}$

The culoss is equal to the wattmeter reading W_{sc}

- Thus, the approximate equivalent circuit of the transformer can be drawn by the calculated values of R_0 and X_0 on LV side and $r_{e(HV)}$ and $x_{e(HV)}$ *on* HV side.
- The efficiency at any load is calculated from the losses W_i and W_{cufl} as

$$\eta_x = \frac{xVA\cos\phi}{xVA\cos\phi + W_i + x^2 W_{FLCu}} \times 100$$

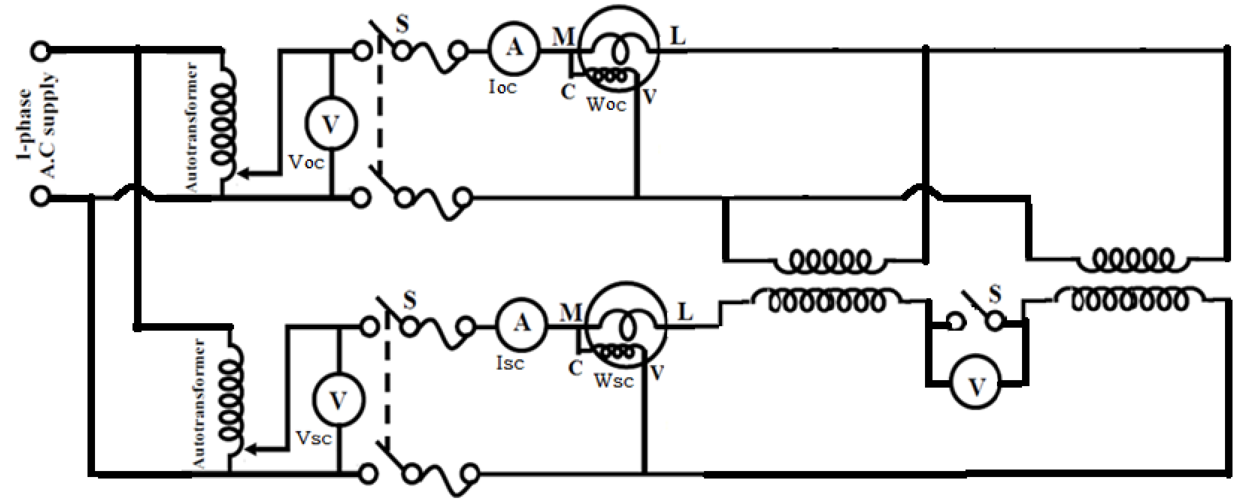
The regulation of the transformer is calculated from the $r_{e(HV)}$ and $x_{e(HV)}$ as

$$\%reg = \frac{I_{HV} r_{eHV}\cos\phi \pm I_{HV} x_{eHV}\sin\phi}{\mathrm{V}_{\mathrm{HV}}} \times 100 \text{ where } + \text{is for lagging pf and} - \text{is for leading pf}$$

Question no. 2 Explain Sumpner’s test or back to back test

Ans: Purpose of Sumpner’s test or back to back test on transformer is to determine efficiency, voltage regulation considering the **heating under loaded** conditions.

1. Two identical transformers are required to conduct the Sumpner's test
2. Both transformers are connected to supply such that one transformer is loaded on another.
3. Both Primaries are connected in parallel and both secondaries are connected in series opposition which is checked by the voltmeter showing zero volts when the switch S is closed.

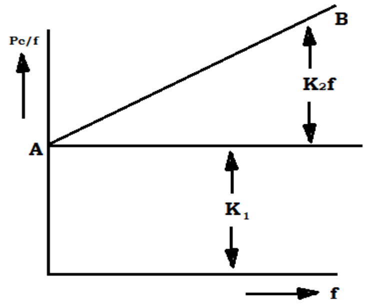


Procedure for sumpner’s test:

1. Both the emf's cancel each other, as transformers are identical. In this case, as per superposition theorem, no current flows through secondary. And thus the no load test is simulated.
2. The current drawn from V_{oc} is $2I_0=I_{oc}$ and the input power measured by wattmeter W_{oc} is equal to iron losses of both transformers. i.e. iron loss per transformer $P_i = W_{oc}/2$.
3. Now, a small voltage V_{sc} is injected into secondary with the help of a low voltage transformer.
4. The voltage V_{sc} is adjusted so that, the rated current I_{sc} flows through the secondary. In this case, both primaries and secondary’s carry rated current.
5. Thus short circuit test is simulated and wattmeter W_{sc} shows total full load copper losses of both transformers. i.e. copper loss per transformer $P_{Cu} = W_{sc}/2$.
6. From above test results, the full load efficiency of each transformer is calculated and is given as

$$\%\eta = \frac{xVA\cos\phi}{xVA\cos\phi + \frac{W_{oc}}{2} + x^2\frac{W_{sc}}{2}} \times 100$$

Question no. 3 Explain the Separation of losses



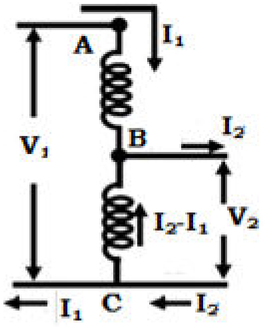
- For a sine flux wave, the transformer core loss is given by

  $P_c = K_h f B_m^x + K_e f^2 B_m^2$
- The transformer core loss consists of hysteresis and eddy current loss.
- Core loss per cycle = $\frac{P_c}{f} = K_h B_m^x + K_e f B_m^2$
- Here $B_m \propto \frac{V}{f}$ is observed as constant, therefore $\frac{P_c}{f} = K_1 + K_2 f$
- For the given transformer conduct the test on it at various frequencies and note down the values of Core loss per cycle (Pc/f)
- From the values of the test plot the graph and is shown in the adjacent Fig., where K_1 and K_2 constants are obtained.
- With constants K_1 and K_2 the core loss can be separated in to hysteresis and eddy current loss at any desired frequency and is given as

  $P_h = K_1 f$ and $P_e = K_2 f^2$

Question no.4 Write a short notes on 1-Φ Auto transformer

1. An auto transformer is a single winding transformer, i.e. both primary and secondary windings are connected to each other both electrically and magnetically.
2. There is one common winding which forms both primary and secondary winding in which voltage is varied by changing the position of secondary tapping on the body of the coil.



3. The primary VA is equal to secondary VA i.e $V_1 I_1 = V_2 I_2$
4. Here the primary VA $V_1 I_1$ is transformed to secondary through both induction and conduction principles
5. The primary winding consists of N_1 turns from A to C and the secondary has N_2 turns
6. Thus the VA from the portion AB will be transformed inductively towards secondary and The VA from the portion BC will be transformed conductively to the secondary
7. The AB has $(N_1 - N_2)$ turns with a current of I_1 from A to B and the portion BC has N_2 turns with a current of $(I_2 - I_1)$ from C to B
8. The primary mmf of the inductively transformed VA in the auto transformer is $(N_1 - N_2) I_1$ which should be same to that of secondary mmf $N_2 (I_2 - I_1)$

9. The primary mmf $(N_1 - N_2) I_1 = N_1 I_1 - N_2 I_1 = N_2 I_2 - N_2 I_1$ since $N_1 I_1 = N_2 I_2$

   $= N_2 (I_2 - I_1)$ equal to the secondary mmf

- ***Transformation ratio*** of the auto transformer is thus the ratio of N_2 to the $N_1 - N_2$

  That is $\dfrac{N_2}{N_1 - N_2} = \dfrac{\frac{N_2}{N_1}}{\frac{N_1 - N_2}{N_1}} = \dfrac{k}{1-k}$

- ***Inductively transformed VA***

  As the inductively transformed VA is equal to the $(V_1\text{-}V_2)*I_1$ from the total VA V_1I_1 Then

$$\frac{\text{inductively transformed VA}}{\text{Total input VA}} = \frac{(V_1 - V_2)I_1}{V_1 I_1} = \frac{V_1 I_1 - V_2 I_1}{V_1 I_1} = 1 - \frac{V_2}{V_1} = 1 - k$$

- ***Conductively transformed VA***

  As the conductively transformed VA is equal to the subtraction of the inductively transformed VA from the total VA. Therefore

$$\text{conductively transformed VA} = V_1 I_1 - (1-k)V_1 I_1 = kV_1 I_1 \;\therefore\; \frac{\text{conductively transformed VA}}{\text{Total input VA}} = k$$

Question no.5 List out the advantages and applications of Autotransformers

1. Its efficiency is more when compared with the conventional one.
2. Its size is relatively very smaller.
3. Voltage regulation of autotransformer is much better.
4. Lower cost
5. Low requirements of excitation current.
6. Less copper is used in its design and construction
7. In conventional transformer the voltage step up or step down value is fixed while in autotransformer, we can vary the output voltage as per out requirements and can smoothly increase or decrease its value as per our requirement.

Applications:

1. Used in both Synchronous motors and induction motors.
2. Used in electrical apparatus testing labs since the voltage can be smoothly and continuously varied.

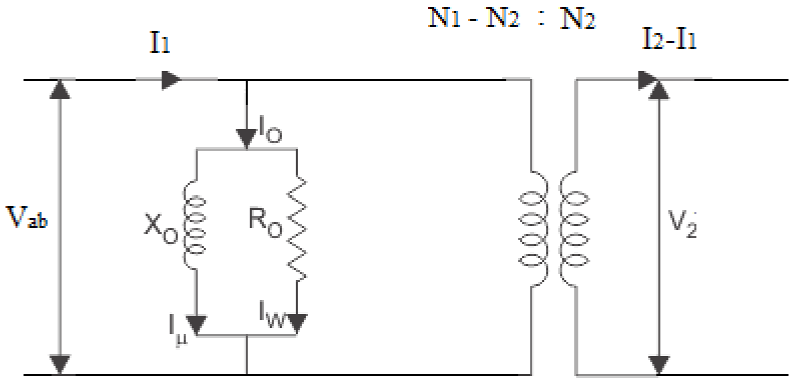
3. They find application as boosters in AC feeders to increase the voltage levels. Then, $\frac{V_2}{V_1}=\frac{N_2}{N_1}=\frac{I_1}{I_2}=K$ Note: The current in the section CB is vector difference of I_2 and I_1. But as the two currents are practically in phase opposition, the resultant current is (I_2-I_1).

Question no.6 Explain the Equivalent circuit of an auto transformer

The equivalennnt circuit of the auto transformer referred to primary is given below with the necessary mathematical equations and diagrams

$V1 = Vab + V_2$

Therefore Vab is the transformed secondary voltage of an auto transformer with a turns ratio of $(N_1-N_2) : N_2$



$V_{ab} = V_2^{'}$ + transformed secondary leakage impedance drop + Primary leakage impedance drop

$$V_{ab}=V_2^{'}+I_1(r_1+jx_1)+(I_2-I_1)(r_2+jx_2)^{'}$$

As voltages are directly proportional to turns and currents are inversely proportional to the turns

$$\frac{V_2^{'}}{V_2}=\frac{N_1-N_2}{N_2} \text{ and } \frac{I_2-I_1}{I_1}=\frac{N_1-N_2}{N_2}$$

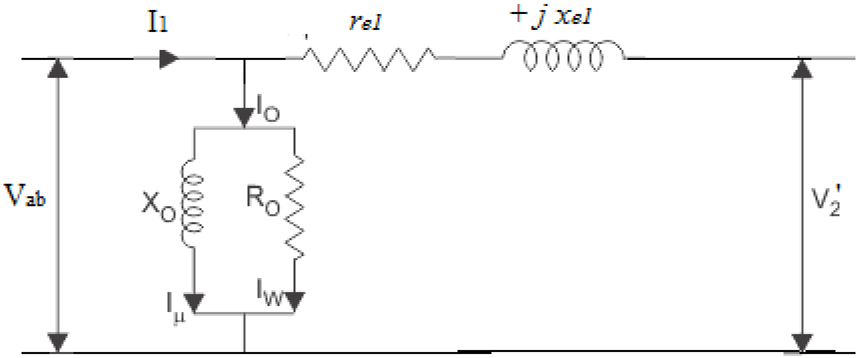
Now, $V_{ab}=V_2\left(\frac{N_1-N_2}{N_2}\right)+I_1(r_1+jx_1)+(I_2-I_1)(r_2+jx_2)\left(\frac{N_1-N_2}{N_2}\right)$

$$V_{ab}=V_2\left(\frac{N_1-N_2}{N_2}\right)+I_1(r_1+jx_1)+I_1(r_2+jx_2)\left(\frac{N_1-N_2}{N_2}\right)^2$$

$$V_{ab}=V_2\left(\frac{N_1-N_2}{N_2}\right)+I_1\left\{\left(r_1+r_2\left(\frac{N_1-N_2}{N_2}\right)^2\right)+j\left(x_1+x_2\left(\frac{N_1-N_2}{N_2}\right)^2\right)\right\}$$

$$V_{ab}=V_2\left(\frac{N_1-N_2}{N_2}\right)+I_1\left[\left(r_1+r_2^{'}\right)+j\left(x_1+x_2^{'}\right)\right]$$

$$V_{ab} = V_2\left(\frac{1-k}{k}\right) + I_1\left[\left(r_1 + r_2^{'}\right) + j\ \left(x_1 + x_2^{'}\right)\right]$$



Therefore $V_1 = V_{ab} + V_2$

$$V_1 = \left\{V_2\left(\frac{1-k}{k}\right) + I_1\left[\left(r_1 + r_2^{'}\right) + j\ \left(x_1 + x_2^{'}\right)\right]\right\} + V_2$$

$$V_1 = \left\{\frac{V_2}{k} - V_2 + I_1\left[\left(r_1 + r_2^{'}\right) + j\ \left(x_1 + x_2^{'}\right)\right]\right\} + V_2$$

$$V_1 = \frac{V_2}{k} + I_1\left[\left(r_1 + r_2^{'}\right) + j\ \left(x_1 + x_2^{'}\right)\right]$$

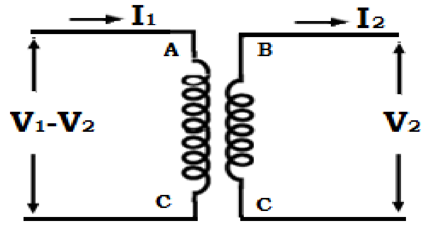
Question no.7 Derive the expression for the saving of cu in auto transformer

- The weight of the copper in the transformer is directly proportional to the MMF in the coil.
- Therefore, the weight of the cu in an ordinary transformer the total MMF is $N_1I_1 + N_2I_2$
- The weight of the cu in an auto transformer is $(N_1 – N_2)\ I_1 + N_2\ (I_2 – I_1)$
- $$\frac{Weight\ of\ the\ cu\ in\ ordinary\ 2\ winding\ transformer}{Weight\ of\ the\ cu\ in\ auto\ transformer} = \frac{N_1I_1 + N_2I_2}{(N_1 - N_2)I_1 + N_2(I_2 - I_1)}$$
- $$= \frac{N_1I_1 + N_2I_2}{(N_1I_1 - N_2I_1) + (N_2I_2 - N_2I_1)} = \frac{1 + \dfrac{N_2I_2}{N_1I_1}}{\left(1 - \dfrac{N_2I_1}{N_1I_1}\right) + \left(\dfrac{N_2I_2}{N_1I_1} - \dfrac{N_2I_1}{N_1I_1}\right)} = \frac{1 + \left(k \times \dfrac{1}{k}\right)}{(1-k) + (1-k)} = \frac{2}{2(1-k)} = \frac{1}{1-k}$$

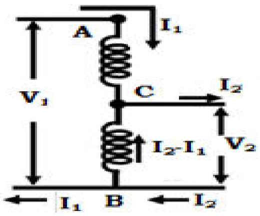
Weight of the cu in an auto transformer = (1-k) * Weight of the cu in an ordinary transformer\

- Saving of the copper in the auto transformer = $W_{2wdg} – W_{auto} = W_{2wdg} – [\ (1-k) * W_{2wdg}]$
- Saving of the copper in the auto transformer = $W_{2wdg} – W_{2wdg} + k\ W_{2wdg} = k\ W_{2wdg}$
- Therefore, saving of cu in auto transformer is ***k times*** the cu used in 2 winding transformer.

Question no.8 Give the Comparison of auto transformer with a two winding transformer



Representation of auto transformer as 2 winding transformer



auto transformer

1. **Ratings:**

- The KVA rating of the auto transformer with primary V_1 and I_1 values are $V_1\ I_1$
- The KVA rating of the auto transformer with transformation principle is the VA available in AC winding as $(V_1 - V_2)\ I_1$

Therefore, $$\frac{KVA_{auto}}{KVA_{2-wdg}} = \frac{V_1 I_1}{(V_1 - V_2)I_1} = \frac{1}{1-K}$$

2. **Losses:**

- The full load losses is same in both the auto transformer and 2 winding
- The per unit losses in the transformer is expressed in terms of rating of the transformer as

$$\frac{\text{Per unit losses}_{auto}}{\text{Per unit losses}_{2-wdg}} = \frac{\text{Full load losses}}{\text{KVA rating as auto T/F}} \times \frac{\text{KVA rating as 2 - wdg T/F}}{\text{Full load losses}} = 1-K$$

3. **Impedance drop:**

- The impedance drop is same in both the auto transformer and 2 winding
- The per unit impedance drop in the transformer is expressed in terms of rated voltage as

$$\frac{\text{Per unit impedance drop}_{auto}}{\text{Per unit impedance drop}_{2-wdg}} = \frac{I_1 z_1 / V_1}{I_1 z_1 / (V_1 - V_2)} = \frac{V_1 - V_2}{V_1} = 1-K$$

4. **Regulation:**

- Regulation of the transformer is directly proportional to impedance drop.

$$\frac{\text{Regulation as auto transformer}}{\text{Regulation as 2 - wdg transformer}} = 1-K$$

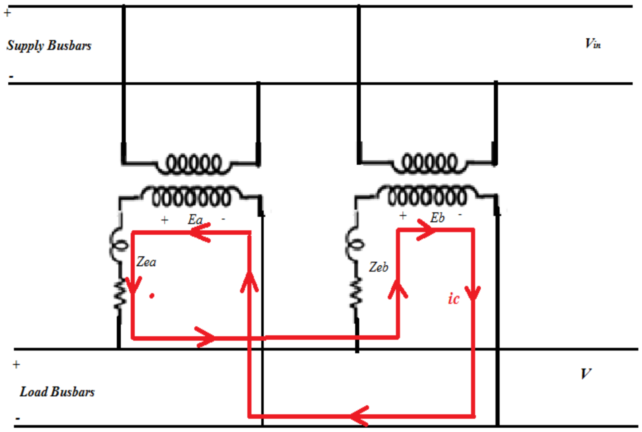
SNo	Auto transformer	2-winding transformer
1	A transformer having one winding, a part of which acts as primary and other as secondary	A static device which transfers electrical energy using two isolated coils
2	Power transfer is partly by Induction and partly by conduction	Through induction completely
3	Smaller in size	Larger in size
4	Gives better voltage regulation	Gives good voltage regulation
5	Less amount of cu is required	More amount of cu is required
6	Low losses and high efficiency	High losses and low efficiency
7	Self induction principle	Mutual induction principle
8	Output voltage can be varied	Output voltage is constant
9	More economical	Less economical
10	Used as starters for induction motors, as voltage regulators in railways and in laboratories	Used in power system for step up and step down the voltages.

Question no.9 List the conditions for Parallel operation of two single phase transformers

Conditions for parallel operation of two single phase transformers

1. The Transformers must have the same voltage ratio i.e. with Primaries of Transformers connected to the same supply; their Secondary's must have the same voltage.
2. The equivalent leakage impedance in Ohm should be inversely proportional to their respective KVA rating. In other words, we can say that all the Transformers should have their per unit leakage impedance based on their own kVA rating equal.
3. The ratio of equivalent leakage reactance to equivalent resistance i.e. $\frac{x_e}{r_e}$ should be same for the two transformers.
4. The primaries and secondary's of the transformers must be connected with the correct polarities.

Question no.10 Explain the parallel operation of the two transformers at No load



The adjacent figure shows the parallel operation connection of the two transformers at no-load and Let

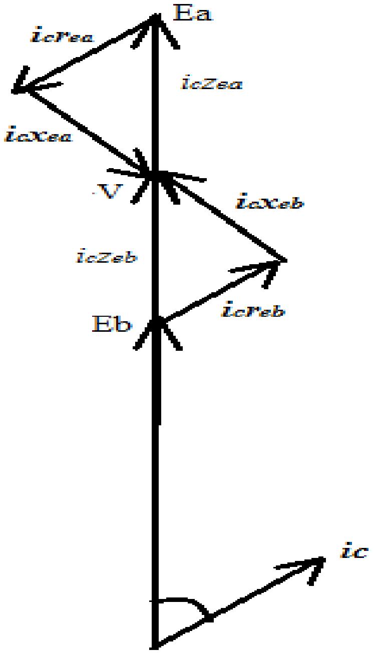
- E_a, E_b = secondary emfs of transformers A & B
- z_{ea} and z_{eb} are the leakage impedances of the transformers referred to secondary.
- i_c is the circulating current in the secondary's of the two transformer
- V is the terminal voltage across the transformers
- There are two cases of the parallel operation of the two transformers namely ***with*** and ***without equal voltage ratios*.**

Case:1 (Equal voltage ratios $E_a = E_b$ at No- load)

- As the two secondary voltages are equal in magnitude, there is no potential difference and hence there is no circulating current.
- Since No- Load operation the no load currents on the secondary's I_a and I_b are zero.
- Therefore $E_a = E_b$ i.e ($E_a - E_b = 0$)

  and $I_a = I_b = 0$

Case:2 (Un equal voltage ratios $E_a > E_b$ at No- load)



- As two secondary voltages are unequal in magnitude, then a potential difference of (E_a - E_b) rises to a circulating current i_c ***(lagging nature)*** against the leakage impedance ($z_{ea} + z_{eb}$)
- Since it is of No- Load operation the no load currents on the secondary's I_a and I_b are zero.
- The KVL for the secondary's loop is $-E_a + i_c z_{ea} + i_c z_{eb} + E_b = 0$

and now $i_c \, (z_{ea} + z_{eb}) = E_a - E_b$ $\quad \therefore i_c = \frac{E_a - E_b}{z_{ea} + z_{eb}}$ and

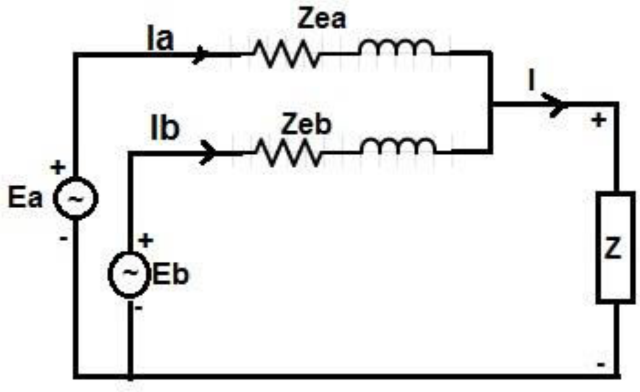
$$i_c = \frac{E_a - E_b}{(r_{ea} + r_{eb}) + j(x_{ea} + x_{eb})} \text{ lags by } \beta = \tan^{-1}\left(\frac{x_{ea} + x_{eb}}{r_{ea} + r_{eb}}\right)$$

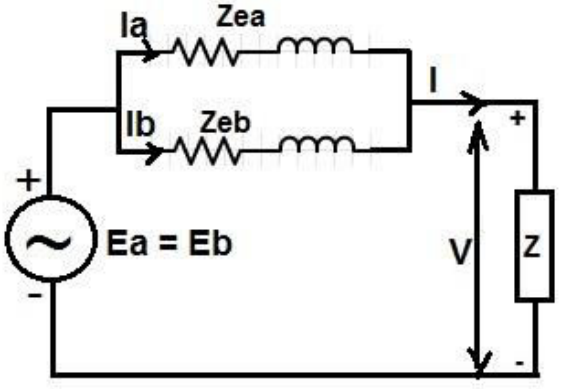
- Also, this circulating current i_c aids to the E_a so that the voltage $i_c z_{ea}$ is dropped from E_a and presents as V at the busbars.

- Also, this circulating current $\boldsymbol{i_c}$ opposes to the $\boldsymbol{E_b}$ so that the voltage $\boldsymbol{i_c\, z_{eb}}$ is added to $\boldsymbol{E_b}$ and presents as V at the busbars.
- The KVLs of the corresponding transformers are
- $V = E_a - i_c z_{ea}$ and $V = E_b + i_c z_{eb}$
- Thus, the circulating current will reduce the voltage of the transformer whose induced voltage is more by subtracting the drop and this same circulating current will boost the voltage of the transformer whose induced voltage is less.

Case:3(Equal voltage ratios $E_a = E_b$ With load)

- With equal voltages on the secondary's of the two transformers there is no circulating current
- Because of the transformers loaded I_a and I_b are the currents supplied by transformers A and B
- Below figure shows the circuit diagram of transformers in parallel with load at equal voltages





Then the KVLs for the two transformers is given by

$V = E_a - I_a z_{ea}$ $\qquad$ $V = E_b - I_b z_{eb}$

As the terminal voltages are same therefore

$E_a - I_a z_{ea} = E_b - I_b z_{eb}$

Since, the induced voltages are same i.e $E_a = E_b$, then the voltage drops across the leakage impedances will also be same.

Therefore $I_a z_{ea} = I_b z_{eb}$ $\qquad \frac{I_a}{I_b} = \frac{z_{eb}}{z_{ea}}$ $\qquad \frac{1000 * VI_a}{1000 * VI_b} = \frac{z_{eb}}{z_{ea}}$ $\qquad \frac{S_a}{S_b} = \frac{z_{eb}}{z_{ea}}$

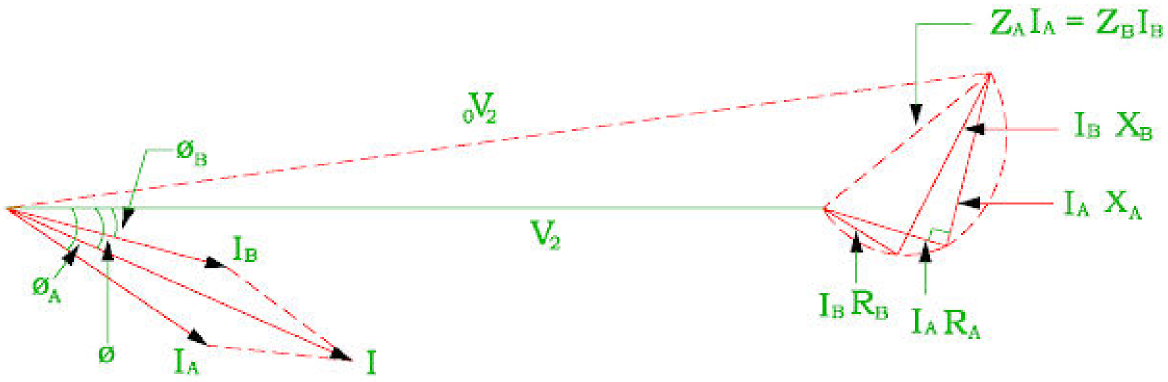
From the above relations

$I_a = I \frac{z_{eb}}{z_{ea} + z_{eb}}$ $\qquad I_b = I \frac{z_{ea}}{z_{ea} + z_{eb}}$ $\qquad S_a = S \frac{z_{eb}}{z_{ea} + z_{eb}}$ $\qquad S_b = S \frac{z_{ea}}{z_{ea} + z_{eb}}$

Thus, the performance of this case depends on the leakage impedances

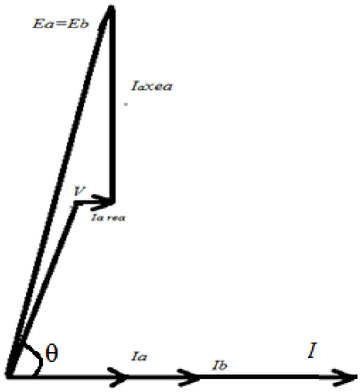
Case a: $|Z_{ea}| = |Z_{eb}|$ ***and*** $\frac{x_{ea}}{r_{ea}} \neq \frac{x_{eb}}{r_{eb}}$

- Let the impedance values are assumed to be same and the leakage impedance angles $\phi_a \neq \phi_b$ and $\phi_a > \phi_b$



- If the X/R ratios of the transformers are different, one transformer will be operating with a higher power factor and the other transformer with a lower power factor that of total load.
- It means that kW load is not proportionally shared by them.

Case b : $|Z_{ea}| > |Z_{eb}|$ ***and*** $\frac{x_{ea}}{r_{ea}} = \frac{x_{eb}}{r_{eb}}$



- The two transformers with this case having unequal impedances and equal load angles will share the load in such a way that, Transformer having greater leakage impedance shares less KVA and lesser leakage impedance shares more KVA.
- Therefore the transformers of different KVA ratings operating in parallel is possible with their leakage impedances in ohms are inversely proportional to their respective KVA ratings

Case:4(Un equal voltage ratios $E_a > E_b$ ***With load)***

- As two secondary voltages are unequal in magnitude, then a potential difference of (E_a - E_b) rises to a circulating current i_c ***(lagging nature)*** against the leakage impedance ($z_{ea} + z_{eb}$)

- As with Loaded operation the load currents on the secondary's are assumed to be equally shared i.e two transformers will supply I/2 currents .
- With circulating current, the currents handled by transformer is $I_a = \frac{I}{2} + i_c$ and $I_b = \frac{I}{2} - i_c$
- The phasor sum of I_a and I_b is equal to the load current I
- The mathematical analysis is

$$V = E_a - I_a z_{ea} = (I_a + I_b)Z \qquad \ldots.(1)$$

$$V = E_b - I_b z_{eb} = (I_a + I_b)Z \qquad \ldots.(2)$$

Equating eq's (1) and (2)

$$E_a - I_a z_{ea} = E_b - I_b z_{eb} \Rightarrow I_a z_{ea} - I_b z_{eb} = E_a - E_b$$

$$I_b = \frac{I_a z_{ea} - (E_a - E_b)}{z_{eb}}$$

Substitution of this I_b in eq .(1)

$$E_a - I_a z_{ea} = \left[I_a + \frac{I_a z_{ea} - (E_a - E_b)}{z_{eb}} \right] Z \qquad E_a - I_a z_{ea} = \left[I_a Z + \frac{I_a Z z_{ea}}{z_{eb}} - \frac{(E_a - E_b)Z}{z_{eb}} \right]$$

$$E_a + \frac{(E_a - E_b)Z}{z_{eb}} = \left[I_a z_{ea} + I_a Z + \frac{I_a Z z_{ea}}{z_{eb}} \right] \qquad E_a + \frac{(E_a - E_b)Z}{z_{eb}} = \left[z_{ea} + Z + \frac{Z z_{ea}}{z_{eb}} \right] I_a$$

$$\frac{E_a}{z_{ea} + Z + \frac{Z z_{ea}}{z_{eb}}} + \frac{(E_a - E_b)Z}{(z_{ea} z_{eb} + Z z_{eb} + Z z_{ea})} = I_a \qquad I_a = \frac{E_a}{z_{ea} + Z + \frac{Z z_{ea}}{z_{eb}}} + \frac{(E_a - E_b)}{\left(\frac{z_{ea} z_{eb}}{Z} + z_{eb} + z_{ea} \right)}$$

Also I_a can be written as

$$I_a = \frac{E_a z_{eb}}{(z_{ea} z_{eb} + Z z_{eb} + Z z_{ea})} + \frac{(E_a - E_b)Z}{(z_{ea} z_{eb} + Z z_{eb} + Z z_{ea})}$$

$$I_a = \frac{E_a z_{eb} + (E_a - E_b)Z}{z_{ea} z_{eb} + Z(z_{ea} + z_{eb})}$$

Similarly

$$I_b = \frac{E_b z_{ea} - (E_a - E_b)Z}{z_{ea} z_{eb} + Z(z_{ea} + z_{eb})}$$

And the total load current is $I_a + I_b = I$

$$I = \frac{E_a z_{eb} + E_b z_{ea}}{z_{ea} z_{eb} + Z(z_{ea} + z_{eb})}$$

And the terminal voltage is V = IZ

$$V = \frac{E_a z_{eb} + E_b z_{ea}}{\frac{z_{ea} z_{eb}}{Z} + \left(z_{ea} + z_{eb}\right)}$$

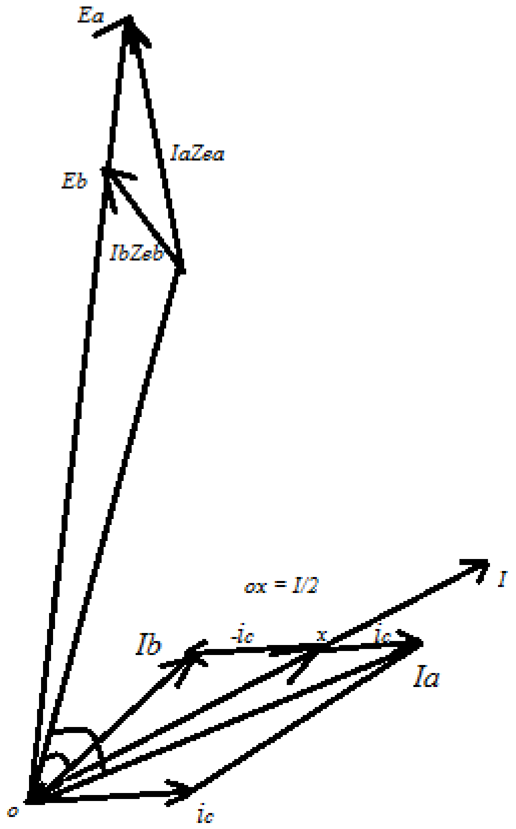
The circulating current $\boldsymbol{i_c}$ is $I_a - I_b$ and is equal to the second terms in I_a and I_b

$$i_c = \frac{\left(E_a - E_b\right)}{\left(\frac{z_{ea} z_{eb}}{Z} + z_{eb} + z_{ea}\right)} \quad \textbf{\textit{also V = I Z}}$$

$$V = \left(\frac{E_a - V}{z_{ea}} + \frac{E_b - V}{z_{eb}}\right) Z$$

$$V\left(\frac{1}{Z} + \frac{1}{z_{ea}} + \frac{1}{z_{eb}}\right) = \frac{E_a}{z_{ea}} + \frac{E_b}{z_{eb}} \quad \textbf{\textit{and}}$$

$$I_a = I \frac{z_{eb}}{z_{ea} + z_{eb}} \qquad I_b = I \frac{z_{ea}}{z_{ea} + z_{eb}}$$



Unit -6

Star – Star connection:

Star-star connection is generally used for small, high-voltage transformers. Because of star connection, number of required turns/phase is reduced (as phase voltage in star connection is 1/√3 times of line voltage only). Thus, the amount of insulation required is also reduced.

Advantages:

- There is no phase displacement between the primary and secondary voltages
- Star points on both sides make it possible to provide neutral connection

Disadvantages:

- If the load on the secondary side of the transformer is unbalanced, the phase voltages of load side change unless the load star point is earthed. The difficulty of shifting neutral can be overcome by connecting the primary star point to the star point of the generator.
- The primary of the transformer draws a magnetizing current which has third and fifth harmonics. If neutral of primary winding is not connected to neutral of generator, the third and fifth harmonics current cannot flow hence the flux in the core cannot be of sinusoidal wave and, therefore, the voltages will be distorted. By connecting primary neutral to the generator neutral, the path for return of these third and fifth harmonics current is provided and, therefore, the trouble of distortion of voltages is overcome.
- Even if neutral point of primary is connected to neutral of generator or earthed, still if third harmonic is present in the alternator voltage form, it will appear on secondary side. Though the secondary line voltages do not contain third harmonic voltage; but the 3rd harmonic voltages are additive in the neutral and causes the current in the neutral of tipple frequency (3rd harmonic) which will cause interference to the nearby communication system. Thus harmonic phase voltages may be high in-shell type 3-phase transformers The star-star connection is rarely used owing to the difficulties associated with the exciting current.

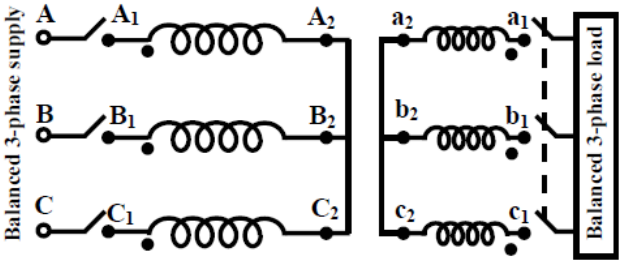


Figure Star/star Connection.

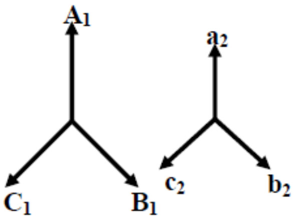


Figure Phasor diagram.

Delta – Delta Connection

Delta – Delta connection is generally used for large, high-current transformers. Because of delta connection, the cross sectional area of the conductor is reduced (as phase current in delta connection is 1/√3 times of line current only). Thus, the amount of cu required is also reduced.

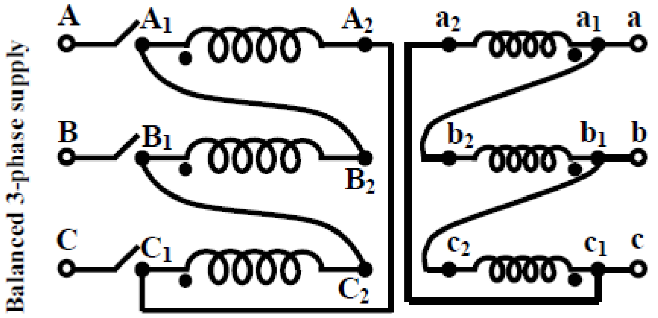
Advantages:

- In order to get secondary voltage as sinusoidal, the magnetizing current of transformer must contain a third harmonic component. The delta connection provides a closed path for circulation of third harmonic component of current. The flux remains sinusoidal which results in sinusoidal voltages.

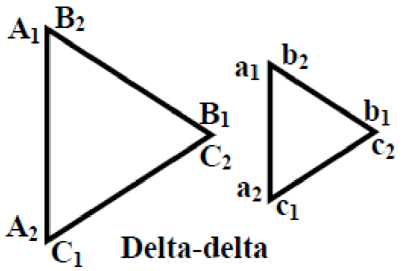
- If there is bank of single phase transformers connected in delta-delta fashion and if one of the transformers is disabled then the supply can be continued with remaining two transformers of course with reduced efficiency.
- Due to delta connection, phase voltage is same as line voltage hence winding have more number of turns. But phase current is (1/√3) times the line current. Hence the cross-section of the windings is very less. This makes the connection economical for low voltages transformers.
- Due to closed delta, third harmonic voltages are absent. The absence of star or neutral point proves to be advantageous in some cases.

Disadvantages:

- Due to the absence of neutral point it is not suitable for three phase four wire system.
- More insulation is required and the voltage appearing between windings and core will be equal to full line voltage in case of earth fault on one phase



Delta-delta Connection.



Delta-delta

Star – Delta connection:

Star – Delta connection is generally used for step down purposes, especially at the end of the power transmission lines that is nearer to the load points

Advantages:

- The primary side is star connected. Hence fewer numbers of turns are required. This makes the connection economical for large high voltage step down power transformers.
- The neutral available on the primary can be earthed to avoid distortion.
- The neutral point allows both types of loads (single phase or three phases) to be met.
- Large unbalanced loads can be handled satisfactory.
- The Y-D connection has no problem with third harmonic components due to circulating currents in Delta. It is also more stable to unbalanced loads since the D partially redistributes any imbalance that occurs.
- The delta connected winding carries third harmonic current due to which potential of neutral point is stabilized. Some saving in cost of insulation is achieved if HV side is star connected. But in practice the HV side is normally connected in delta so that the three phase loads like motors

and single phase loads like lighting loads can be supplied by LV side using three phase four wire system.

- As Grounding Transformer: In Power System Mostly grounded Y- Δ transformer is used for no other purpose than to provide a good ground source in ungrounded Delta system. Take, for example, a distribution system supplied by Δ connected (i.e., ungrounded) power source. . If it is required to connect phase-to-ground loads to this system a grounding bank is connected to the system.

Disadvantages:

- In this type of connection, the secondary voltage is not in phase with the primary. Hence it is not possible to operate this connection in parallel with star-star or delta-delta connected transformer.
- One problem associated with this connection is that the secondary voltage is shifted by 30^0 with respect to the primary voltage. This can cause problems when paralleling 3-phase transformers since transformers secondary voltages must be in-phase to be paralleled. Therefore, we must pay attention to these shifts.

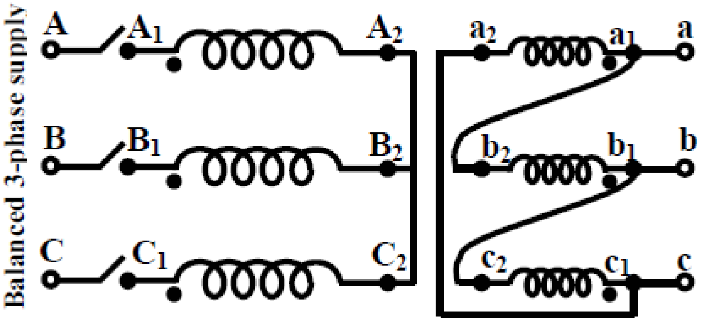


Figure: Star/delta Connection.

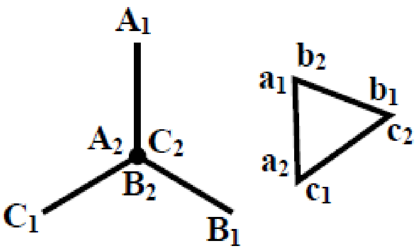


Figure: Phasor diagram.

Delta – Star connection:

Delta– Star connection is generally used for step up purposes, especially at the starting of the power transmission lines that is nearer to the generating stations.

Advantages:

- *Cross section area of winding is less at Primary side:*
  On primary side due to delta connection winding cross-section required is less.
- **Used at Three phase four wire System:**
  On secondary side, neutral is available, due to which it can be used for 3-phase, 4 wire supply system.
- *No distortion of Secondary Voltage:*
  No distortion due to third harmonic components.
- *Handled large unbalanced Load:*
  Large unbalanced loads can be handled without any difficulty.
- *Grounding Isolation between Primary and Secondary:*
  Assuming that the neutral of the Y-connected secondary circuit is grounded, a load connected phase-to-neutral or a phase-to-ground fault produces two equal and opposite currents in two phases in the primary circuit without any neutral ground current in the primary circuit.

Therefore, in contrast with the Y-Y connection, phase-to-ground faults or current unbalance in the secondary circuit will not affect ground protective relaying applied to the primary circuit. This feature enables proper coordination of protective devices and is a very important design consideration.

The neutral of the Y grounded is sometimes referred to as a grounding bank, because it provides a local source of ground current at the secondary that is isolated from the primary circuit.

- *Harmonic Suppression:*

  The magnetizing current must contain odd harmonics for the induced voltages to be sinusoidal and the third harmonic is the dominant harmonic component. In a three-phase system the third harmonic currents of all three phases are in phase with each other because they are zero-sequence currents. In the Y-Y transformer connection, the only path for third harmonic current is through the neutral.

  In the Δ -Y connection, however, the third harmonic currents, being equal in amplitude and in phase with each other, are able to circulate around the path formed by the Δ connected winding. The same thing is true for the other zero-sequence harmonics.

- *Grounding Bank:*

  It provides a local source of ground current at the secondary that is isolated from the primary circuit. For suppose an ungrounded generator supplies a simple radial system through Δ-Y transformer with grounded Neutral at secondary as shown Figure. The generator can supply a single-phase-to-neutral load through the -grounded Y transformer.

Disadvantages:

- In this type of connection, the secondary voltage is not in phase with the primary. Hence it is not possible to operate this connection in parallel with star-star or delta-delta connected transformer.
- If secondary of this transformer should be paralleled with secondary of another transformer without phase shift, there would be a problem.

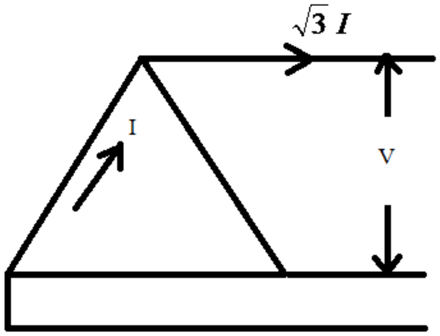
Open delta or V-V connection:

- If one of the transformers of a D – D is removed and 3-phase supply is connected to the primaries, then three equal 3-phase voltages will be available at the secondary terminals on no-load. This method of transforming 3-phase power by means of only two transformers is called the open – D or V – V connection.

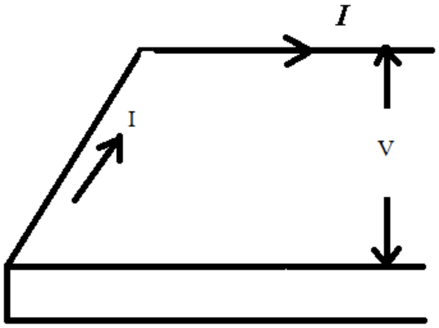
  This open delta connection is employed
- When the three-phase load is too small to warrant the installation of full three-phase transformer bank.
- When one of the transformers in a D – D bank is disabled, so that service is continued although at reduced capacity, till the faulty transformer is repaired or a new one is substituted.
- When it is anticipated that in future the load will increase necessitating the closing of open delta.
- One important point to note is that the total load that can be carried by a V – V bank is not two-third of the capacity of a D – D bank but it is only 57.7% of it. That is a reduction of 15% (strictly, 15.5%) from its normal rating.

Example:

Suppose there is D – D bank of three 10-kVA transformers. When one transformer is removed, then it runs in V – V. The total rating of the two transformers is 20 kVA. But the capacity of the V – V bank is not the sum of the transformer kVA ratings but only 0.866 of it i.e. 20 ´ 0.866 = 17.32 (or 30 ´ 0.57 = 17.3 kVA). The fact that the ratio of V-capacity to D-capacity is 1/ 3 = 57.7% (or nearly 58%) instead of 66 .2 per cent can be proved as follows:



Delta connection



Open Delta connection

From the above diagram

$$\frac{V-Vcapacity}{\Delta-\Delta Capacity}=\frac{\sqrt{3}VI}{\sqrt{3}V\sqrt{3}I}=\frac{1}{\sqrt{3}}=0.577=58\%$$

Therefore, V-V capacity = 57.7% of the Delta – Delta connection capacity.

It is obvious from above that when one transformer is removed from a D – D bank.

1. The bank capacity is reduced from 30 kVA to 30* 0.577 = 17.3 kVA and not to 20 kVA as might be thought off-hand.
2. Only 86.6% of the rated capacity of the two remaining transformers is available (i.e. 20 *0.866 = 17.3 KVA). In other words, ratio of operating capacity to available capacity of an open-D is 0.866. This factor of 0.866 is sometimes called the utility factor.
3. Each transformer will supply 57.7% of load and not 50% when operating in V – V

However, it is worth noting that if three transformers in a D – D bank are delivering their rated load and one transformer is removed, the overload on each of the two remaining transformers is 73.2% because

The disadvantages of this connection are :

1. The average power factor at which the V-bank operates is less than that of the load. This power factor is actually 86.6% of the balanced load power factor. Another significant point to note is that, except for a balanced unity power factor load, the two transformers in the V – V bank operate at different power factors.
2. Secondary terminal voltages tend to become unbalanced to a great extent when the load is increased, this happens even when the load is perfectly balanced.

It may, however, be noted that if two transformers are operating in V – V and loaded to rated capacity (in the above example, to 17.3 kVA), the addition of a third transformer increases the total capacity by 3 or 173.2% (i.e. to 30 kVA). It means that for an increase in cost of 50% for the third transformer, the increase in capacity is 73.2% when converting from a V – V system to a D – D system.

Power Supplied by V – V Bank

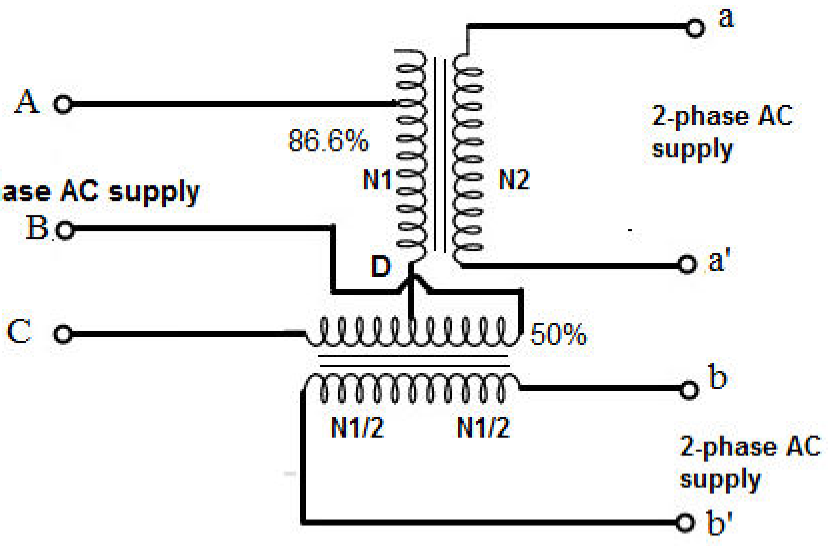
When a V – V bank of two transformers supplies a balanced 3-phase load of power factor cos f, then one transformer operates at a p.f. of cos (30° – Φ) and the other at cos (30° + Φ). Consequently, the two transformers will not have the same voltage regulation.

Scott Connection:

- Scott connection is also known as the three phase to two phase converter.

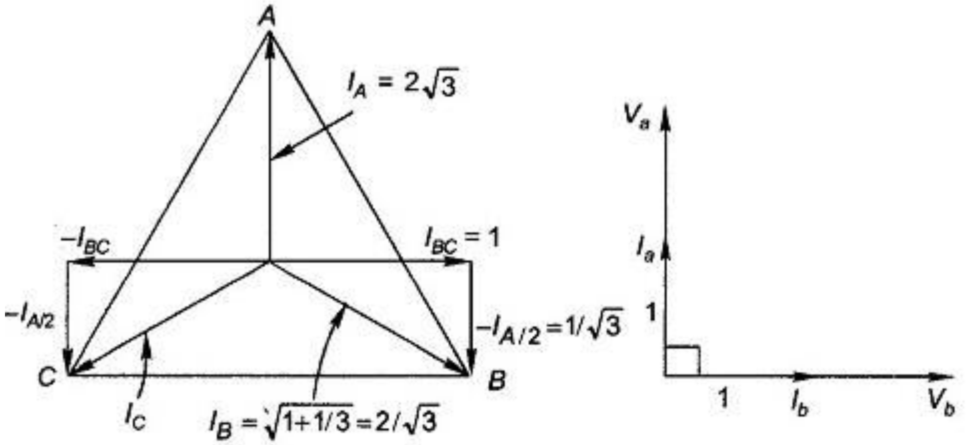
  Two phase supply is needed for following applications
    - To supply power to two phase electric furnaces
    - To interlink two phase and three phase system.
    - To supply power to 2 phase source from three phase equipment and vice-versa.

Circuit Description:



- Two single phase transformers with suitable tapping on both side, are required for the scott connection. They are
- Transformer 1: Main transformer with 50% tapping.
- Transformer 2: Teaser transformer with 86.6% tapping.
- Teaser transformer is connected between center tapping D and A.
- D is center tapping of main transformer and it connected across the line B and C of three phase side. It has secondary bb’ and primary BC.
- 3 phase input power is required for the scott connection. It provides two single phase outputs. These output are at 90° out of phase and must not be connected in series or in parallel as it creates a vector current on the primary side. Teaser transformer has secondary aa’ and primary AD.

  Fig shows the phasor diagram of the voltage across primary and secondary.



Scott connection, balanced load (unity power factor)

Mathematical Formulas:

The loads on the two single phase transformers are known as P_{2M} , $\cos\Phi_M$ and P_{2T} , $\cos\Phi_T$ on main and teaser transformers respectively.

Calculate the load currents

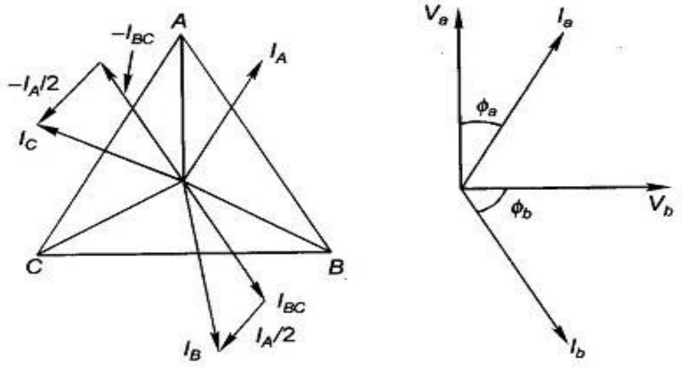
$$I_{2M} = \frac{P_{2M}}{V_2 \cos\phi_M} \qquad I_{2T} = \frac{P_{2T}}{V_2 \cos\phi_T}$$

Let , Transformation ratio of main transformer is $K = \frac{N_2}{N_1} = \frac{V_2}{V_1}$

Transformation ratio of Teaser transformer is $K_T = \frac{N_2}{0.866N_1} = \frac{V_2}{0.866V_1} = 1.15K$

Then, the primary side currents are

$$I_A = I_{1T} = 1.15KI_{2T} \qquad I_B = I_C = I_{1M} = \sqrt{(I_{1M})^2 + \left(\frac{I_{1T}}{2}\right)^2} \qquad I_{1M} = KI_{2M}$$



Scott connection, unbalanced load

Problem on Scott connection:

In a Scott connection, calculate the values of line currents on the three phase side, if the loads on 2 phase side are 300 kW and 400 kW, both at 125 V and 0.707 p.f. and the three phase line voltage is 3300 V. The 300 kW load is on the leading phase of the 2 phase side. Neglect the losses.

Given Data:

Primary line voltage $V_1 = 3300$ V

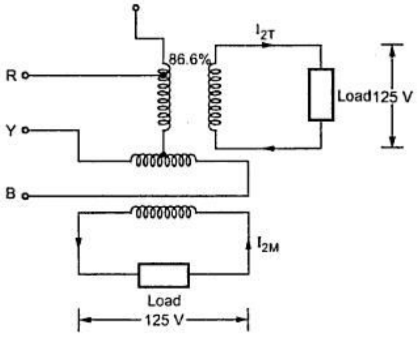
Secondary voltage $V_2 = 125$ V

Load on teaser secondary

$P_{2T} = 300$ kW $\quad$ $\cos\varnothing_T = 0.707$ p.f.

Load on main secondary

$P_{2M} = 400$ kW $\quad$ $\cos\varnothing_M = 0.707$ p.f.



To find: I_R, I_Y & I_B = ?

Solution:

Transformation ratio for main transformer, $K_M = V_2/V_1 = 125/3300 = 0.0378$

Transformation ratio for teaser transformer, $K_T = 1.15 \times K_M = 1.15 \times 0.0378 = 0.0436$

Secondary current in teaser transformer, $I_{2T} = (300 \times 10^3) / (125 \times 0.707) = 3394.62$ A

Primary current in teaser transformer, $= K_T \times I_{2T} = 0.0436 \times 3394.62 = 148$ A

∴ Line current, $\mathbf{I_R = 148}$ **A**

Secondary current in main transformer, $I_{2M} = (400 \times 10^3) / (125 \times 0.707) = 4526.16$ A

Primary current in main transformer, $I_{1M} = K_M \cdot I_{2M} = 0.0378 \times 452.16 = 171.08$ A

In addition to this I_{1M} current each half of the primary winding of the main transformer carries half of teaser primary current. i.e $I_{1T}/2 = 148/2 = 74$ A

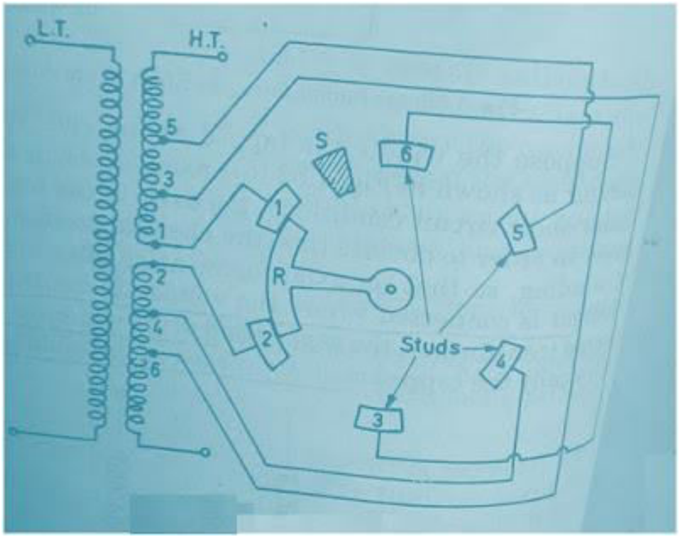
Total current carried $= \sqrt{((I_{1M})^2 + (I_{1T}/2)^2)} = \sqrt{((171.08)^2 + (74)^2)} = 186.39$ A

∴ Line currents, $\mathbf{I_Y = I_B = 186.39}$ **A**

The three line currents are $\mathbf{I_R = 148}$ **A,** $\mathbf{I_Y = 186.39}$ **A,** $\mathbf{I_B = 186.39}$ **A**

OFF Load Tap Changer:

- This type of Tap changer is used for seasonal voltage variations
- As in summers load on HT increases leads to drop in HT voltage so output can be increased by using Transformer Tap changer
- In Winters when load get reduced, HT voltage restores to normal levels then tap changer position can be done according to value of HT Voltage.



There are six studs marked as 1-6. The winding is tapped at six points equal to no. of studs. The tapping leads are connected to six corresponding marked stationary studs arranged in circle. The face plate carrying six studs can be mounted anywhere on Transformer. The Rotatable arm R can be rotated by means of Hand-wheel from outside transformer tank.

Winding taps are provided as per requirement if there are huge fluctuations in voltages then %age of tap changer interval vary accordingly.

Let's assume transformer winding is tapped at 2.5% interval, then with we can get following tapings with Rotatable arm provided outside Transformer tank :-

At Studs 1,2 : Full winding of Transformer will remains in circuit

At studs 2,3 : 97.5% Transformer winding will remain in the circuit.

At Studs 3,4 : 95% of transformer winding will be in circuit

At Stud 4,5 : 92.5 % of Transformer winding will be in circuit

At Stud 5,6 : 90% of Transformer winding will be in circuit

Usually in Transformer for 11 KV HT voltage Transformer tapping will be on 3 No. and then O/P voltage tapping will be done according to requirement and Variation in HT voltage.

Stop S fixes the final position and prevents the arm R from being rotated clockwise. In the absence of Stop S the Arm R may be come in contact with Stud 1 and 6. In-Such case only lower part of winding will be cut out of circuit which will leads to mechanical stress which is completely undesirable.

There are following taps available in 11KV/433 V Transformer as below:-

11550 HT Voltage – 433 V

11275 HT Voltage- 433 V

11000 HT Voltage – 433 V

10725 HT Voltage- 433 V

10450 HT Voltage- 433 V

Initially Transformer tapping is places at 11KV i.e. 3 No. at Rotating Handle.

Due care required when shifting Tap is that Transformer must be disconnected from line

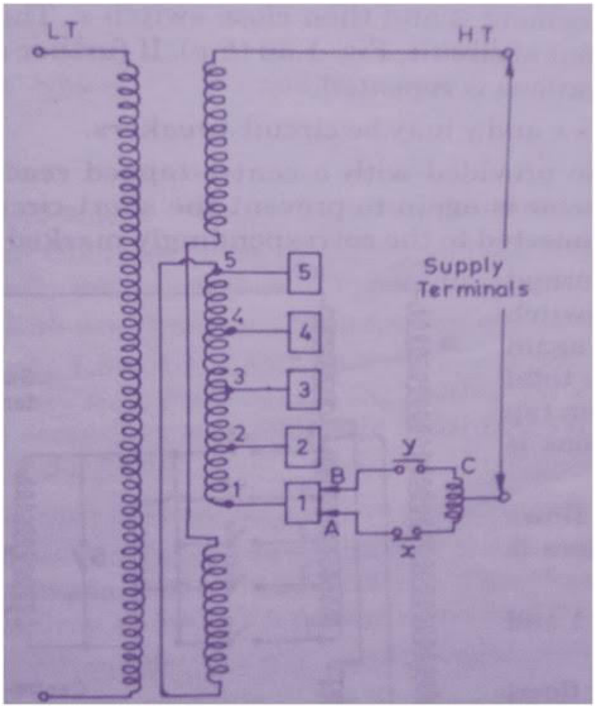
ON Load Tap Changer:

A tap changer designed to operate while the Transformer is charged is called On Load Tap Changer (OLTC). On Load Tap Changer, OLTC is Make Before Break type which ensures that no sparking will take place while operating OLTC. On Load Tap Changer (OLTC) is used for short period or daily voltage regulation. During the operation of On Load Tap Changer, OLTC following two things shall be taken care:

- The main circuit shall not be opened else heavy sparking will take place.
- No part of tapped winding shall be shorted.

The below figure shows the elementary form of On Load tap Changer, OLTC. As shown in the figure the center tapped Reactor C prevents the tapped winding from getting short circuited. The Transformer tappings are connected to the segments 1 to 5. Two fingers A and B are movable and connected with the Centre Tapped Reactor through switches x and y and make contact with any one of the segment 1 to 5 under normal operation.

- As clear from the figure, both the fingers A and B are connected with segment 1 and switches x and y are closed, thereby whole winding is in circuit. As both the fingers are connected with the segment 1, half of the current will flow from the lower part of the Center tapped Reactor C and half of the current will flow from upper part. As the Reactor is wound in the same direction, therefore mmf because of upper half of the current and lower half of the current will cancel out in Reactor. Therefore Reactor will be behave like non inductive and will not offer any impedance. Therefore, the voltage drop in the center tapped Reactor will be very small.



- Now to change the tapping of the Transformer, following sequence of operation will take place one by one. The steps to be taken for changing the tap are shown in figure below.

Step1: Open Switch y

The entire current now will flow through switch x and lower half of the Reactor C. It therefore becomes highly inductive and there will be a large voltage drop across it. Reactor C shall be designed to withstand full load current momentarily.

Step2: Move Finger B

As the Switch y is open therefore finger B is not carrying any current and can be moved to segment 2 without any sparking.

Step3: Close Switch y

Now close Switch y. Now the Transformer winding between tap 1 and 2 get connected through the Reactor C. As the impedance offered by the Reactor is high for current flowing only in one direction, the circulating current through the Reactor, finger A and finger B will be very less. In this manner the Reactor prevents the tapped winding from getting short circuited.

Step4: Open Switch x

Now the entire current will flow from the upper half portion of the Reactor causing a high voltage drop across it.

Step5: Move Finger A

As the Switch x is open therefore finger A is not carrying any current and can be moved to segment 2 without any sparking.

In this way, the winding between segment 1 and 2 is completely cut out. If further change in voltage is required, above sequence of operation is repeated.