$$X(\omega) \longrightarrow \boxed{H(\omega)} \longrightarrow Y(\omega)$$

Freq. resp.
(fourier xform of impulse resp.)

mul. things in Freq. domain & inverse F.T is lot easier than time dom. convolution

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$Y(\omega) = \int_{-\infty}^{\infty} y(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} \left(\int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau \right) e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} x(\tau) \left[\int_{-\infty}^{\infty} h(t-\tau)\, e^{-j\omega t}\, dt \right] d\tau$$

$$= \int_{-\infty}^{\infty} x(\tau)\, \mathcal{F}(h(t-\tau))\, d\tau$$

$$= \int_{-\infty}^{\infty} x(\tau) .\, H(\omega)\, e^{-j\omega\tau}\, d\tau \qquad H(\omega) = \mathcal{F}(h(t))$$

$$= H(\omega) \int_{-\infty}^{\infty} x(\tau)\, e^{-j\omega\tau}\, d\tau$$

$$= H(\omega) .\, X(\omega)$$

$h \to$ imp. resp. of sys.

$$\delta(t) \to \boxed{\text{LTI sys}} \to h(t)$$ Impulse resp.

$$\mathcal{F}(h(t)) = H(\omega)$$ Frequency Response

Sometimes,

$$x(t) \to \boxed{h(t)} \to y(t)$$

for

$$x(t) \to \boxed{H(\omega)} \to y(t)$$

$$x(t) \to \boxed{h_1(t)} \to \boxed{h_2(t)} \to y(t)$$

$$x(t) \to \boxed{H_1(\omega)} \to \boxed{H_2(\omega)} \to y(t)$$

$$Y(\omega) = X(\omega)\, H_1(\omega)\, H_2(\omega)$$

simple multiplication no problem in interchanging

$$x(t) \to \boxed{H_2(\omega)} \to \boxed{H_1(\omega)} \to y(t)$$

i.e the same as saying if I change the impulse resp. I'll get the same result.

$$x(t) \to \boxed{h_2(t)} \to \boxed{h_1(t)} \to y(t)$$

Changing the order of LTI sys. doesn't change the o/p

Scanned by CamScanner

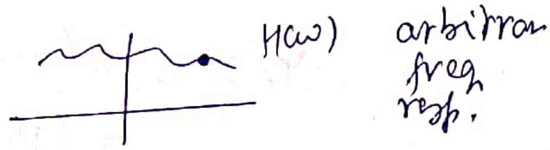
What does the frequency response mean?

$x(t) = e^{j\omega_0 t}$ complex exp. (Phase shift of a const.(1))

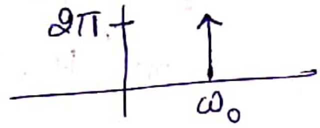
$X(\omega) = 2\pi\, \delta(\omega - \omega_0)$

How does my system respond to complex sinusoid.

$Y(\omega) = \underbrace{H(\omega)}_{\text{Freq. resp.}} \cdot X(\omega)$



$= H(\omega) \cdot 2\pi\, \delta(\omega - \omega_0)$



$= \underbrace{H(\omega_0)}_{\text{some const.}} \cdot \underbrace{2\pi\, \delta(\omega - \omega_0)}$

$y(t) = H(\omega_0) \cdot e^{j\omega_0 t}$

This says if I put a complex exponential in with a certain freq. ω_0 what comes out is the same exponential just multiplied by some complex no. → not changing anything about freq. in this signal

The only thing i.e happening is that complex exp. is multiplied by some scaler (that scaler has a magnitude & an ∡), not changing fundamental freq. content

i.e **Key idea:-**

The system cannot introduce new freq. into the O/P that were'nt present in I/P

Let's suppose that difficult to visualize.

$h(t)$ is real,

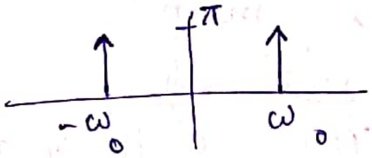
$x(t)$ is a cosine $= \cos\omega_0 t$

$H(\omega) = (H(-\omega))^*$

$Y(\omega) = X(\omega)\, H(\omega)$

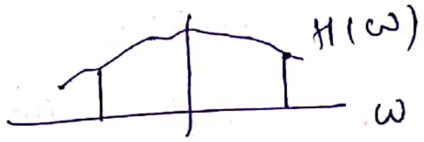
F.T of cosine

Scanned by CamScanner



$\because \mathcal{F}(e^{j\omega_0 t}) = 2\pi\,\delta(\omega-\omega_0)$

$\cos\omega_0 t = \dfrac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}$



$$Y(\omega) = \pi\left(\delta(\omega-\omega_0) + \delta(\omega+\omega_0)\right) H(\omega)$$
$$= \pi\left(H(\omega_0)\,\delta(\omega-\omega_0) + H(-\omega_0)\,\delta(\omega+\omega_0)\right)$$
$$= \pi\left(H(\omega_0)\,\delta(\omega-\omega_0) + (H(\omega_0))^*\,\delta(\omega+\omega_0)\right)$$
$$= \pi\left(|H(\omega_0)|\,e^{j\angle H(\omega_0)}\,\delta(\omega-\omega_0) + |H(\omega_0)|\,e^{-j\angle H(\omega_0)}\,\delta(\omega+\omega_0)\right)$$

$e^{j\omega_0 t} \leftrightarrow \delta(\omega-\omega_0)\,2\pi$

$$y(t) = |H(\omega_0)|\cos(\omega_0 t + \angle H(\omega_0))$$

If I put a cosine into this real value I.R sys. what comes out is the very same cosine multiplied by the mag. & freq. response & phase shifted by the ∡

same cosine : → amplified / attenuated

eg: $\cos 3t \longrightarrow \boxed{} \longrightarrow \cos 5t$

- This is not LTI sys.
- introduced a diff. freq. that wasn't present in the original signal

Freq. resp. tells what is happ. to every freq. in the signal

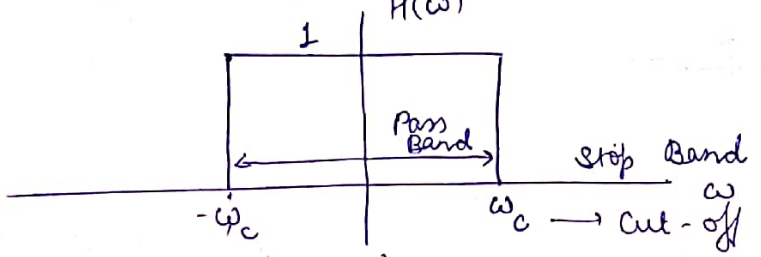
One of the most imp. freq. resp. or types of filters is a Low Pass Filter

We often / usually interpret $H(\omega)$ as what the system does to freq. of the I/P signal → often interchanged with "FILTER"

- We're taking freq. in the I/P & damping down / removing certain freq.
- The most imp. of these is the LOW

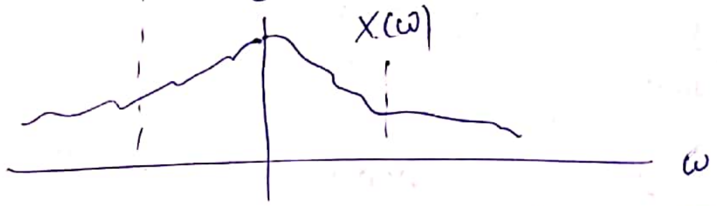
PASS filter.

Sketch freq. resp.



$\omega_c \rightarrow$ cut-off freq.

Result of applying this to some input signal which has some arbitrary F.T



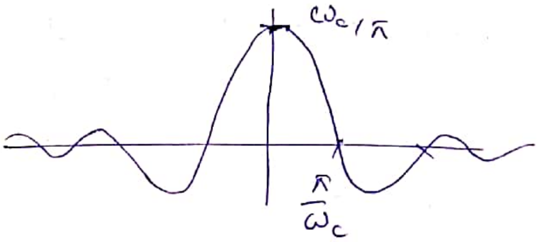
it cuts out all the freq. that're higher than ω_c

At the end

corr. impulse funcn. is .

$$h(t) = \frac{\omega_c}{\pi} \sin(\omega_c t)$$

related to cut off frequency



Imp. trade-off in filter-design. This is often exactly what we want in the freq. domain something that cuts out only what we want

How to implement in T.D ?

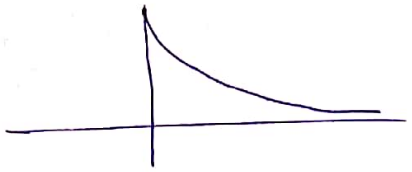
Sinc is not a good filter

① not causal (goes in both direction; looking into the future to filter the signal corr. to pieces of impulse resp. that are left to y-axis

② goes ∞-far (wiggling) ⟶ cause wiggling in O/P

Scanned by CamScanner

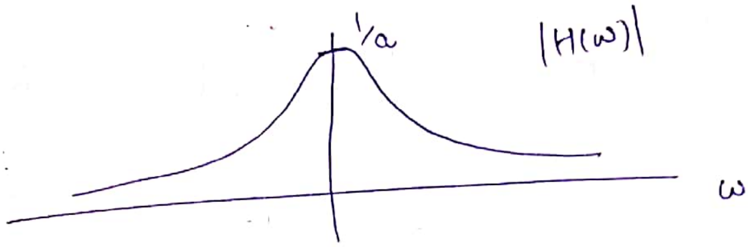
Consider $h(t) = e^{-at} u(t)$ $\quad a > 0$



$$H(\omega) = \frac{1}{a+j\omega}$$

Typically for filtering 1st thing we care about is the magnitude of the freq. response (abs. value!)

$$|H(\omega)| = \frac{1}{\sqrt{a^2+\omega^2}} = \cancel{\frac{\sqrt{a^2+\omega^2}}{a^2+\omega^2}}$$



At $\omega = 0$

fairly crude L.P.F

How to make the fall-off wider / thiner → depends on a

path of solving LTI systems

1. compute $X(\omega)$ from $x(t)$
2. multiply $X(\omega) \cdot H(\omega)$ to get $Y(\omega)$
3. Inverse F.T to get $y(t)$

eg. $x(t) = e^{-5t} u(t)$, $\quad h(t) = e^{-3t} u(t)$

$$X(\omega) = \frac{1}{5+j\omega} \qquad H(\omega) = \frac{1}{3+j\omega}$$

$$Y(\omega) = X(\omega) \cdot H(\omega) = \frac{1}{(5+j\omega)(3+j\omega)} = \frac{A}{5+j\omega} + \frac{B}{3+j\omega}$$

Partial fractions or

$$= \frac{1}{-2(5+j\omega)} + \frac{1}{2(3+j\omega)}$$

$$= -\frac{1}{2} e^{-5t} + \frac{1}{2} e^{-3t}$$

Scanned by CamScanner

(2) $x(t) = e^{-t}u(t)$ $\qquad H(\omega) = \dfrac{2+j\omega}{(1+j\omega)(3+j\omega)}$

$$X(\omega) = \frac{1}{1+j\omega}$$

$$Y(\omega) = \frac{2+j\omega}{(1+j\omega)(1+j\omega)(3+j\omega)}$$

$$= \frac{2+j\omega}{(1+j\omega)^2(3+j\omega)} = \frac{A}{(1+j\omega)^2} + \frac{B}{1+j\omega} + \frac{C}{3+j\omega}$$

$$\frac{(2+s)}{(1+s)(1+s)(3+s)}$$

$$\frac{(2+s)}{(1+s)^2(3+s)} = \frac{K_1}{(1+s)^2} + \frac{K_2}{(1+s)} + \frac{K_3}{(3+s)}$$

$$K_3 = \frac{-1}{4}$$

$$\left(K_1 = \frac{2-1}{3-1} = \frac{1}{2}\right) \rightarrow \frac{(2+s)}{(3+s)} = K_1 + K_2(1+s) + \frac{K_3(1+s)^2}{(3+s)}$$

$$\frac{(3+s)-(2+s)}{(3+s)^2} = K_2 + K_3\left[\frac{(3+s)2(1+s) - (1+s)^2}{(3+s)^2}\right]$$

$$s=-1 \Big/ \frac{1}{(3+s)^2} = K_2 + K_3(1+s)\left[\frac{6+2s-1-s}{(3+s)^2}\right]$$

$$\cancel{1 = K_3(-4) = K_3 = -1/4} \qquad K_2 = \frac{1}{4}$$

$$= \frac{1}{2}\cdot\frac{1}{(1+s)^2} + \frac{1}{4}\frac{1}{(1+s)} - \frac{1}{4}\cdot\frac{1}{3+s}$$

$$= \frac{1}{2}\cdot te^{-t}u(t) + \frac{1}{4}e^{-t}u(t) - \frac{1}{4}\cdot e^{-3t}u(t)$$

$$y(t) = \frac{d}{dt}[x(t)] \qquad Y(\omega) = j\omega X(\omega)$$

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 3y = \frac{dx}{dt} + 2x$$

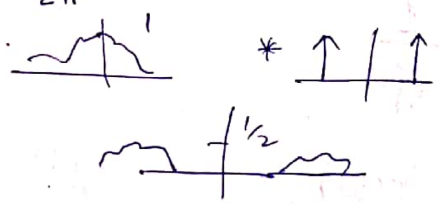
$$(j\omega)^2Y(\omega) + 4j\omega Y(\omega) + 3Y(\omega) = j\omega X(\omega) + 2X(\omega)$$

$$H(\omega) = Y(\omega)/X(\omega) = \frac{(2+j\omega)}{(1+j\omega)(3+j\omega)}$$

Scanned by CamScanner

$z(t) = x(t)\,y(t)$ $\quad Z(\omega) = \frac{1}{2\pi} X(\omega) * Y(\omega)$ Amp. Mod.

$x(t)\cos(\omega_0 t)$

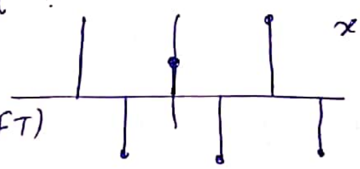


We talked about

Continous Time F.T (CTFT)

$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

what if I've a discrete signal ? $x[n]$



we can define the D.T Fou. Trans (DTFT) (intuition:- turn integral to sum)

as

$$\boxed{X(\omega) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n}} \quad \text{(DTFT)}$$

no more t

still a continous funcn. of ω

In C.T I can've ∞ no freq.

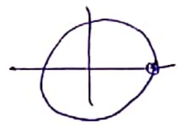
but in D.T only a fixed set of freq.

CTFTs have a freq. range $\omega \in (-\infty, \infty)$

DTFTs have a freq. range of width $\underline{2\pi}$ why is it?

$$X(\omega + 2\pi) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j(\omega + 2\pi)n}$$

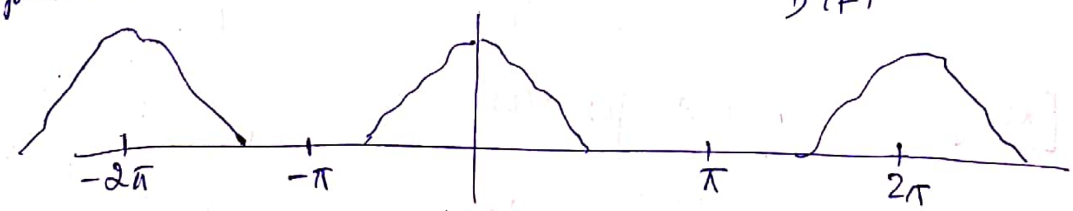
$$= \underbrace{\boxed{\sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n}}}_{} \; e^{-j2\pi n} \leftarrow \text{Integer}$$



$$= X(\omega)$$

- DTFT can be evaluated at any ω but its 2π-periodic

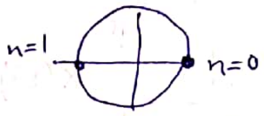
DTFT

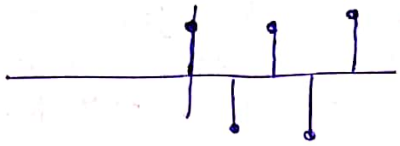


$\therefore$ In practice we usually restrict our attention in this range $(-\pi, \pi)$ & only draw DTFT in this range with the understanding DTFT actually has copies at other ω.

why I chose $-\pi$ & π as boundaries (highest discrete time freq. I can create)

$$e^{j\pi n} = \cos \pi n + j \sin \pi n$$
$$= \pm 1 = (-1)^n$$





There're some symmetries that when everything is real, I often just draw from 0 to π because I can predict what left side would be.

CT IFT :-
$$\frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega)\, e^{j\omega t}\, d\omega$$

DT IFT (by analogy)
$$x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\omega)\, e^{j\omega n}\, d\omega$$

Proof:- consider $\int_{-\pi}^{\pi} \underline{X(\omega)}\, e^{j\omega n}\, d\omega$

$$= \int_{-\pi}^{\pi} \sum_{m=-\infty}^{\infty} x[m]\, e^{-j\omega m} \cdot e^{j\omega n}\, d\omega$$

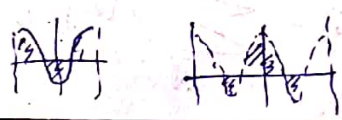
A sufficient condition to switch the order...
$$\sum_{m=-\infty}^{\infty} |x[n]| < \infty$$ (sum converges!)

Above
$$= \sum_{m=-\infty}^{\infty} x[m] \int_{-\pi}^{\pi} e^{j\omega(n-m)}\, d\omega$$

$$\int_{-\pi}^{\pi} \cos \omega(n-m) + j \sin \omega(n-m)\, d\omega$$

$n-m = 0$ $\int_{-\pi}^{\pi} 1\, d\omega = 2\pi$

$n-m \neq 0$ $\int_{-\pi}^{\pi}$



$\to 2\pi\, x[n]$

Only at $m=n$ the integral is 2π else 0

Scanned by CamScanner

DTFT:

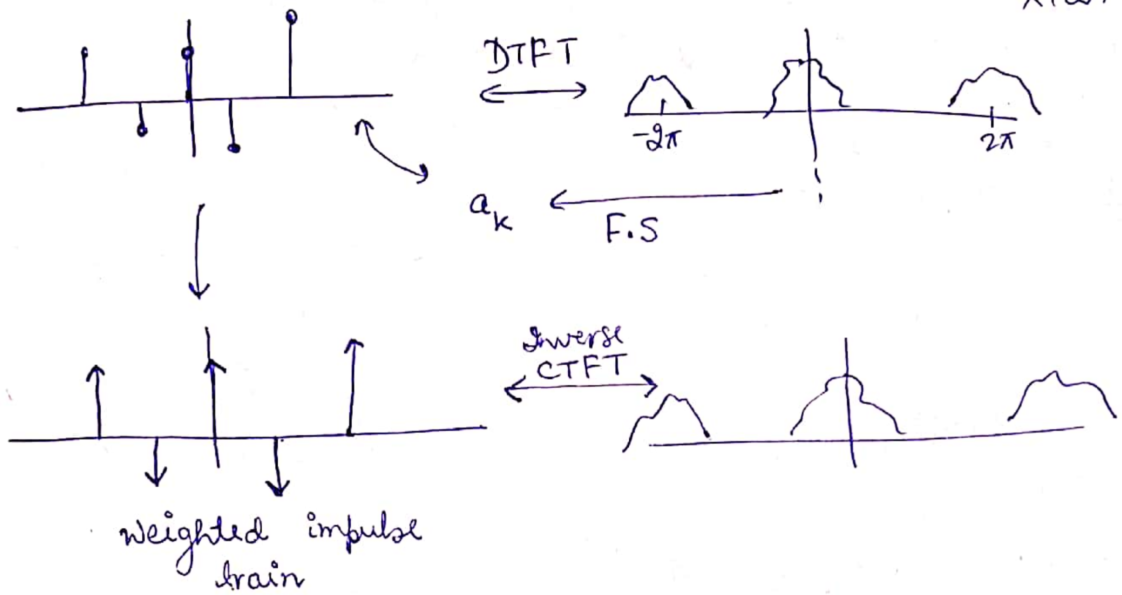
$$X(\omega) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n}$$

$$\underbrace{x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\omega)\, e^{j\omega n}\, d\omega}$$

This looks quite familiar
↳ Fourier Series expansion for a signal with period 2π

$$a_k = \frac{1}{T} \int_{-T/2}^{T/2} x(t)\, e^{-j\frac{2\pi}{T}kt}\, dt$$

continous signal
$X(\omega)$ is periodic



All this are tied together

	Continous	Discrete
Periodic	Fourier Series	? (DFT) ↳ After Z-transform
non-periodic	Fourier Transform	DTFT

If $\sum_{n=-\infty}^{\infty} |x[n]| < \infty$

$$\lim_{N\to\infty} \sup_{\omega} |X(\omega) - X_N(\omega)| = 0$$

As N gets big, $X_N(\omega) = \sum_{n=-N}^{n=N} x[n] \cdot e^{-j\omega n}$

Scanned by CamScanner

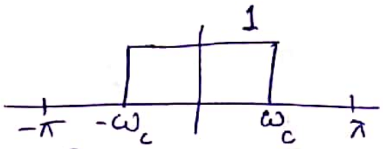
converges at each point to $X(\omega)$

If $\sum |x[n]|^2 < \infty$ (looser condn.)

$$\lim_{n\to\infty} \int_{-\pi}^{\pi} |X(\omega) - X_N(\omega)|^2 \, d\omega = 0$$ (mean sq. converg.)

eg. (low pass filter)

$X(\omega) =$



$$x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\omega) \cdot e^{j\omega n} \, d\omega$$

$$= \frac{1}{2\pi} \int_{-\omega_c}^{\omega_c} e^{j\omega n} \, d\omega = \frac{1}{2\pi j n} \left. e^{j\omega n} \right]_{-\omega_c}^{\omega_c}$$

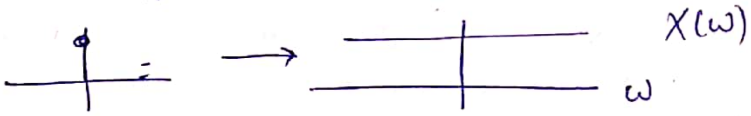
$$= \frac{1}{2\pi j n} \left(e^{j\omega_c n} - e^{-j\omega_c n} \right) = \frac{1}{\pi n} \sin \omega_c n$$

$$= \frac{\omega_c}{\pi} \operatorname{sinc}(\omega_c n)$$

satisfies $\sum | \;|^2 < \infty$
not $\sum | \;| < \infty$

(3) $x[n] = \delta[n]$

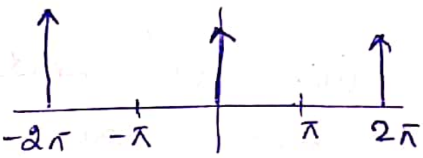
$$X(\omega) = \sum_{n=-\infty}^{\infty} \delta[n] e^{-j\omega n} = 1$$

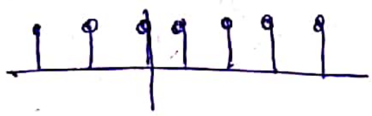


what if $X(\omega) = \delta(\omega)$ (little careful what this δ-funcn means in DTFT)

$$x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\omega) \, e^{j\omega n}$$

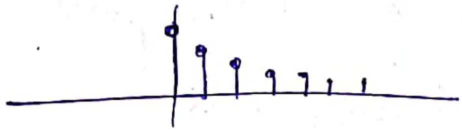
$$= \frac{1}{2\pi}$$





Scanned by CamScanner

$x[n] = \alpha^n u[n]$ $\qquad |\alpha| < 1$



$$X(\omega) = \sum_{n=-\infty}^{\infty} \alpha^n u[n]\, e^{-j\omega n}$$

$$= \sum_{n=0}^{\infty} \alpha^n e^{-j\omega n}$$

$$= \sum_{n=0}^{\infty} \left(\alpha e^{-j\omega}\right)^n = \frac{1}{1-\alpha e^{-j\omega}}$$

with real signal I and if with complex (signal) F.T

magnitude Response $|X(\omega)|$

Phase Response $\measuredangle X(\omega)$

mag. resp. is what defines the character of the filter whether its a low pass, etc.

$$|X(\omega)| = \left|\frac{1}{1-\alpha e^{-j\omega}}\right| = \frac{1}{|(1-\alpha\cos\omega) + j\alpha\sin\omega)} \qquad \alpha < 1$$

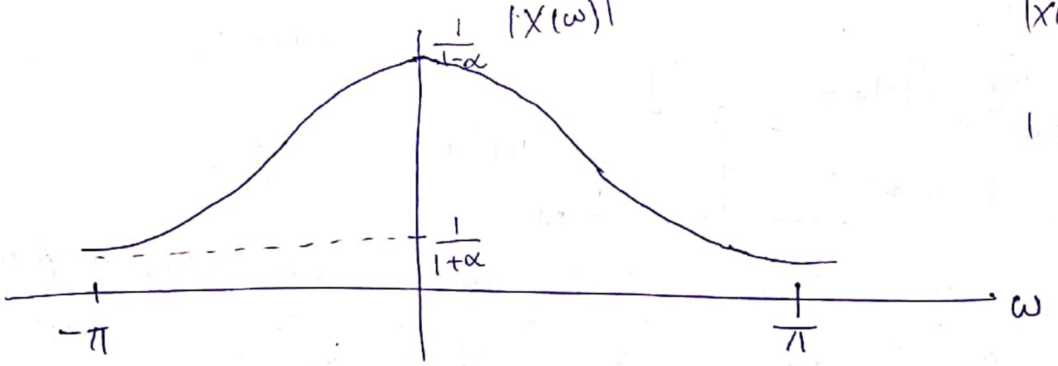
$$= \frac{1}{\sqrt{(1-\alpha\cos\omega)^2 + (\alpha\sin\omega)^2}}$$

$\omega = 0$

$|X(\omega)| = \frac{1}{(1-\alpha)} > 1$

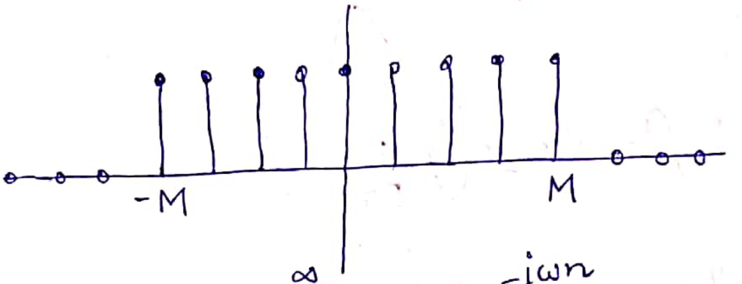
$\omega = \pi$

$|X(\omega)| = \frac{1}{(1+\alpha)}$



(looks like a crude low pass filter stuff in the middle is getting passed through & stuff at edges is getting attenuated)

Pulse in Time Domain



$$X(\omega) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$$

$$= \boxed{\sum_{n=-M}^{M} e^{-j\omega n}}$$

(2M+1) non-zero elements

$$= e^{j\omega M} \sum_{n=0}^{2M} e^{-j\omega n}$$

when $n=0$
$e^{j\omega M}$
when $n=2M$
$e^{-j\omega M}$
same terms in sum

G.P

$$= e^{j\omega M}\left(\frac{1-e^{-j\omega(2M+1)}}{1-e^{-j\omega}}\right)$$

when $\omega = 0$,
Adding a bunch of 1s
$X(\omega) = 2M+1$

cannot solve for $\omega = 0$ here!

when $\omega \neq 0$

$$= \underbrace{e^{j\omega M}\, \frac{e^{-j\omega\frac{(2M+1)}{2}}}{e^{-j\omega/2}}}_{\text{cancels to } 1} \cdot \frac{\left(e^{+j\omega\frac{(2M+1)}{2}} - e^{-j\omega\frac{(2M+1)}{2}}\right)}{\left(e^{j\omega/2} - e^{-j\omega/2}\right)}$$

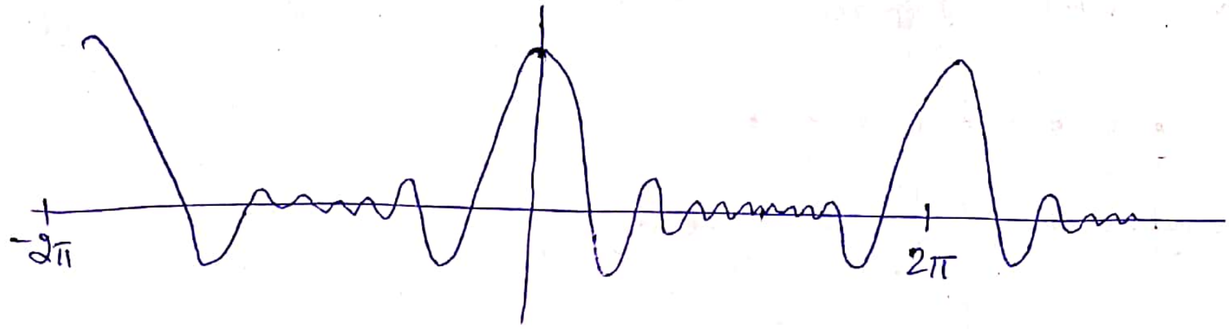
take half of what u see in $(1-e^{-x})$

$$= \frac{\sin \omega \frac{(2M+1)}{2}}{\sin \omega/2}$$

This is not a sinc function!

Rem. whatever I do in D.T FT world has to be periodic & sinc funcn. is not periodic

Scanned by CamScanner



- Intuition is still that a pulse converts to a sinc but the kind of sinc that I get in freq. domain has to necessarily be modified to make it periodic

Why we care about DTFTs?

usually the DTFT that we're going to take is for the Impulse response

- We want to use the DTFT to study LTI systems.
- Our convolution prop. still holds

$$y[n] = x[n] * h[n]$$

DTFT,

$$Y(\omega) = \sum_{n=-\infty}^{\infty} \underbrace{y[n]} e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]\, e^{-j\omega n}$$

$$= \sum_{k=-\infty}^{\infty} x[k] \sum_{n=-\infty}^{\infty} h[n-k]\, e^{-j\omega n}$$

∴ Summing up all the values of h

Rename
$m = n - k$
$n = m + k$

$$= \sum_{k=-\infty}^{\infty} x[k] \sum_{m=-\infty}^{\infty} h[m]\, e^{-j\omega(m+k)}$$

$$= \left(\sum_{k=-\infty}^{\infty} x[k]\, e^{-j\omega k}\right)\left(\sum_{m=-\infty}^{\infty} h[m]\, e^{-j\omega m}\right)$$

$$Y(\omega) = X(\omega)\; H(\omega)$$

Scanned by CamScanner

Freq. Response :-

$$x[n] = Ae^{j\omega_0 n}$$

$$y[n] = x[n] * h[n]$$ (Arbitrary Impulse Resp.)

$$= \sum_{k=-\infty}^{\infty} h[k]\ x[n-k]$$

$$= \sum_{k=-\infty}^{\infty} h[k].\ Ae^{j\omega_0(n-k)}$$

$$= Ae^{j\omega_0 n} \sum_{k=-\infty}^{\infty} h[k]\ e^{-j\omega_0 k}$$

$$= H(\omega_0) \cdot Ae^{j\omega_0 n}$$

so, if this complex exp. comes in what comes out is the same complex exp. mul. by this complex no. which is the freq. response evaluated at that freq.

$$Ae^{j\omega_0 n} \longrightarrow \underbrace{A\,|H(\omega_0)|}_{\text{Amplitude scaling}}\ e^{j(\overbrace{\measuredangle H(\omega_0)}^{\text{Phase shifting}} + \omega_0 n)}$$

In a same way (did this in CTFT) ; if $h[n]$ is real

$$\cos(\omega_0 n + \phi) \longrightarrow \underline{|H(\omega_0)|}\ \cos(\omega_0 n + \phi + \underline{\angle H(\omega_0)})$$

I can't introduce new freq. into the sys. with an LTI sys.

amplify, attenuate or move them around!

eg. $h[n] = \left(\frac{1}{3}\right)^n u[n]$

$$x[n] = 2e^{j\pi/3\, n}$$

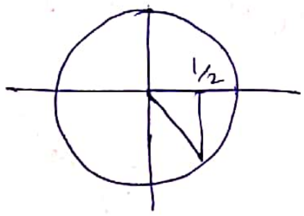
what is the O/P ?

1st thing $\rightarrow$ $x[n]$ is a single sinusoid at freq. $\pi/3$

Scanned by CamScanner

$\therefore H\left(\frac{\pi}{3}\right)$ now, $H(\omega) = \dfrac{1}{1 - \frac{1}{3}e^{-j\omega}}$

$$H\left(\frac{\pi}{3}\right) = \frac{1}{1 - \frac{1}{3}e^{-j\pi/3}}$$



$$= \frac{1}{1 - \frac{1}{3}\left(\frac{1}{2} - \frac{\sqrt{3}}{2}j\right)}$$

$$= \frac{1}{\frac{5}{6} + \frac{\sqrt{3}}{6}j} = \frac{1}{\frac{5}{6} + \frac{\sqrt{3}}{6}j} = \left|\frac{3}{\sqrt{7}}\right| \angle -\tan^{-1}\sqrt{3}/5$$

$$= 2\left|H\left(\frac{\pi}{3}\right)\right| e^{j\left(\frac{\pi}{3}n + \angle H\left(\frac{\pi}{3}\right)\right)}$$

no need to take $X(\omega)$ & so on

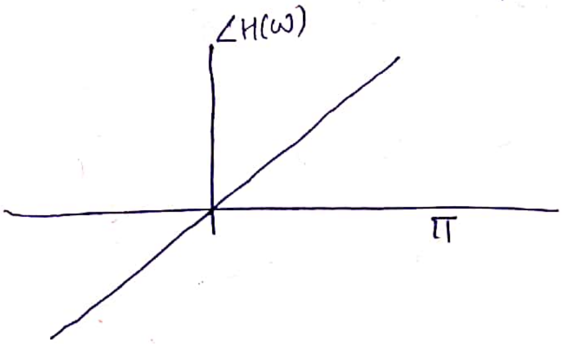
- when I've a pure combination of cosines & sines then I can only get attenuated / amplified versions of those sines & cosines.

<u>magnitude response</u>

- when $h[n]$ is real, the magnitude resp. is even.

$\omega = 0 \to DC$
$\omega = \pi \to$ highest freq.

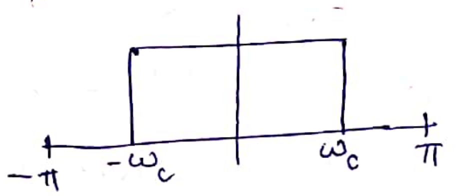
- when $h[n]$ is real, the phase resp. is odd ($\angle H(\omega)$)



straight line is a desirable thing!

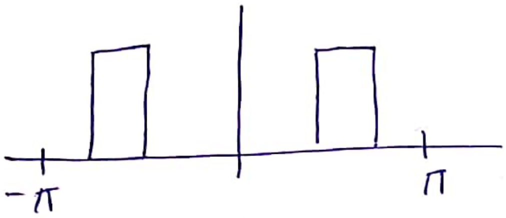
Scanned by CamScanner

"Ideal" Filters

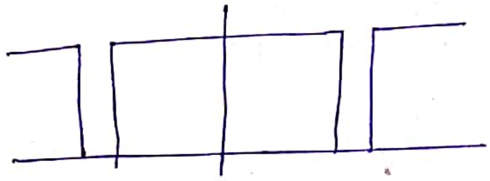


upper bound on how high a freq. could get
Low Pass filter, cutoff ω_c

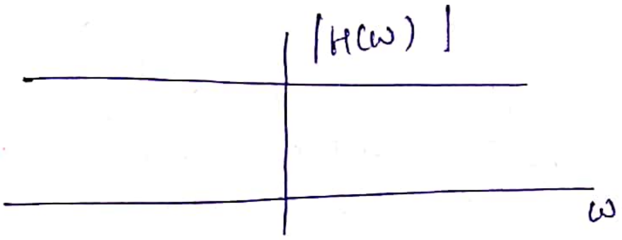
High Pass



Band Pass



Band Stop



All Pass
↳ freq. content isn't getting changed but the phases of individual sines & cosines are getting changed.

Phase Resp.

why is $\angle H(\omega) = -c\omega$ desirable?

Say, $|H(\omega)|$ is piece-wise constant (above) in the pass region ($|H(\omega)| = 1$)

$$Y(\omega) = X(\omega)\, H(\omega)$$
$$= X(\omega)\, |H(\omega)|\, e^{j\angle H(\omega)}$$
$$= X(\omega)\, e^{-jc\omega}$$

for a linear phase filter
↳ $\angle H(\omega) = -c\omega$

This is basically a phase shift in freq. domain

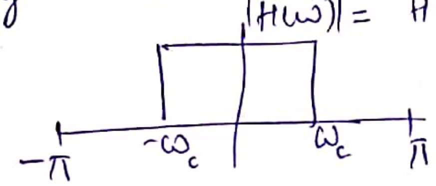
$y[n] = x[n-c]$ delay in T.D)

This is good because when I apply this linear phase filter to a signal all I'm really doing is delaying the O/P (entirity)
(not - distortion!)

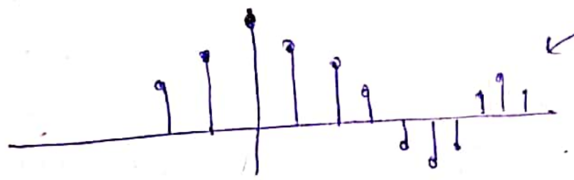
Pure delay of the I/P → not considered distortion

→ Why can't we've just zero delay?

$|H(\omega)| = H(\omega)$ for 0 delay



$$h[n] = \frac{\sin \omega_c \pi n}{\pi n}$$



not absolutely summable!
$\sum |h| = \infty$ not stable

∞ - long

non - causal

imp. resp. not desirable

However, I can make a nice impulse resp. that comes close to pulse above & has linear phase (only delaying O/P)

So to filter a signal → tolerate a little bit of delay