Module 2

DC Circuit

Lesson
3
Introduction of Electric Circuit

Objectives

- Familiarity with and understanding of the basic elements encountered in electric networks.
- To learn the fundamental differences between linear and nonlinear circuits.
- To understand the Kirchhoff's voltage and current laws and their applications to circuits.
- Meaning of circuit ground and the voltages referenced to ground.
- Understanding the basic principles of voltage dividers and current dividers.
- Potentiometer and loading effects.
- To understand the fundamental differences between ideal and practical voltage and current sources and their mathematical models to represent these source models in electric circuits.
- Distinguish between independent and dependent sources those encountered in electric circuits.
- Meaning of delivering and absorbing power by the source.

L.3.1 Introduction

The interconnection of various electric elements in a prescribed manner comprises as an electric circuit in order to perform a desired function. The electric elements include controlled and uncontrolled source of energy, resistors, capacitors, inductors, etc. Analysis of electric circuits refers to computations required to determine the unknown quantities such as voltage, current and power associated with one or more elements in the circuit. To contribute to the solution of engineering problems one must acquire the basic knowledge of electric circuit analysis and laws. Many other systems, like mechanical, hydraulic, thermal, magnetic and power system are easy to analyze and model by a circuit. To learn how to analyze the models of these systems, first one needs to learn the techniques of circuit analysis. We shall discuss briefly some of the basic circuit elements and the laws that will help us to develop the background of subject.

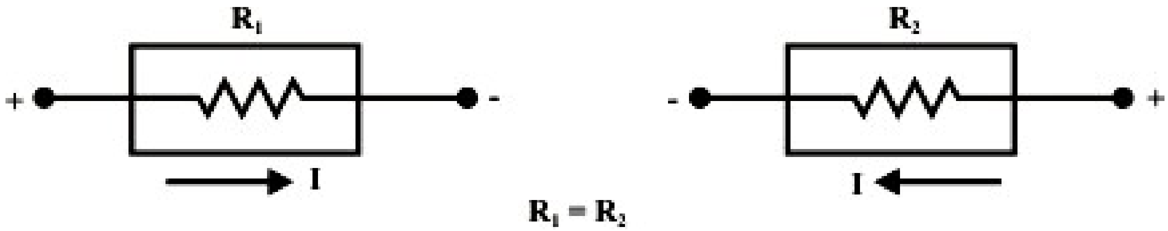
L-3.1.1 Basic Elements & Introductory Concepts

Electrical Network: A combination of various electric elements (Resistor, Inductor, Capacitor, Voltage source, Current source) connected in any manner what so ever is called an electrical network. We may classify circuit elements in two categories, passive and active elements.

Passive Element: The element which receives energy (or absorbs energy) and then either converts it into heat (R) or stored it in an electric (C) or magnetic (L) field is called passive element.

Active Element: The elements that supply energy to the circuit is called active element. Examples of active elements include voltage and current sources, generators, and electronic devices that require power supplies. A transistor is an active circuit element, meaning that it can amplify power of a signal. On the other hand, transformer is not an active element because it does not amplify the power level and power remains same both in primary and secondary sides. Transformer is an example of passive element.

Bilateral Element: Conduction of current in both directions in an element (example: Resistance; Inductance; Capacitance) with same magnitude is termed as bilateral element.



Unilateral Element: Conduction of current in one direction is termed as unilateral (example: Diode, Transistor) element.

Meaning of Response: An application of input signal to the system will produce an output signal, the behavior of output signal with time is known as the response of the system.

L-3.2 Linear and Nonlinear Circuits

Linear Circuit: Roughly speaking, a linear circuit is one whose parameters do not change with voltage or current. More specifically, a linear system is one that satisfies (i) homogeneity property [response of $\alpha\, u(t)$ equals α times the response of $u(t)$, $S(\alpha\, u(t))$ = $\alpha\, S(u(t))$ for all α; and $u(t)$] (ii) additive property [that is the response of system due to an input $(\alpha_1 u_1(t) + \alpha_2\, u_2(t))$ equals the sum of the response of input $\alpha_1\, u_1(t)$ and the response of input $\alpha_2\, u_2(t)$, $S(\alpha_1\, u_1(t) + \alpha_2\, u_2(t))$ = $\alpha_1\, S(u_1(t)) + \alpha_2\, S(u_2(t))$.] When an input $u_1(t)$ *or* $u_2(t)$ is applied to a system "S", the corresponding output response of the system is observed as $S(u_1(t)) = y_1(t)$ *or* $S(u_2(t)) = y_2(t)$ respectively. Fig. 3.1 explains the meaning of homogeneity and additive properties of a system.

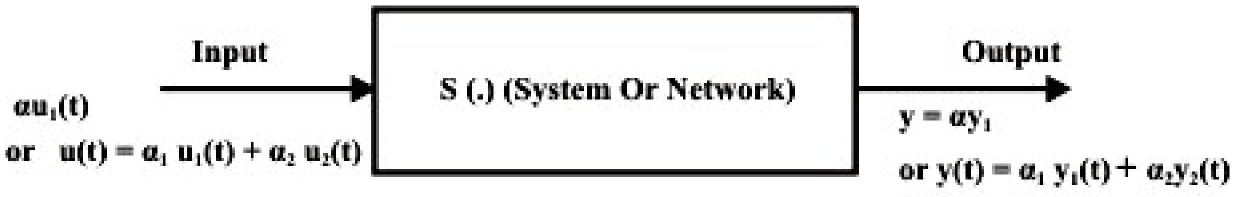


Fig. 3.1: Input output behavior of a system

Non-Linear Circuit: Roughly speaking, a non-linear system is that whose parameters change with voltage or current. More specifically, non-linear circuit does not obey the homogeneity and additive properties. Volt-ampere characteristics of linear and non-linear elements are shown in figs. 3.2 - 3.3. In fact, a circuit is linear if and only if its input and output can be related by a straight line passing through the origin as shown in fig.3.2. Otherwise, it is a nonlinear system.

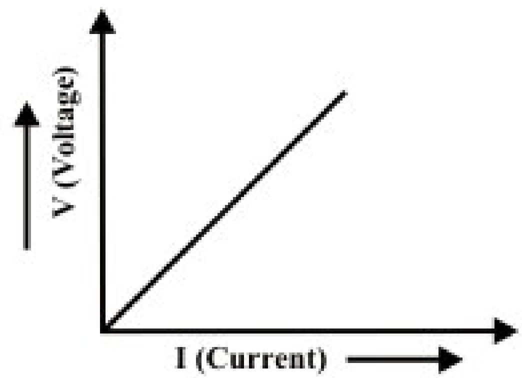


Fig. 3.2: V-I characteristics of linear element.

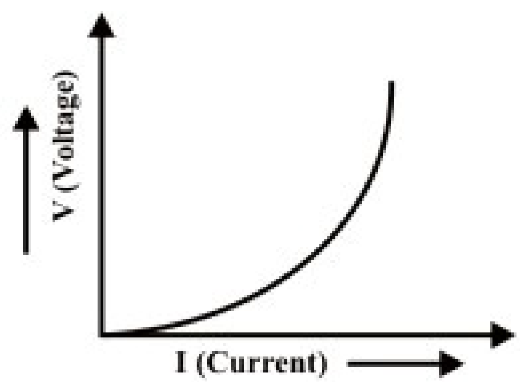


Fig. 3.3: V-I characteristics of non-linear element.

Potential Energy Difference: The voltage or potential energy difference between two points in an electric circuit is the amount of energy required to move a unit charge between the two points.

L-3.3 Kirchhoff's Laws

Kirchhoff's laws are basic analytical tools in order to obtain the solutions of currents and voltages for any electric circuit; whether it is supplied from a direct-current system or an alternating current system. But with complex circuits the equations connecting the currents and voltages may become so numerous that much tedious algebraic work is involve in their solutions.

Elements that generally encounter in an electric circuit can be interconnected in various possible ways. Before discussing the basic analytical tools that determine the currents and voltages at different parts of the circuit, some basic definition of the following terms are considered.

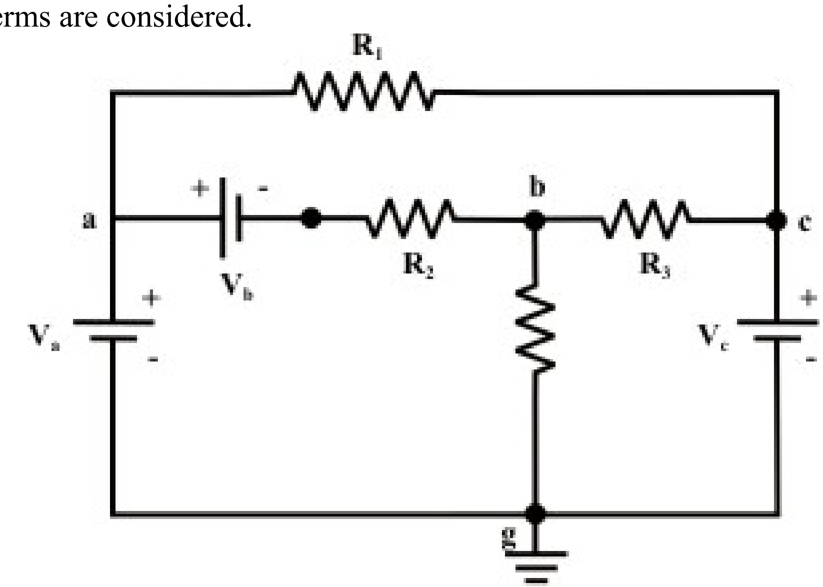
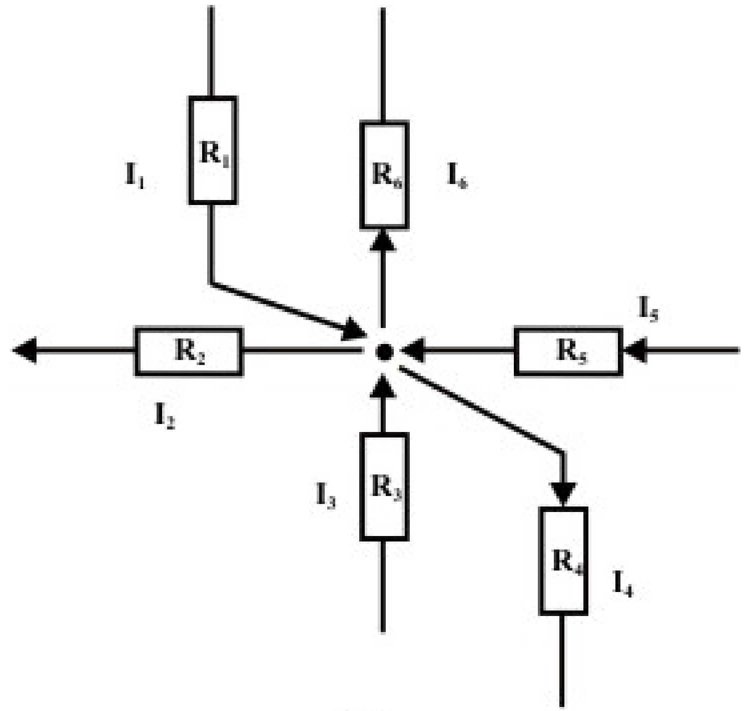


Fig. 3.4: A simple resistive network

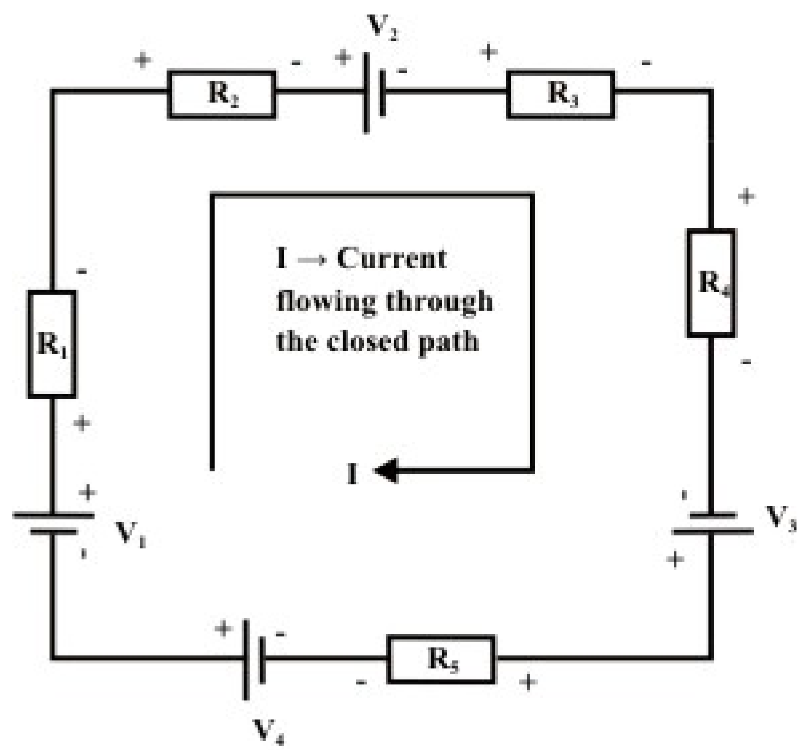
- Node- A node in an electric circuit is a point where two or more components are connected together. This point is usually marked with dark circle or dot. The circuit in fig. 3.4 has nodes a, b, c, and g. Generally, a point, or a node in an circuit specifies a certain voltage level with respect to a reference point or node.
- Branch- A branch is a conducting path between two nodes in a circuit containing the electric elements. These elements could be sources, resistances, or other elements. Fig.3.4 shows that the circuit has six branches: three resistive branches (a-c, b-c, and b-g) and three branches containing voltage and current sources (a-, a-, and c-g).
- Loop- A loop is any closed path in an electric circuit i.e., a closed path or loop in a circuit is a contiguous sequence of branches which starting and end points for tracing the path are, in effect, the same node and touches no other node more than once. Fig. 3.4 shows three loops or closed paths namely, a-b-g-a; b-c-g-b; and a-c-b-a. Further, it may be noted that the outside closed paths a-c-g-a and a-b-c-g-a are also form two loops.
- Mesh- a mesh is a special case of loop that does not have any other loops within it or in its interior. Fig. 3.4 indicates that the first three loops (a-b-g-a; b-c-g-b; and a-c-b-a) just identified are also 'meshes' but other two loops (a-c-g-a and a-b-c-g-

a) are not.

With the introduction of the Kirchhoff's laws, a various types of electric circuits can be analyzed.



(a)



(b)

Fig. 3.5: Illustrates the Kirchhoff's laws

Kirchhoff's Current Law (KCL): KCL states that at any node (junction) in a circuit the algebraic sum of currents entering and leaving a node at any instant of time must be equal to zero. Here currents entering(+ve sign) and currents leaving (-ve sign) the node must be assigned opposite algebraic signs (see fig. 3.5 (a), $I_1 - I_2 + I_3 - I_4 + I_5 - I_6 = 0$).

Kirchhoff's Voltage Law (KVL): It states that in a closed circuit, the algebraic sum of all source voltages must be equal to the algebraic sum of all the voltage drops. Voltage drop is encountered when current flows in an element (resistance or load) from the higher-potential terminal toward the lower potential terminal. Voltage rise is encountered when current flows in an element (voltage source) from lower potential terminal (or negative terminal of voltage source) toward the higher potential terminal (or positive terminal of voltage source). Kirchhoff's voltage law is explained with the help of fig. 3.5(b).

KVL equation for the circuit shown in fig. 3.5(b) is expressed as (we walk in clockwise direction starting from the voltage source V_1 and return to the same point)

$$V_1 - IR_1 - IR_2 - V_2 - IR_3 - IR_4 + V_3 - IR_5 - V_4 = 0$$

$$V_1 - V_2 + V_3 - V_4 = IR_1 + IR_2 + IR_3 + IR_4 + IR_5$$

Example: L-3.1 For the circuit shown in fig. 3.6, calculate the potential of points $A, B, C,$ *and* E with respect to point D. Find also the value of voltage source V_1.

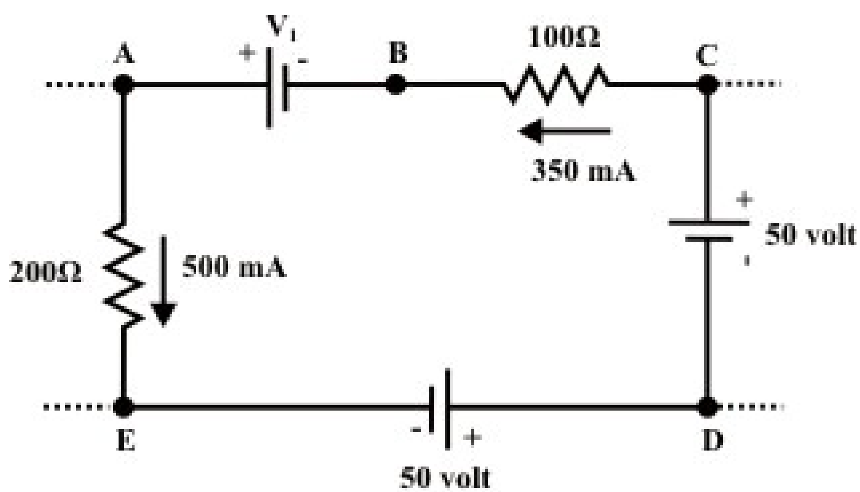


Fig. 3.6: A part of dc resistive circuit is presented

Solution Let us assume we move in clockwise direction around the close path D-E-A-B-C-D and stated the following points.

- 50 *volt* source is connected between the terminals D & E and this indicates that the point E is lower potential than D. So, V_{ED} (i.e., it means potential of E with respect to D) is -50 *volt* and similarly $V_{CD} = 50\,volt$ or $V_{DC} = -50\,volt$.
- $500\,mA$ current is flowing through $200\,\Omega$ resistor from A *to* E and this implies that point A is higher potential than E. If we move from lower potential (E) to

higher potential (A), this shows there is a rise in potential. Naturally, $V_{AE} = 500 \times 10^{-3} \times 200 = 100\,volt$ and $V_{AD} = -50 + 100 = 50\,volt$.
Similarly, $V_{CB} = 350 \times 10^{-3} \times 100 = 35\,volt$

- V_1 voltage source is connected between A & B and this indicates that the terminal B is lower potential than A i.e., $V_{AB} = V_1\,volt$ or $V_{BA} = -V_1\,volt.$. One can write the voltage of point B with respect to D is $V_{BD} = 50 - V_1\,volt.$
- One can write KVL law around the closed-loop D-E-A-B-C-D as $V_{ED} + V_{AE} + V_{BA} + V_{CB} + V_{DC} = 0$

$-50 + 100 - V_1 + 35 - 50 = 0 \Rightarrow V_1 = 35\,volt.$

Now we have $V_{ED} = -50\,volt$, $V_{AD} = -50 + 100 = 50\,volt, V_{BD} = 50 - 35 = 15\,volt,$
$V_{CD} = 15 + 35 = 50\,volt.$

L-3.4 Meaning of Circuit Ground and the Voltages referenced to Ground

In electric or electronic circuits, usually maintain a reference voltage that is named "ground voltage" to which all voltages are referred. This reference voltage is thus at ground potential or zero potential and each other terminal voltage is measured with respect to ground potential, some terminals in the circuit will have voltages above it (positive) and some terminals in the circuit will have voltages below it (negative) or in other words, some potential above or below ground potential or zero potential.
Consider the circuit as shown in fig. 3.7 and the common point of connection of elements V_1 & V_3 is selected as ground (or reference) node. When the voltages at different nodes are referred to this ground (or reference) point, we denote them with double subscripted voltages V_{ED}, V_{AD}, V_{BD}, *and* V_{CD}. Since the point D is selected as ground potential or zero potential, we can write V_{ED} as V_E, V_{AD} as V_A and so on.

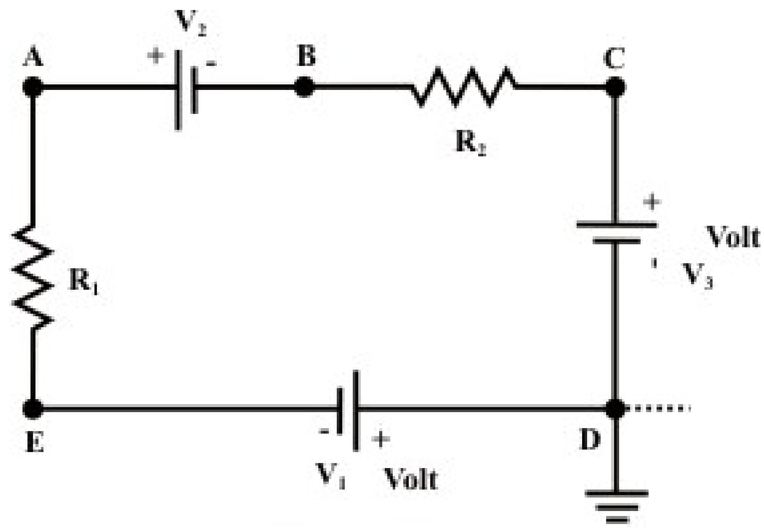


Fig. 3.7: A Simple dc resistive circuit

In many cases, such as in electronic circuits, the chassis is shorted to the earth itself for safety reasons.

L-3.5 Understanding the Basic Principles of Voltage Dividers and Current dividers

L-3.5.1 Voltage Divider

Very often, it is useful to think of a series circuit as a voltage divider. The basic idea behind the voltage divider is to assign a portion of the total voltage to each resistor. In Figure 3.8 (a), suppose that the source voltage is E. By the circuit configuration shown one can divide off any voltage desired (V_{out}), less than the supply voltage E, by adjusting R_1, R_2 *and* R_3.

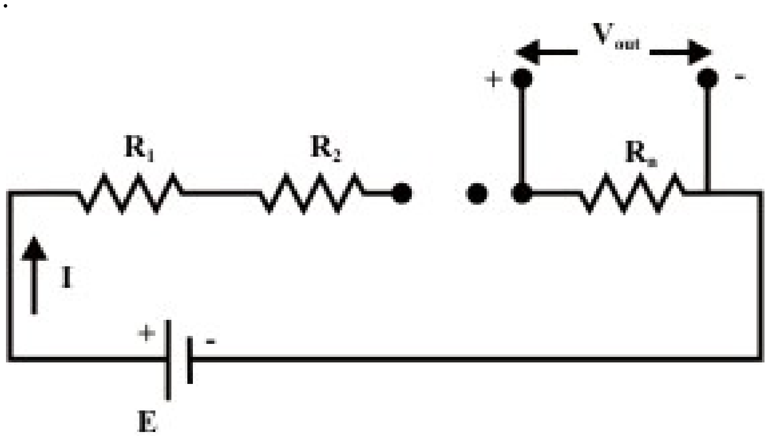


Fig. 3.8(a): Voltage Divider

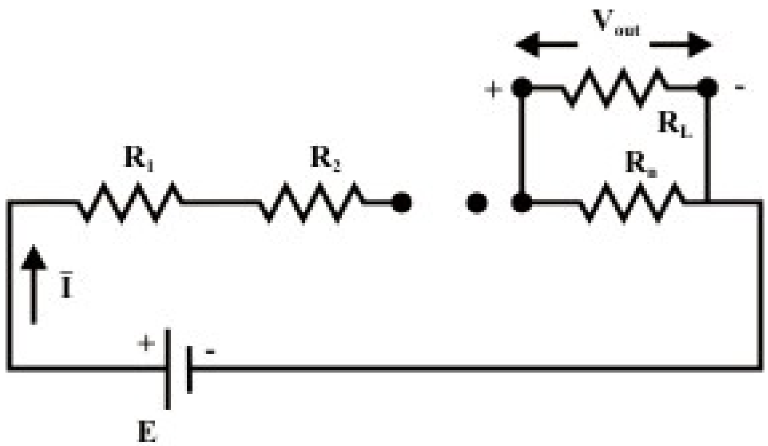


Fig. 3.8(b): Voltage Divider With Load

From figure 3.8(a) the output of the voltage divider V_{out} is computed by the relation

$$V_{out} = I\, R_n = \frac{E}{R_1 + R_2 + ... + R_n} R_n \tag{3.1}$$

Equation (3.1) indicates that the voltage across any resistor R_i ($R_i \; i = 1, 2, ...n$) in a series circuit is equal to the applied voltage (E) across the circuit multiplied by a factor $\frac{R_i}{\sum_{j=1}^{n} R_j}$. It should be noted that this expression is only valid if the same current I flows through all the resistors. If a load resistor R_L is connected to the voltage divider (see figure 3.8(b)), one can easily modify the expression (3.1) by simply combining R_L & R_n in parallel to find a new $\overline{R}_n$ and replacing R_n by $\overline{R}_n$ in equation (3.1).

<u>Example</u>: L-3.2 For the circuit shown in Figure 3.9,

(i) Calculate V_{out}, ignoring the internal resistance R_s of the source E. Use voltage division.

(ii) Recalculate V_{out} taking into account the internal resistance R_s of the source. What percent error was introduced by ignoring R_s in part (i)?

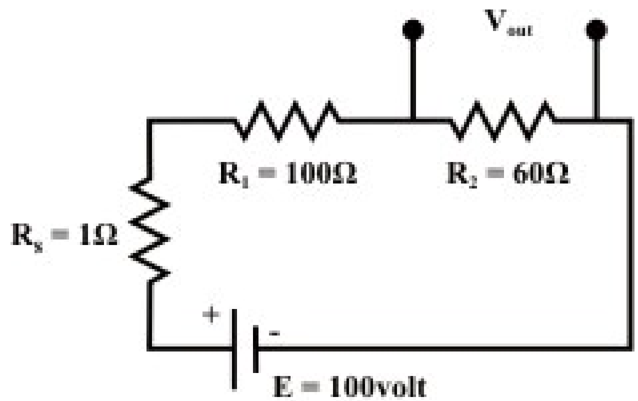


Fig. 3.9

Solution: Part (i): From equation (3.1) the output voltage V_{out} across the resistor $R_2 = \frac{E}{R_1 + R_2} R_2 = \frac{100}{100+60} \times 60 = 37.9\, volt$ (when the internal resistance R_s of the source is considered zero.) Similarly, V_{out} $= 37.27\ volt$ when R_s is taken into account for calculation. Percentage error is computed as $= \frac{37.9 - 37.27}{37.27} \times 100 = 1.69\%$

L-3.5.2 Current divider

Another frequently encountered in electric circuit is the current divider. Figure 3.10 shows that the current divider divides the source current I_s between the two resistors.

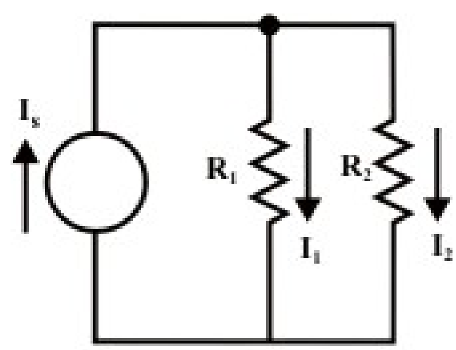


Fig. 3.10: Current Divider

The parallel combination of two resistors is sometimes termed as current divider, because the supply current is distributed between the two branches of the circuit. For the circuit, assume that the voltage across the branch is V and the current expression in R_1 resistor can be written as

$$\frac{I_1}{I_s} = \frac{\frac{V}{R_1}}{V\left(\frac{1}{R_1}+\frac{1}{R_2}\right)} = \frac{R_2}{R_1+R_2} \; or \; I_1 = \frac{R_2}{R_1+R_2}\times I_s$$

. Similarly, the current flowing through the R_2 can be obtained as $I_2 = \frac{R_1}{R_1+R_2}\times I_s$. It can be noted that the expression for I_1 has R_2 on its top line, that for I_2 has R_1 on its top line.

Example: L-3.3 Determine $I_1, I_2, I_3 \,\&\, I_5$ using only current divider formula when $I_4 = 4$A.

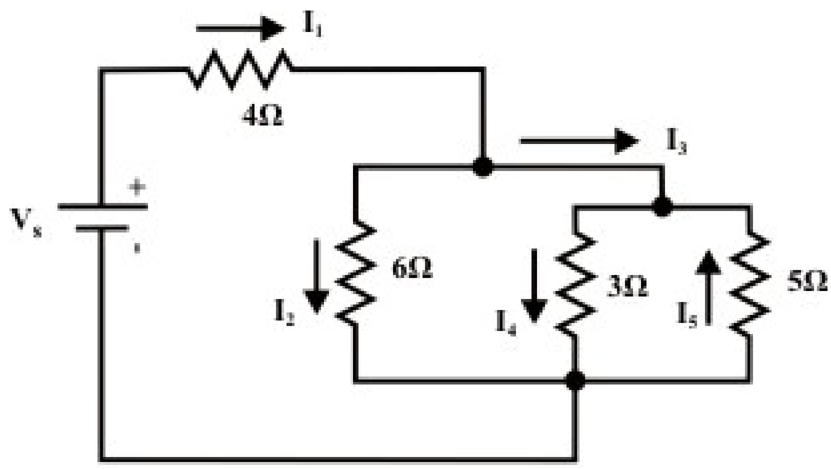


Fig. 3.11

Solution- Using the current division formula we can write $I_4 = \frac{5}{5+3}I_3 = \frac{5}{8}I_3 \rightarrow I_3 = \frac{4\times 8}{5} = 6.4\,A$. Similarly, $-I_5 = \frac{3}{8}\times I_3 \rightarrow I_5 = \frac{3}{8}\times 6.4 = 2.4\,A$.
Furthermore, we can write $I_3 = \frac{6}{6+(3\|5)}I_1 = \frac{6}{6+1.879}I_1 \rightarrow I_1 = \frac{7.879}{6}\times 6.4 = 8.404\,A$ and

$$I_2 = \frac{1.879}{6+1.879}\times I_1 = 2.004\,A.$$

L-3.6 Potentiometer and its function

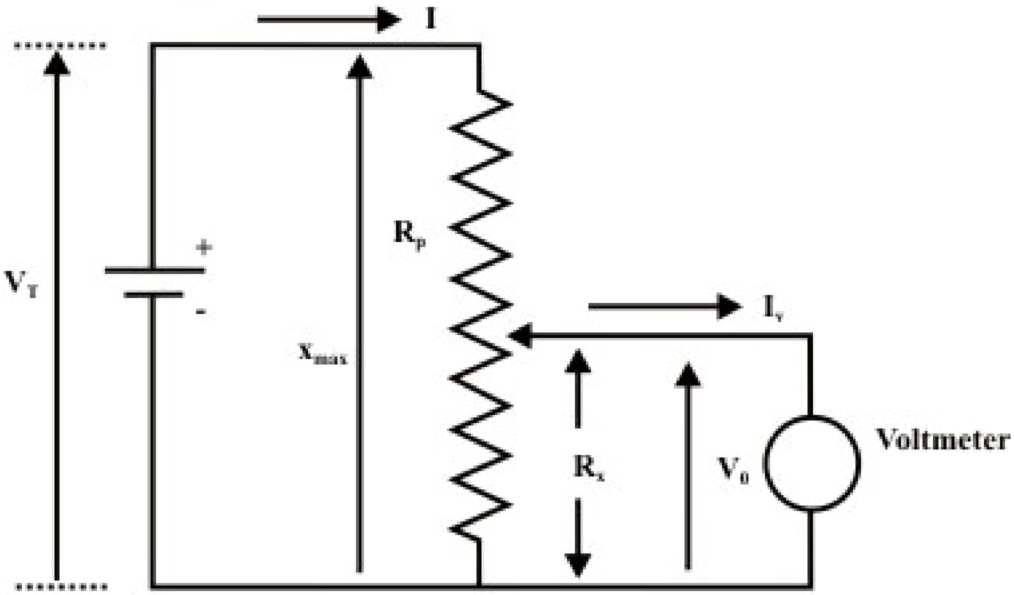


Fig. 3.12: A voltmeter is connected across the output terminals of potentiometer

The potentiometer has a resistance R_p and its wiper can move from top position $x = x_{\max}$ to bottom position $x = 0$. The resistance R_x corresponds to the position x of the wiper such that

$\frac{R_x}{x} = \frac{R_p}{x_{\max}} \Rightarrow R_x = \left(\frac{R_p}{x_{\max}}\right) x$ (assumed that the per unit length resistance of the potentiometer is same through out its length). Figure 3.12 represents a potentiometer whose output is connected to a voltmeter. In true sense, the measurement of the output voltage V_o with a voltmeter is affected by the voltmeter resistance R_V and the relationship between V_O and x (x= wiper distance from the bottom position) can easily be established. We know that the voltmeter resistance is very high in $M\,\Omega$ range and practically negligible current is flowing through the voltmeter. Under this condition, one can write the expression for voltage between the wiper and the bottom end terminal of the potentiometer as

$$V_{out}\,(= I\,R_x) = \frac{V_T\,(= I\,R_p)}{x_{\max}} \times x \;\Rightarrow\; V_{out} = V_T \times \frac{x}{x_{\max}} = V_{out} = V_T \times \frac{R_x}{R_p}$$

It may be noted that depending on the position of movable tap terminal the output voltage (V_{out}) can be controlled. By adjusting the wiper toward the top terminal, we can increase V_{out}. The opposite effect can be observed while the movable tap moves toward the bottom terminal. A simple application of potentiometer in real practice is the volume control of a radio receiver by adjusting the applied voltage to the input of audio amplifier of a radio set. This audio amplifier boosts this voltage by a certain fixed factor and this voltage is capable of driving the loudspeaker.

Example- L-3.4 A $500-k\Omega$ potentiometer has 110 V applied across it. Adjust the position of R_{bot} such that 47.5 V appears between the movable tap and the bottom end terminal (refer fig.3.12).

Solution- Since the output voltage (V_{bot}) is not connected to any load, in turn, we can write the following expression

$$V_{out} = V_T \times \frac{x}{x_{\max}} \frac{V_{bot}}{V_T} = \frac{R_{bot}}{R_T} \rightarrow R_{bot} = \frac{V_{bot}}{V_T} \times R_T = \frac{47.5}{110} \times 500000 = 216 - k\Omega.$$

L-3.7 Practical Voltage and Current Sources

L-3.7.1 Ideal and Practical Voltage Sources

- An ideal voltage source, which is represented by a model in fig.3.13, is a device that produces a constant voltage across its terminals ($V = E$) no matter what current is drawn from it (terminal voltage is independent of load (resistance) connected across the terminals)

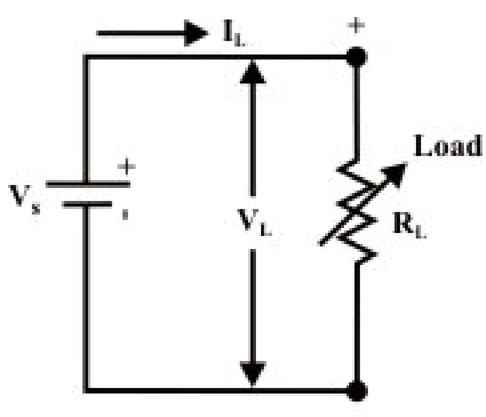


Fig. 3.13: Ideal dc voltage source

For the circuit shown in fig.3.13, the upper terminal of load is marked plus (+) and its lower terminal is marked minus (-). This indicates that electrical potential of upper terminal is V_L volts higher than that of lower terminal. The current flowing through the load R_L is given by the expression $V_s = V_L = I_L R_L$ and we can represent the terminal $V - I$ characteristic of an ideal dc voltage as a straight line parallel to the x-axis. This means that the terminal voltage V_L remains constant and equal to the source voltage V_s irrespective of load current is small or large. The $V - I$ characteristic of ideal voltage source is presented in Figure 3.14.

- However, real or practical dc voltage sources do not exhibit such characteristics (see fig. 3.14) in practice. We observed that as the load resistance R_L connected across the source is decreased, the corresponding load current I_L increases while the terminal voltage across the source decreases (see eq.3.1). We can realize such voltage drop across the terminals with increase in load current provided a resistance element (R_s) present inside the voltage source. Fig. 3.15 shows the model of practical or real voltage source of value V_s.

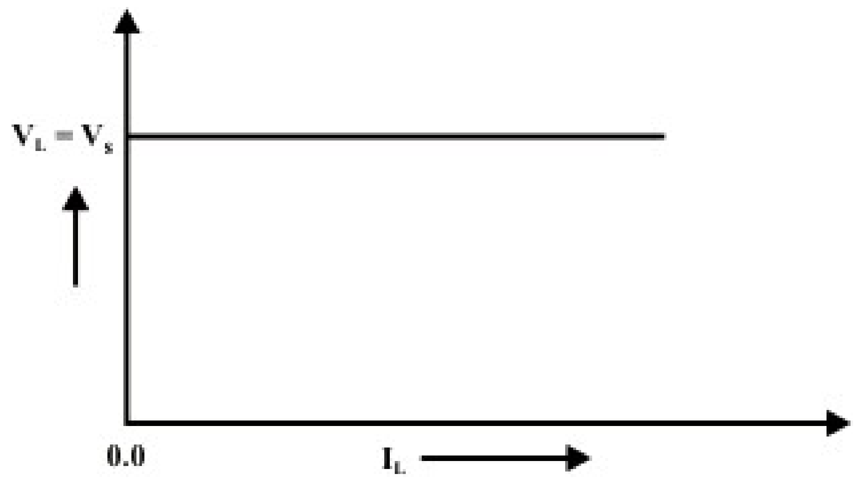


Fig. 3.14: V-I characteristics of ideal voltage source

The terminal $V-I$ characteristics of the practical voltage source can be described by an equation

$$V_L = V_s - I_L R_s \tag{3.1}$$

and this equation is represented graphically as shown in fig.3.16. In practice, when a load resistance R_L more than 100 times larger than the source resistance R_s, the source can be considered approximately ideal voltage source. In other words, the internal resistance of the source can be omitted. This statement can be verified using the relation $R_L = 100 R_s$ in equation (3.1). The practical voltage source is characterized by two parameters namely known as (i) Open circuit voltage (V_s) (ii) Internal resistance in the source's circuit model. In many practical situations, it is quite important to determine the source parameters experimentally. We shall discuss briefly a method in order to obtain source parameters.

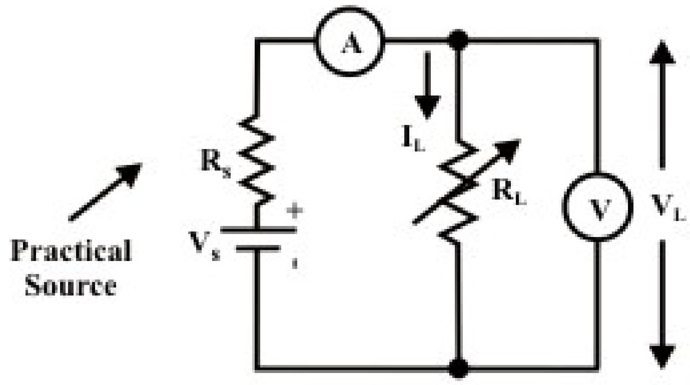


Fig. 3.15: Practical dc voltage source model

Method-: Connect a variable load resistance across the source terminals (see fig. 3.15). A voltmeter is connected across the load and an ammeter is connected in series with the load resistance. Voltmeter and Ammeter readings for several choices of load resistances are presented on the graph paper (see fig. 3.16). The slope of the line is $-R_s$, while the curve intercepts with voltage axis (at $I_L = 0$) is the value of V_s.

The $V - I$ characteristic of the source is also called the source's <u>**"regulation curve" or "load line"**</u>. The open-circuit voltage is also called the "no-load" voltage, V_{oc}. The maximum allowable load current (rated current) is known as full-load current I_{Fl} and the corresponding source or load terminal voltage is known as "full-load" voltage V_{FL}. We know that the source terminal voltage varies as the load is varied and this is due to internal voltage drop inside the source. The percentage change in source terminal voltage from no-load to full-load current is termed the "voltage regulation" of the source. It is defined as

$$\textit{Voltage regulation } (\%) = \frac{V_{oc} - V_{FL}}{V_{FL}} \times 100$$

For ideal voltage source, there should be no change in terminal voltage from no-load to full-load and this corresponds to "zero voltage regulation". For best possible performance, the voltage source should have the lowest possible regulation and this indicates a smallest possible internal voltage drop and the smallest possible internal resistance.

Example:-L-3.5 A practical voltage source whose short-circuit current is 1.0A and open-circuit voltage is 24 Volts. What is the voltage across, and the value of power dissipated in the load resistance when this source is delivering current 0.25A?

Solution: From fig. 3.10, $I_{sc} = \frac{V_s}{R_s} = 1.0\,A$ (short-circuit test) $V_{oc} = V_s = 24\ volts$ (open-circuit test). Therefore, the value of internal source resistance is obtained as $R_s = \frac{V_s}{I_{sc}} = 24\,\Omega$. Let us assume that the source is delivering current $I_L =$ 0.25A when the load resistance R_L is connected across the source terminals. Mathematically, we can write the following expression to obtain the load resistance R_L .

$$\frac{24}{24+R_L}=0.25 \rightarrow R_L=72\Omega .$$

Now, the voltage across the load $R_L = I_L R_L = 0.25\times 72 = 18$ *volts.*, and the power consumed by the load is given by $P_L = I_L{}^2 R_L = 0.0625\times 72 = 4.5$ *watts.*

Example-L-3.6 (Refer fig. 3.15) A certain voltage source has a terminal voltage of 50 V when I= 400 mA; when I rises to its full-load current value 800 mA the output voltage is recorded as 40 V. Calculate (i) Internal resistance of the voltage source (R_s). (ii) No-load voltage (open circuit voltage V_s). (iii) The voltage Regulation.

Solution- From equation (3.1) ($V_L = V_s - I_L R_s$) one can write the following expressions under different loading conditions.

$50 = V_s - 0.4 R_s$ & $40 = V_s - 0.8 R_s$ $\rightarrow$ solving these equations we get, $V_s = 60V$ & $R_s = 25\Omega$.

$$\textit{Voltage regulation } (\%) = \frac{V_{oc} - V_{FL}}{V_{FL}}\times 100 = \frac{60-40}{60}\times 100 = 33.33\%$$

L-3.7.2 Ideal and Practical Current Sources

- Another two-terminal element of common use in circuit modeling is `current source` as depicted in fig.3.17. An ideal current source, which is represented by a model in fig. 3.17(a), is a device that delivers a constant current to any load resistance connected across it, no matter what the terminal voltage is developed across the load (i.e., independent of the voltage across its terminals across the terminals).

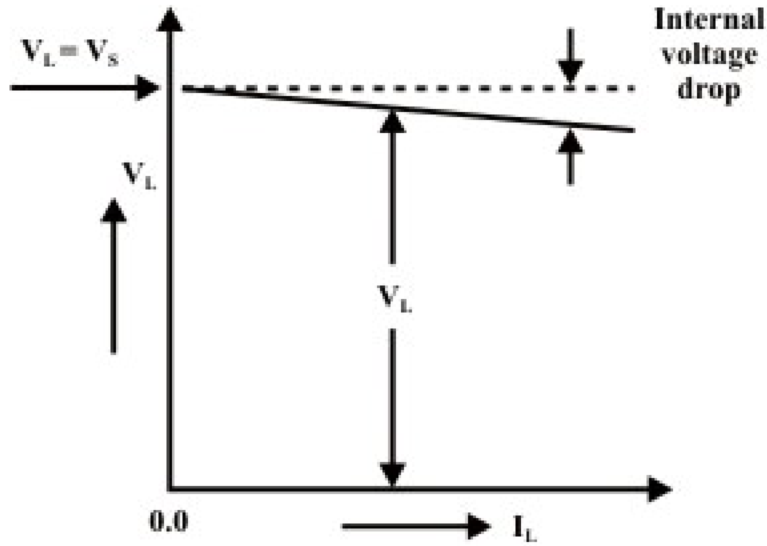


Fig. 3.16: V-I characteristics of practical voltage source

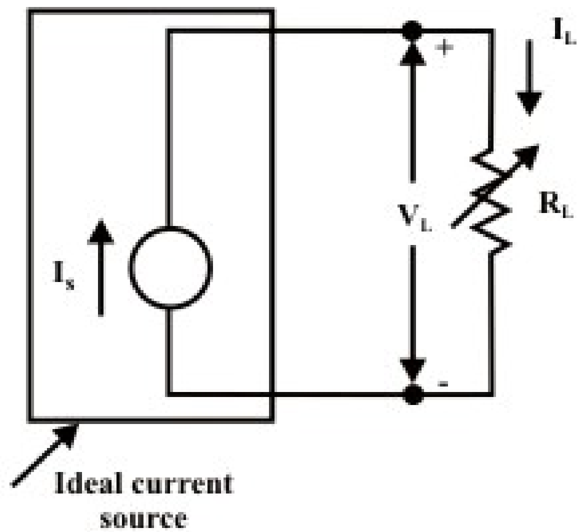


Fig. 3.17(a): Ideal current source with variable load

It can be noted from model of the current source that the current flowing from the source to the load is always constant for any load resistance (see fig. 3.19(a)) i.e. whether R_L is small (V_L is small) or R_L is large (V_L is large). The vertical dashed line in fig. 3.18 represents the $V-I$ characteristic of ideal current source. In practice, when a load R_L is connected across a practical current source, one can observe that the current flowing in load resistance is reduced as the voltage across the current source's terminal is increased, by increasing the load resistance R_L. Since the distribution of source current in two parallel paths entirely depends on the value of external resistance that connected across the source (current source) terminals. This fact can be realized by introducing a parallel resistance R_s in parallel with the practical current source I_s, as shown in fig. 3.17(b). The dark lines in fig. 3.18 show the $V - I$ characteristic (load-line) of practical current source. The slope of the curve represents the internal resistance of the source. One can apply KCL at the top terminal of the current source in fig. 3.17(b) to obtain the following expression.

$$I_L = I_s - \frac{V_L}{R_s} \; Or \; V_L = I_s R_s - R_s I_L = V_{oc} - R_s I_L \tag{3.2}$$

The open circuit voltage and the short-circuit current of the practical current source are given by $V_{oc} = I_s R_s$ and $I_{short} = I_s$ respectively. It can be noted from the fig.3.18 that source 1 has a larger internal resistance than source 2 and the slope the curve indicates the internal resistance R_s of the current source. Thus, source 1 is closer to the ideal source. More specifically, if the source internal resistance $R_s \geq 100\, R_L$ then source acts nearly as an ideal current source.

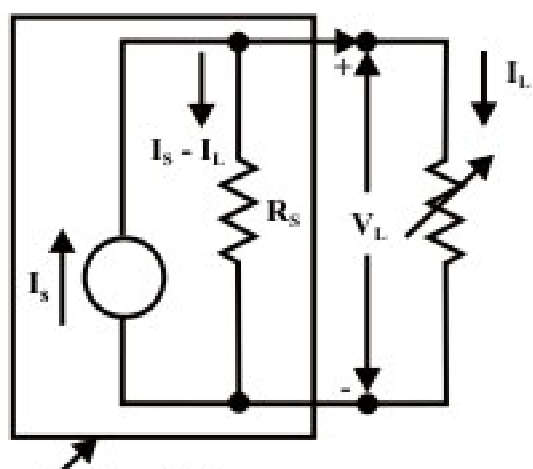


Fig. 3.17(b): Practical current source with variable load

L-3.7.3 Conversion of a Practical Voltage Source to a Practical Current source and vise-versa

- Voltage Source to Current Source

For the practical voltage source in fig. 3.19(a), the load current is calculated as

$$I_L = \frac{V_s}{R_s + R_L} \tag{3.3}$$

Note that the maximum current delivered by the source when $R_L = 0$ (under short-circuit condition) is given by $I_{\max} = I_s = \frac{V_s}{R_s}$. From eq.(3.3) one can rewrite the expression for load current as

$$I_L = \frac{I_s \times R_s}{R_s + R_L} \tag{3.4}$$

A simple current divider circuit having two parallel branches as shown in fig.3.19 (b) can realize by the equation (3.4).

Note: A practical voltage source with a voltage V_s and an internal source resistance R_s can be replaced by an equivalent practical current source with a current $I_s = \frac{V_s}{R_s}$ and a source internal resistance R_s (see fig. 3.19(b)).

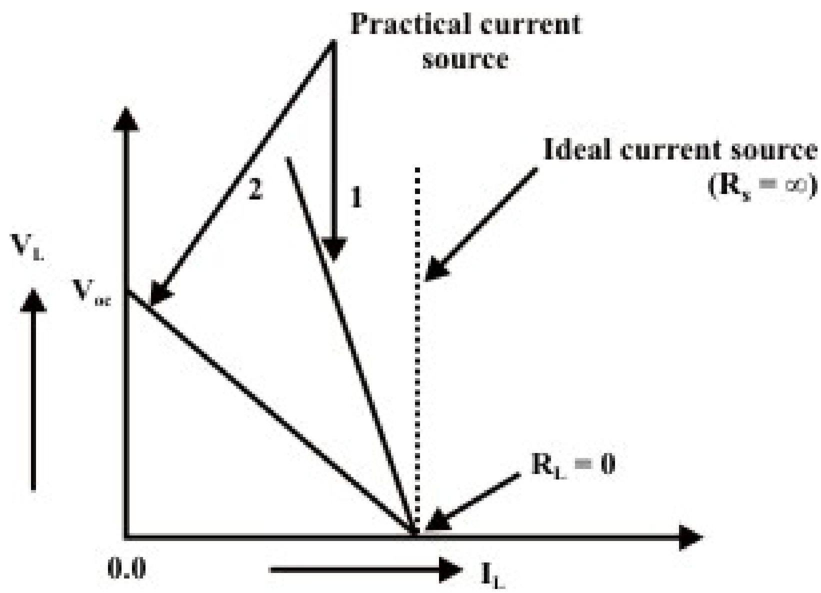


Fig. 3.18: V-I characteristic of practical current source

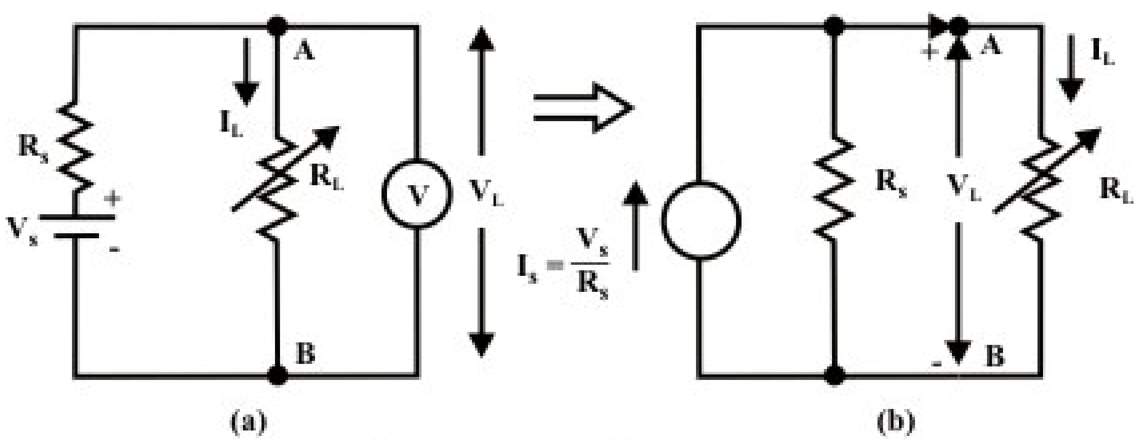


Fig. 3.19: Source Conversions

- Current source to Voltage Source

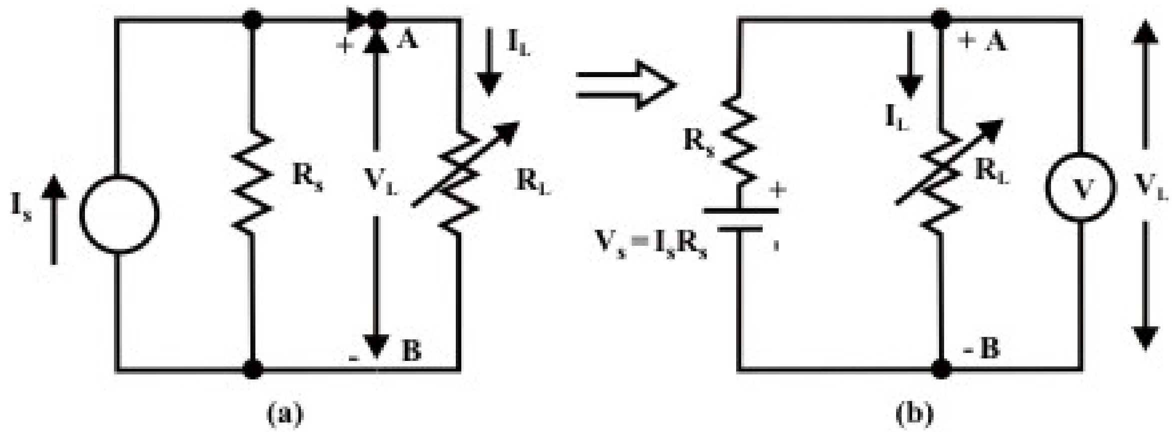


Fig. 3.20: Current Source to Voltage source conversion

For the circuit in fig. 3.15(a), the load voltage V_L is given by

$$V_L = I_L R_L = \left(\frac{R_s}{R_s + R_L} \times I_s \right) R_L = I_s R_s \left(\frac{R_L}{R_s + R_L} \right) = V_s \left(\frac{R_L}{R_s + R_L} \right) \qquad (3.5)$$

Equation (3.5) represents output from the voltage source across a load resistance and this act as a voltage divider circuit. Figure 3.20(b) describes the situation that a voltage source with a voltage value $V_s = I_s R_s$ and an internal source resistance R_s has an equivalent effect on the same load resistor as the current source in figure 3.20(a).

Note: A current source with a magnitude of current I_s and a source internal resistance R_s can be replaced by an equivalent voltage source of magnitude $V_s = I_s R_s$ and an internal source resistance R_s (see fig. 3.20(b)).

Remarks on practical sources: (i) The open circuit voltage that appears at the terminals A & B for two sources (voltage & current) is same (i.e., V_s).

(ii) When the terminals A & B are shorted by an ammeter, the shot-circuit results same in both cases (i.e., I_s).

(iii) If an arbitrary resistor (R_L) is connected across the output terminals A & B of either source, the same power will be dissipated in it.

(iv) The sources are equivalent only as concerns on their behavior at the external terminals.

(v) The internal behavior of both sources is quite different (i.e., when open circuit the voltage source does not dissipate any internal power while the current source dissipates. Reverse situation is observed in short-circuit condition).

L-3.8 Independent and Dependent Sources that encountered in electric circuits

- Independent Sources

So far the voltage and current sources (whether ideal or practical) that have been discussed are known as independent sources and these sources play an important role to drive the circuit in order to perform a specific job. The internal values of these sources (either voltage source or current source) – that is, the generated voltage V_s or the generated current I_s (see figs. 3.15 & 3.17) are not affected by the load connected across the source terminals or across any other element that exists elsewhere in the circuit or external to the source.

- Dependent Sources

Another class of electrical sources is characterized by dependent source or controlled source. In fact the source voltage or current depends on a voltage across or a current through some other element elsewhere in the circuit. Sources, which exhibit this dependency, are called dependent sources. Both voltage and current types of sources may be dependent, and either may be controlled by a voltage or a current. In general, dependent source is represented by a diamond (◇)-shaped symbol as not to confuse it with an independent source. One can classify dependent voltage and current sources into four types of sources as shown in fig.3.21. These are listed below:

(i) Voltage-controlled voltage source (VCVS) (ii) Current-controlled voltage source (ICVS) (iii) Voltage-controlled current source(VCIS) (iv) Current-controlled current source(ICIS)

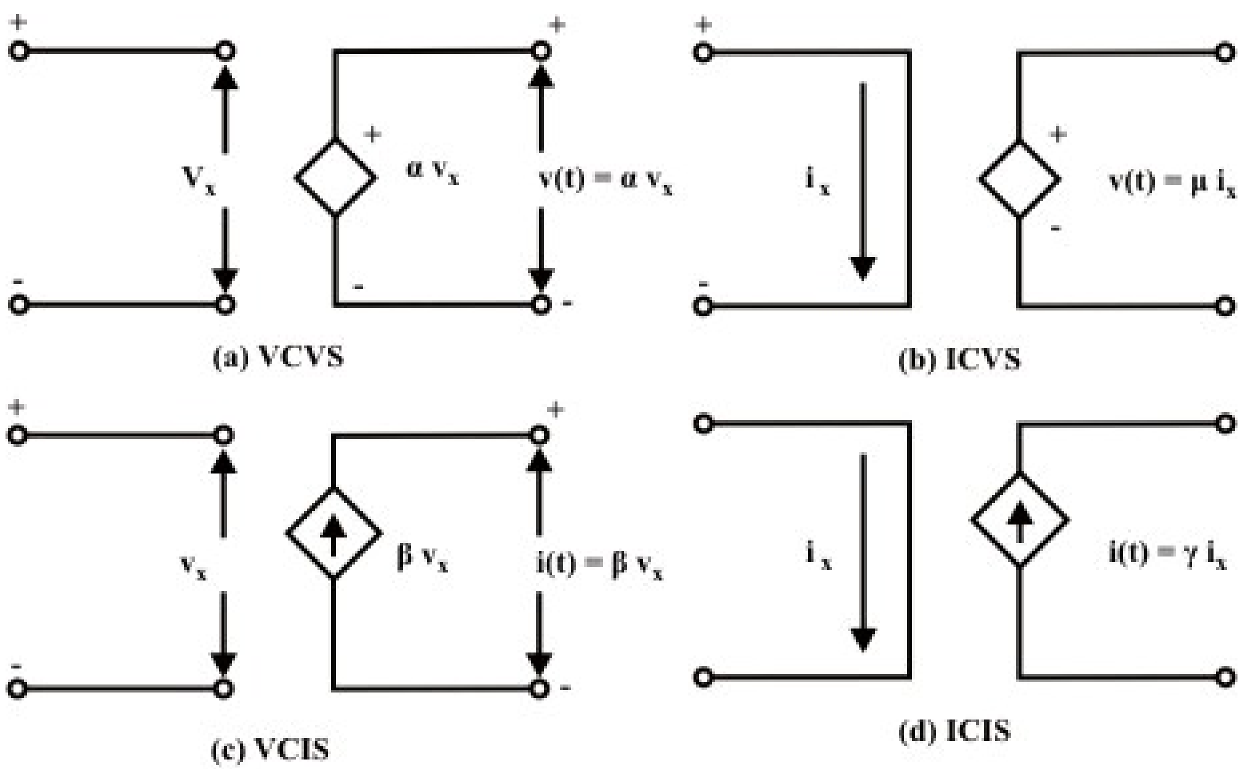
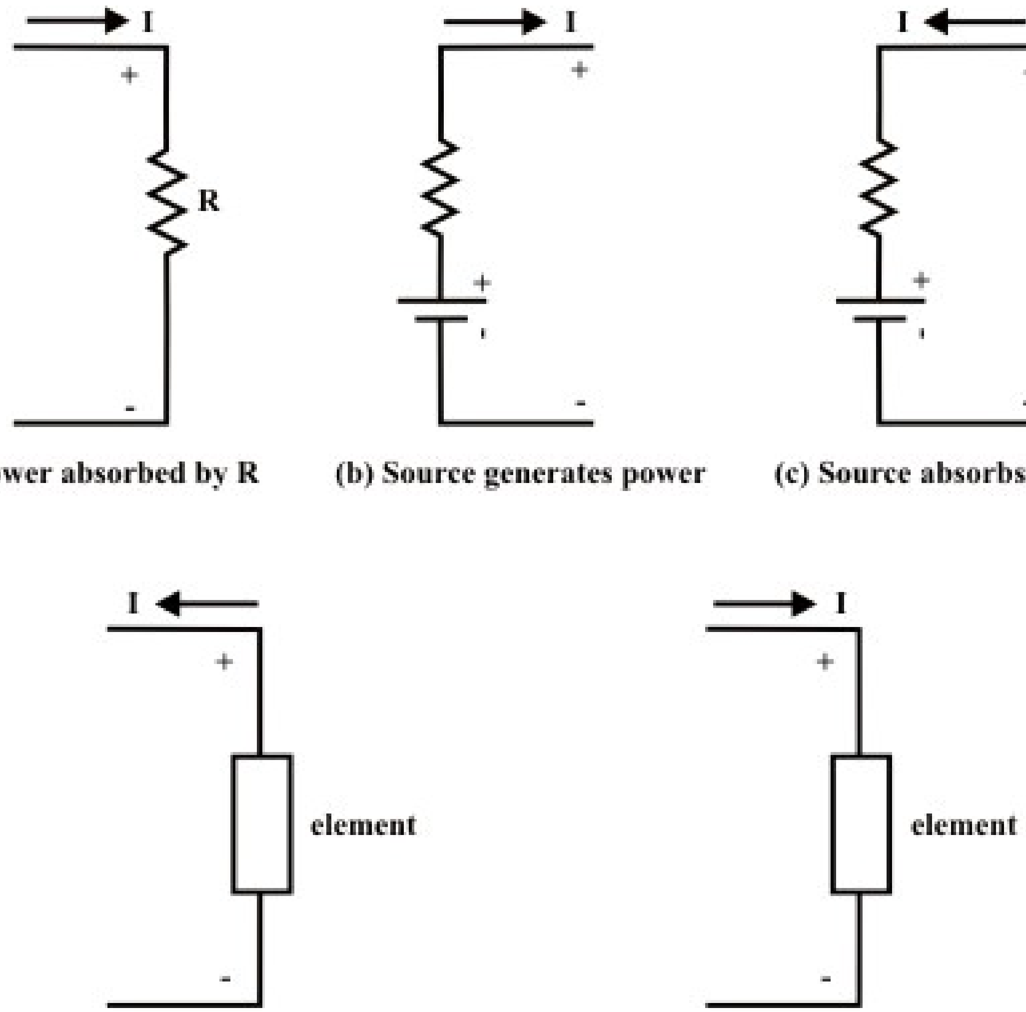


Fig. 3.21: Ideal dependent (controlled) sources

Note: When the value of the source (either voltage or current) is controlled by a voltage (v_x) somewhere else in the circuit, the source is said to be voltage-controlled source. On the other hand, when the value of the source (either voltage or current) is controlled by a current (i_x) somewhere else in the circuit, the source is said to be current-controlled source. KVL and KCL laws can be applied to networks containing such dependent sources. Source conversions, from dependent voltage source models to dependent current source models, or visa-versa, can be employed as needed to simplify the network. One may come across with the dependent sources in many equivalent-circuit models of electronic devices (transistor, BJT(bipolar junction transistor), FET(field-effect transistor) etc.) and transducers.

L-3.9 Understanding Delivering and Absorbing Power by the Source.

It is essential to differentiate between the absorption of power (or dissipating power) and the generating (or delivering) power. The power absorbed or dissipated by any circuit element when flows in a load element from higher potential point (i.e +ve terminal) toward the lower terminal point (i.e., -ve terminal). This situation is observed when charging a battery or source because the source is absorbing power. On the other hand, when current flows in a source from the lower potential point (i.e., -ve terminal) toward the higher potential point (i.e., +ve terminal), we call that source is generating power or delivering power to the other elements in the electric circuit. In this case, one can note that the battery is acting as a "source" whereas the other element is acting as a "sink". Fig.3.22 shows mode of current entering in a electric element and it behaves either as source (delivering power) or as a sink (absorbing or dissipating power).



(a) Power absorbed by R (b) Source generates power (c) Source absorbs power

(d) Power generated by this element (e) Power absorbed by this element

Fig. 3.22: Source and sink configurations

L.3.10 <u>**Test Your Understanding**</u> [marks distribution shown inside the bracket]

T.1 If a 30 V source can force 1.5 A through a certain linear circuit, how much current can 10 V force through the same circuit? (Ans. 500 mA.) [1]

T.2 Find the source voltage V_s in the circuit given below [1]

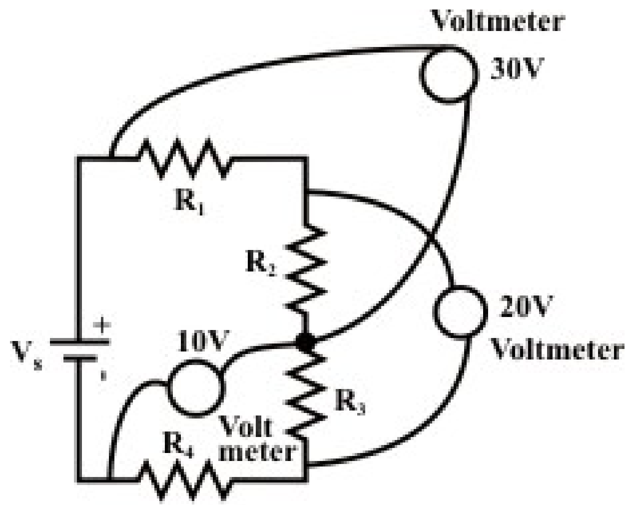


Fig. 3.33

(Ans. 40 V)

T.3 For the circuit shown in Figure T.3 [1x4]

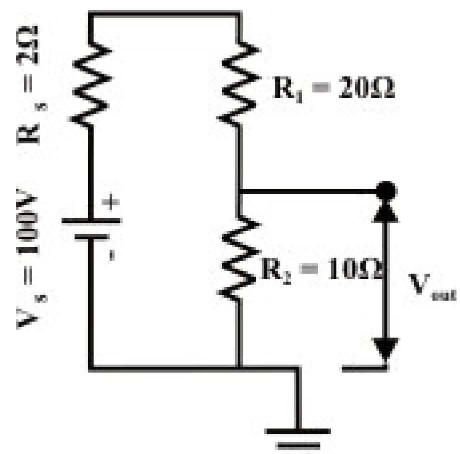


Fig. 3.34

(a) Calculate V_{out}, ignoring the internal resistance of the source R_s (assuming it's zero). Use Voltage division method. (Ans.33.333 V)

(b) Recalculate V_{out}, taking into account R_s. What percentage error was introduced by ignoring R_s in part (a). (Ans. 31.29 V , 6.66%)

(c) Repeat part (a) & (b) with the same source and replacing $R_1 = 20\Omega \; by \; 20k\Omega$ & $R_2 = 10\Omega \; by \; 1k\Omega$. Explain why the percent error is now so much less than in part (b). (Ans. 33.333 V, 33.331 V, 0.006%)

T.4 For the circuit shown in figure T.4 [1x6]

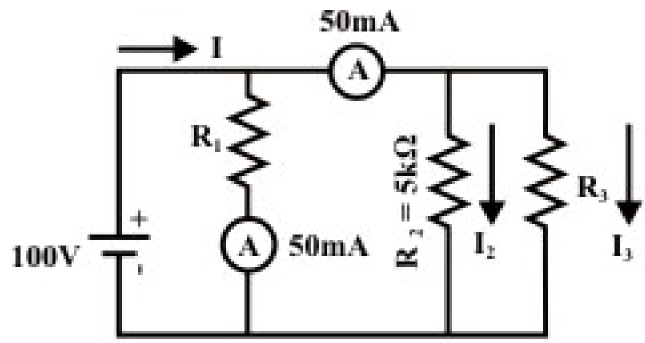


Fig. 3.35

(a) Find , in any order, I_2, I_3, *and* I (b) Find, in any order, R_1, R_3, *and* R_{eq}.
(Ans. (a) 20 mA, 30 mA and 100 mA (b) $2k\Omega$, 3.33 $k\Omega$ and $1k\Omega$.)

T.5 Refer to the circuit shown in Figure T.5 [1x4]

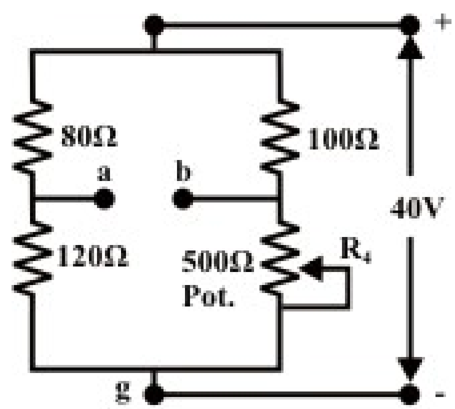


Fig. 3.36

(a) What value of R_4 will balance the bridge (i.e., $V_{ab} = 0.0$) (b) At balanced condition, find the values of V_{ag} & V_{bg}. (Ans. 150Ω, $24V$ (a is higher potential than 'g', since current is flowing from 'a' to 'b'), $24V$ (b is higher potential than 'g')
(b) Does the value of V_{ag} depend on whether or not the bridge is balanced? Explain this. (Ans. No., since flowing through the 80Ω branch will remain same and hence potential drop across the resistor remains same.)
(c) Repeat part (b) for V_{bg}. (Ans. Yes . Suppose the value of R_4 is increased from its balanced condition, this in turn decreases the value of current in that branch and subsequently voltage drop across the 100Ω is also decreases. The indicates that the voltage across V_{bg} will increase to satisfy the KVL.)
(d) If the source voltage is changed to 50 V will the answer to part (a) change? Explain this. (Ans. No.)

T.6 If an ideal voltage source and an ideal current source are connected in parallel, then the combination has exactly the same properties as a voltage source alone. Justify this statement. [1]

T.7 If an ideal voltage source and an ideal current source are connected in series, the combination has exactly the same properties as a current source alone. Justify this statement. [1]

T.8 When ideal arbitrary voltage sources are connected in parallel, this connection violates KVL. Justify. [1]

T.9 When ideal arbitrary current sources are connected in series, this connection violates KCL. Justify.
[1]

T.10 Consider the nonseries-parallel circuit shown in figure T.10. Determine R and the equivalent resistance R_{eq} between the terminals "a" & "b" when $v_1 = 8V$.

(Appling basic two Kirchhoff's laws) (Ans. $R = 4\Omega$ & $R_{eq} = 4\Omega$) [3]

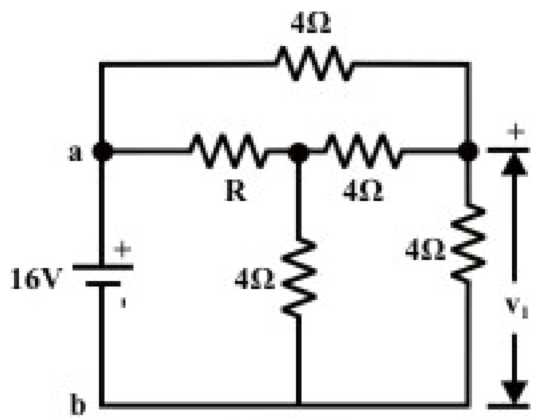


Fig. 3.37

T.11 A 20 V voltage source is connected in series with the two series-resistors $R_1 = 5\Omega$ & $R_2 = 10\Omega$. (a) Find I, V_{R1}, V_{R2}. (Ans. 1.333 A, 6.6667 V, 13.33 V)

(b) Find the power absorbed or generated by each of the three elements. (8.88 W (absorbed), 17.76 W (absorbed), 26.66 W delivered or generated (since current is leaving the plus terminal of that source.) [2]

T.12 Consider the circuit of figure T.12

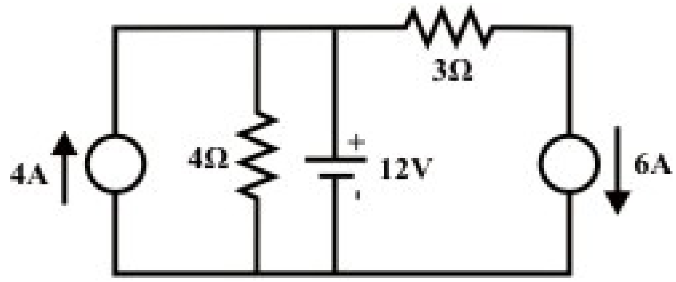


Fig. 3.38

Find powers involved in each of the five elements and whether absorbed or generated. (Ans. 48 W (G), 36 W (A), 60 W (G), 108 W (A) and 36 W (G). (results correspond to elements from left to right, CS, R, VS, R, CS). [4]

T.13 For the circuit of Figure T.13 Suppose $V_{in} = 20V$.

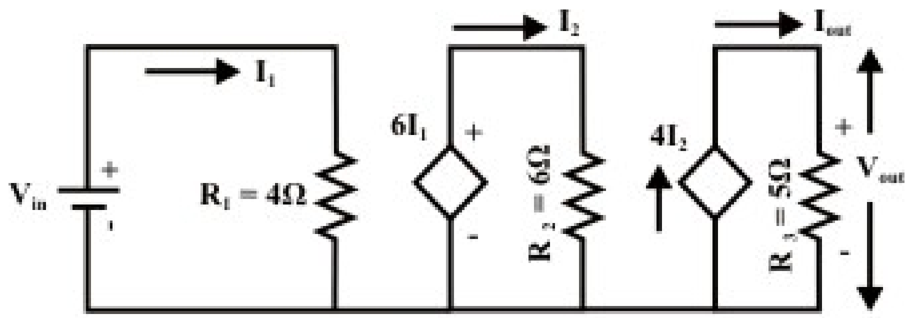


Fig. 3.39

(a) Find the output voltage and output current. [2]

(b) Find the ratio of output voltage (V_{out}) to input voltage (V_{in}) i.e. $\frac{V_{out}}{V_{in}}$ = voltage gain. [1]

(c) Find the power delivered by each source(dependent & independent sources).[2]

(Ans. (a) 100 V, 20 A (note that $6I_1$ is the value of dependent voltage source with the polarity as shown in fig. T.13 whereas $4I_2$ represents the value of dependent current source) (b) 5 (voltage gain). (c) 100 W (VS), 150 W (DVS), 2000 W (DCS)).

T.14 Find the choice of the resistance R_2 (refer to Fig. T.13) so that the voltage gain is 30. (Ans. $R_2 = 1\Omega$ [1]

T.15 Find equivalent resistance between the terminals 'a' & 'b' and assume all resistors values are 1Ω. [2]

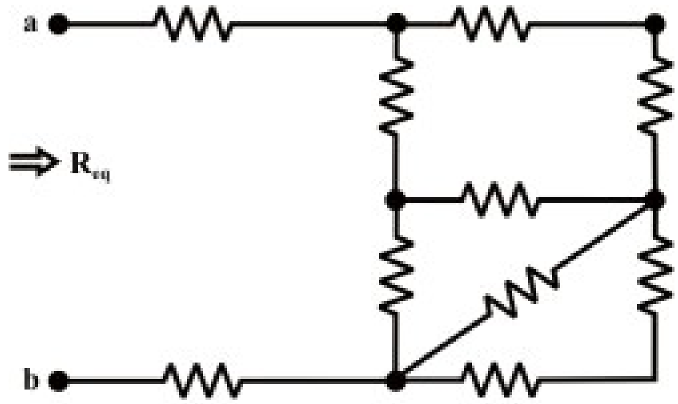


Fig. 3.40

Module 2

DC Circuit

Lesson 4

Loop Analysis of resistive circuit in the context of dc voltages and currents

Objectives

- Meaning of circuit analysis; distinguish between the terms mesh and loop.
- To provide more general and powerful circuit analysis tool based on Kirchhoff's voltage law (KVL) only.

L.4.1 Introduction

The Series-parallel reduction technique that we learned in lesson-3 for analyzing DC circuits simplifies every step logically from the preceding step and leads on logically to the next step. Unfortunately, if the circuit is complicated, this method (the simplify and reconstruct) becomes mathematically laborious, time consuming and likely to produce mistake in calculations. In fact, to elevate these difficulties, some methods are available which do not require much thought at all and we need only to follow a well-defined faithful procedure. One most popular technique will be discussed in this lesson is known as 'mesh or loop' analysis method that based on the fundamental principles of circuits laws, namely, Ohm's law and Kirchhoff's voltage law. Some simple circuit problems will be analyzed by hand calculation to understand the procedure that involve in mesh or loop current analysis.

L.4.1.1 Meaning of circuit analysis

The method by which one can determine a variable (either a voltage or a current) of a circuit is called analysis. Basic difference between 'mesh' and 'loop' is discussed in lesson-3 with an example. A 'mesh' is any closed path in a given circuit that does not have any element (or branch) inside it. A mesh has the properties that (i) every node in the closed path is exactly formed with two branches (ii) no other branches are enclosed by the closed path. Meshes can be thought of a resembling window partitions. On the other hand, 'loop' is also a closed path but inside the closed path there may be one or more than one branches or elements.

L.4.2 Solution of Electric Circuit Based on Mesh (Loop) Current Method

Let us consider a simple dc network as shown in Figure 4.1 to find the currents through different branches using Mesh (Loop) current method.

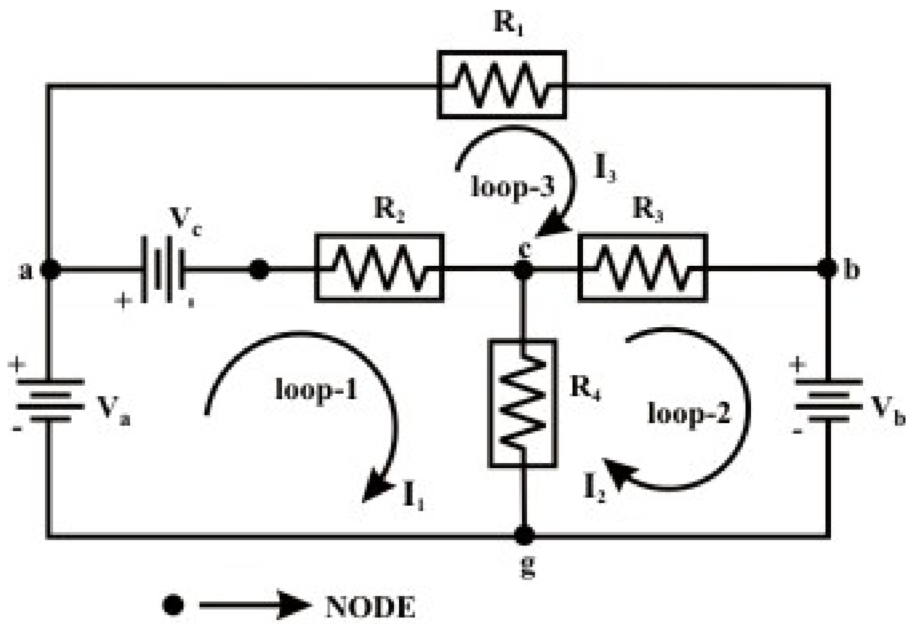


Figure 4.1

Applying KVL around mesh (loop)-1:(note in mesh-1, I_1 is known as local current and other mesh currents I_2 & I_3 are known as foreign currents.)

$$V_a - V_c - \left(I_1 - I_3\right)R_2 - \left(I_1 - I_2\right)R_4 = 0$$

$$V_a - V_c = \left(R_2 + R_4\right)I_1 - R_4 I_2 - R_2 I_3 = R_{11}I_1 - R_{12}I_2 - R_{13}I_3 \qquad (4.1)$$

Applying KVL around mesh (loop)-2:(similarly in mesh-2, I_2 is local current and I_1 & I_3 are known as foreign currents)

$$-V_b - \left(I_2 - I_3\right)R_3 - \left(I_2 - I_1\right)R_4 = 0$$

$$-V_b = -R_4 I_1 + \left(R_3 + R_4\right)I_2 - R_3 I_3 = -R_{21}I_1 + R_{22}I_2 - R_{23}I_3 \qquad (4.2)$$

Applying KVL around mesh (loop)-3:

$$V_c - I_3 R_1 - \left(I_3 - I_2\right)R_3 - \left(I_3 - I_1\right)R_2 = 0$$

$$V_c = -R_2 I_1 - R_3 I_2 + \left(R_1 + R_2 + R_3\right)I_3 = -R_{31}I_1 - R_{32}I_2 + R_{33}I_3 \qquad (4.3)$$

** In general, we can write for i^{th} mesh (for $i = 1, 2, \ldots.. N$)

$$\sum V_{ii} = -R_{i1}I_1 - R_{i2}I_2 \ldots\ldots.. + R_{ii}I_i - R_{i,i+1}I_{i+1} - \ldots. R_{iN}\ I_N$$

$\sum V_{ii} \rightarrow$ simply means to take the algebraic sum of all voltage sources around the i^{th} mesh.

R_{ii} $\rightarrow$ means the total self resistance around the i^{th} mesh.

R_{ij} $\rightarrow$ means the mutual resistance between the and j^{th} meshes.

Note: Generally, $R_{ij} = R_{ji}$ (true only for linear bilateral circuits)

$I_i \rightarrow$ the unknown mesh currents for the network.

Summarize:

Step-I: Draw the circuit on a flat surface with no conductor crossovers.

Step-2: Label the mesh currents (I_i) carefully in a clockwise direction.

Step-3: Write the mesh equations by inspecting the circuit (No. of independent mesh (loop) equations=no. of branches (b) - no. of principle nodes (n) + 1).

Note:

To analysis, a resistive network containing voltage and current sources using ‘mesh’ equations method the following steps are essential to note:

- If possible, convert current source to voltage source.
- Otherwise, define **the voltage** across the current source and write the mesh equations as if these source voltages were known. Augment the set of equations with one equation for each current source expressing a known mesh current or difference between two mesh currents.
- Mesh analysis is valid only for circuits that can be drawn in a two-dimensional plane in such a way that no element crosses over another.

Example-L-4.1: Find the **current** through 'ab-branch' (I_{ab}) and **voltage (**V_{cg}**)** across the current source using Mesh-current method.

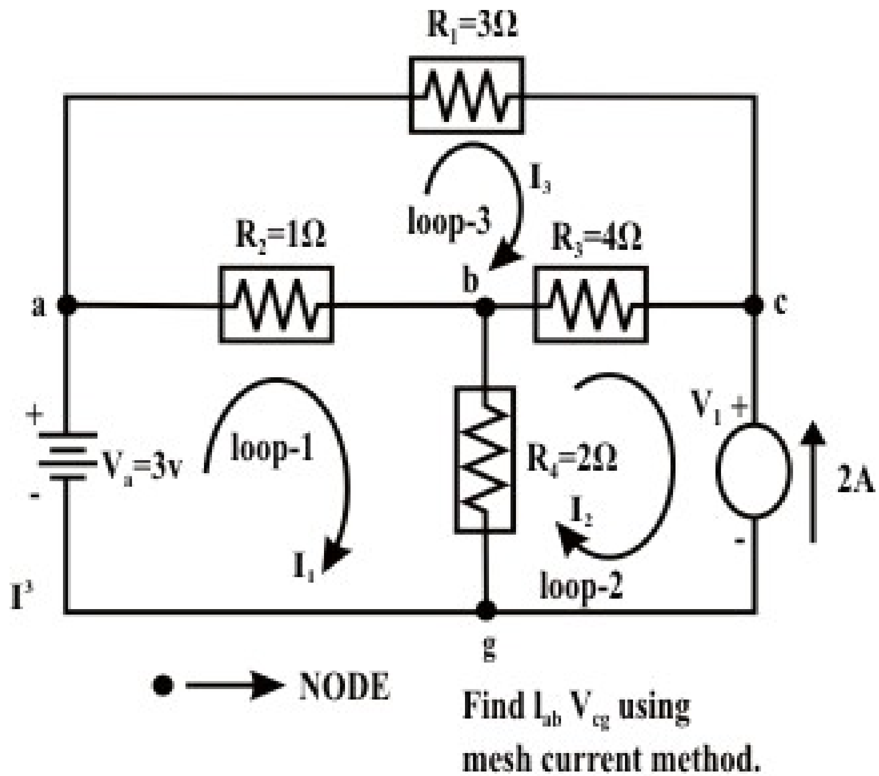


Figure 4.2

Solution: Assume voltage across the current source is v_1 ('c' is higher potential than 'g' (ground potential and assumed as zero potential) and note *I_2 = -2A* (since assigned current direction (I_2) is opposite to the source current)

Loop - 1: (Appling KVL)

$$V_a - (I_1 - I_3)\,R_2 - (I_1 - I_2)R_4 = 0 \Rightarrow 3 = 3I_1 - 2I_2 - I_3$$

$$3I_1 - I_3 = -1 \qquad (4.4)$$

Loop - 2: (Appling KVL)

Let us assume the voltage across the current source is v_1 and its top end is assigned with a positive sign.

$$-v_1 - (I_2 - I_1)R_4 - (I_2 - I_3)R_3 = 0 \;\Rightarrow\; -v_1 = -2I_1 + 6I_2 - 4I_3$$

$$2I_1 + 12 + 4I_3 = v_1 \qquad \text{(note: } I_2 = -2A\text{)} \qquad (4.5)$$

Loop - 3: (Appling KVL)

$$-I_3\,R_1 - (I_3 - I_2)R_3 - (I_3 - I_1)R_2 = 0 \Rightarrow -I_1 - 4I_2 + 8I_3 = 0$$

$$I_1 - 8I_3 = 8 \qquad \text{(Note, } I_2 = -2A) \qquad (4.6)$$

Solving equations (4.4) and (4.6), we get $I_1 = -\frac{48}{69} = -0.6956A$ and

$I_3 = -\frac{25}{23} = -1.0869A$, $I_{ab} = I_1 - I_3 = 0.39A$, $I_{bc} = I_2 - I_3 = -0.913A$ and

$I_{bg} = I_1 - I_2 = 1.304A$

- ve sign of current means that the current flows in reverse direction (in our case, the current flows through 4Ω resistor from 'c' to 'b' point). From equation (4.5), one can get v_1 == *6.27 volt.*

Another way: $-v_1 + v_{bg} + v_{bc} = 0 \Rightarrow v_1 = v_{cg} = 6.27\,volt$.

Example-L-4.2 For the circuit shown Figure 4.3 (a) find V_x using the mesh current method.

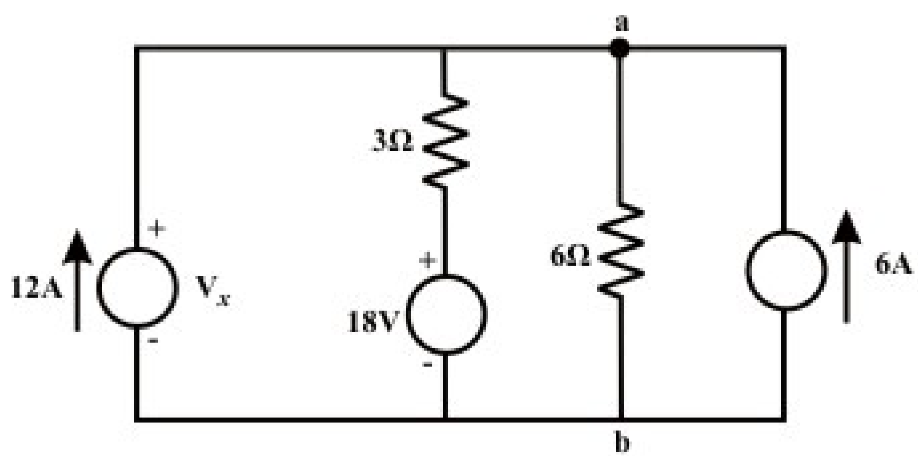


Fig. 4.3(a)

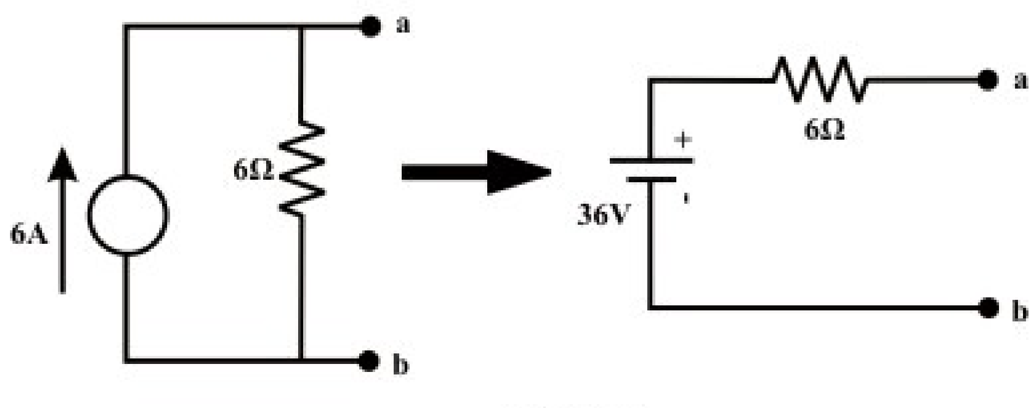


Fig. 4.3(b)

Solution: One can easily convert the extreme right current source (6 A) into a voltage source. Note that the current source magnitude is 6 A and its internal resistance is $6\,\Omega$. The given circuit is redrawn and shown in Figure 4.3 (c)

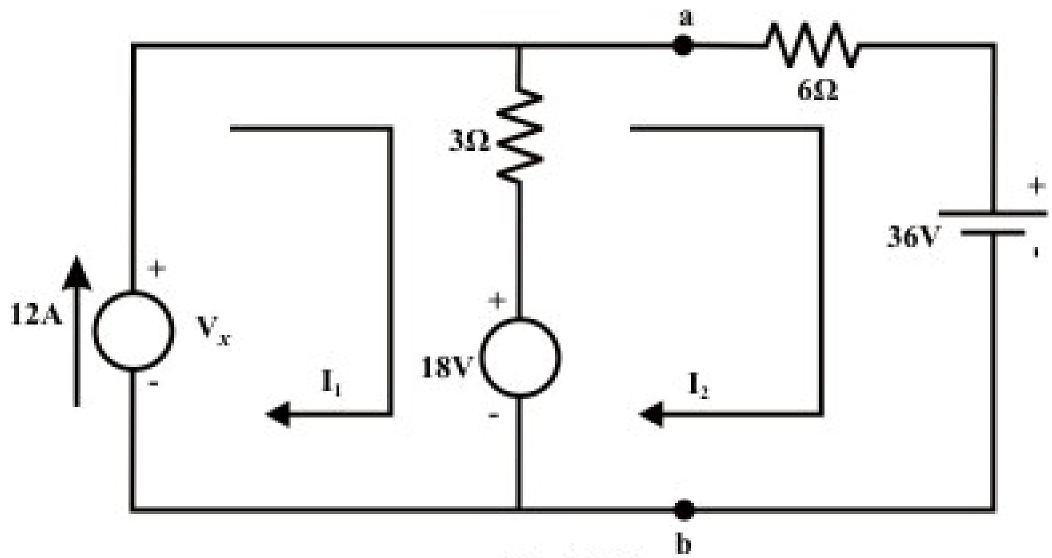


Fig. 4.3(c)

Loop-1: (Write KVL, note $I_1 = 12\,A$)

$V_x - (I_1 - I_2) \times 3 - 18 = 0 \Rightarrow V_x + 3I_2 = 54$ (4.7)

Loop-2: (write KVL)

$18 - (I_2 - I_1) \times 3 - I_2 \times 6 - 36 = 0 \Rightarrow 9I_2 = 18 \Rightarrow I_2 = 2\,A$

Using the value of $I_2 = 2\,A$ in equation (4.7), we get $V_x = 48\,volt$.

Example-L-4.3 Find v_R for the circuit shown in figure 4.4 using 'mesh current method. Calculate the power absorbed or delivered by the sources and all the elements.

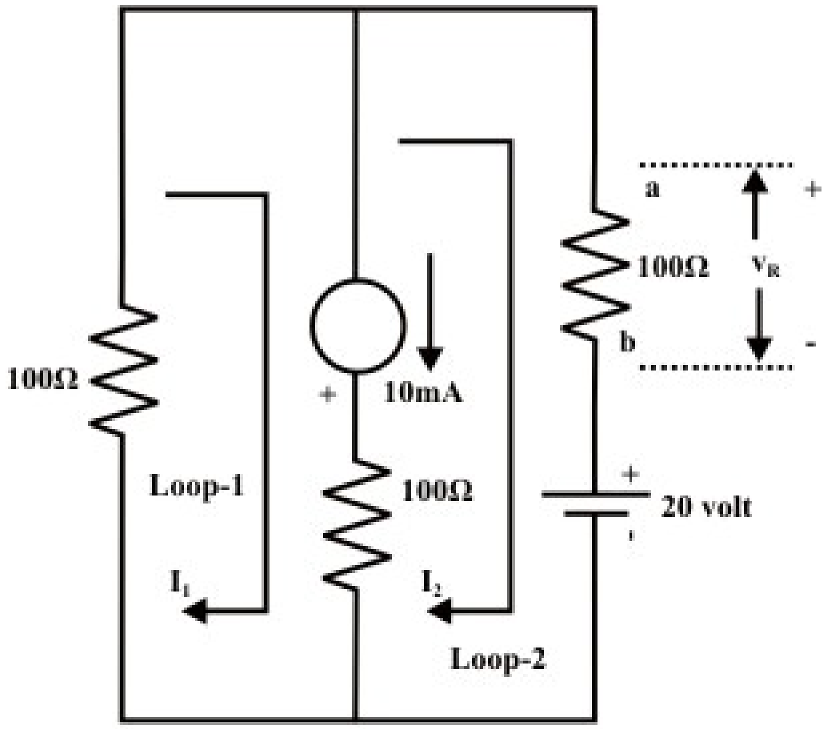


Figure 4.4

Solution: Assume the voltage across the current source is 'v' and the bottom end of current source is marked as positive sign.

For loop No. 1: (KVL equation)

$$v-(I_1-I_2)\times 100-I_1\times 100=0 \Rightarrow v-200\,I_1+100\,I_2=0 \qquad (4.8)$$

It may be noted that from the figure that the current flowing through the $100\,\Omega$ resistor (in the middle branch) is $10\,mA$. More specifically, one can write the following expression

$$I_1-I_2=10\times 10^{-3} \qquad (4.9)$$

For loop No. 2: (KVL equation)

$$-20-(I_2-I_1)\times 100-v-I_2\times 100=0 \Rightarrow v+200\,I_2-100\,I_1=-20 \qquad (4.10)$$

Solving equations (4.8)–(4.10), one can obtained the loop currents as $I_1=-0.095\;=-\,95\,mA$ (-ve sign indicates that the assigned loop current direction is not correct or in other words loop current (I_1) direction is anticlockwise.) and $I_2=-0.105=-105\,mA$ (note, loop current (I_2) direction is anticlockwise). Now the voltage across the $100\,\Omega$ resistor (extreme right branch) is given by $v_R=I_2\times 100=-0.105\times 100=-10.5\,volt$. .This indicates that the resistor terminal (b) adjacent to the voltage source is more positive than the other end of the resistor terminal (a). From equation (4.8) $v = -8.5\,volt$ and this implies that the 'top' end of the current source is more positive than the bottom 'end'.

Power delivered by the voltage source = $20 \times 0.105 = 2.1W$ (note that the current is leaving the positive terminal of the voltage source). On the other hand, the power received or absorbed by the current source = $8.5 \times 0.01 = 0.085W$ (since current entering to the positive terminal (top terminal) of the current source). Power absorbed by the all resistance is given

$= (0.105)^2 \times 100 + (0.095)^2 \times 100 + (10 \times 10^{-3})^2 \times 100 = 2.015W$.

Further one can note that the power delivered ($P_d = 2.1W$) = power absorbed ($P_{ab} = 0.085 + 2.015 = 2.1W$) $= 2.1W$

L.4.3 Test Your Understanding [Marks:50]

T.4.1 To write the Kirchhoff's voltage law equation for a loop, we proceed clockwise around the loop, considering voltage rises into the loop equation as ------- terms and voltage drops as -------- terms. [2]

T.4.2 When writing the Kirchhoff's voltage law equation for a loop, how do we handle the situation when an ideal current source is present around the loop? [2]

T.4.3 When a loop current emerges with a positive value from mathematical solution of the system of equations, what does it mean? What does it mean when a loop current emerges with a negative value? [2]

T.4.4 In mesh current method, the current flowing through a resistor can be computed with the knowledge of ------ loop current and ---------- loop current. [2]

T.4.5 Find the current through 6Ω resistor for the circuit Figure 4.5 using 'mesh current' method and hence calculate the voltage across the current source. [10]

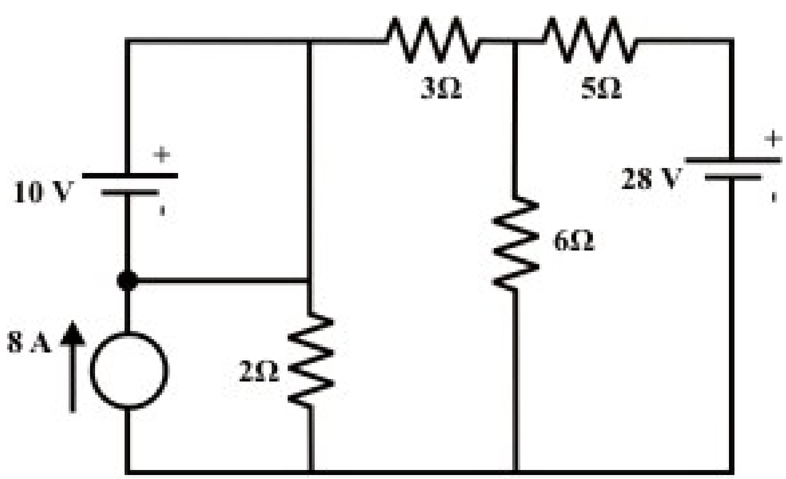


Figure 4.5

(Answer: $3.18\,A$; $13.22\,V$)

T.4.6 For the circuit shown in Figure 4.6, find the current through I_{AB} , I_{AC} , I_{CD} *and* I_{EF} using 'mesh current' method. [12]

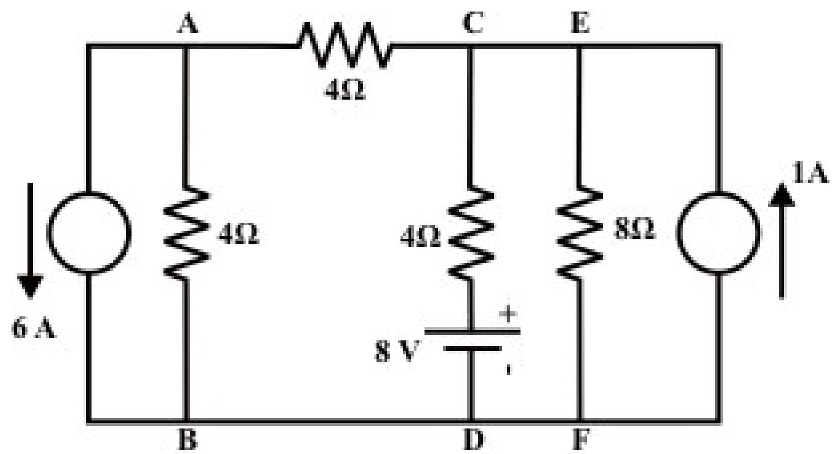


Figure 4.6

(Answer: $I_{AB} = -3\,A; I_{AC} = -3\,A; I_{CD} = -2\,A$ *and* $I_{EF} = 0\,A.$)

T.4.7 Find the current flowing through the $R_L = 1k\Omega$ resistor for the circuit shown in Figure 4.7 using 'mesh current' method. What is the power delivered or absorbed by the independent current source? [10]

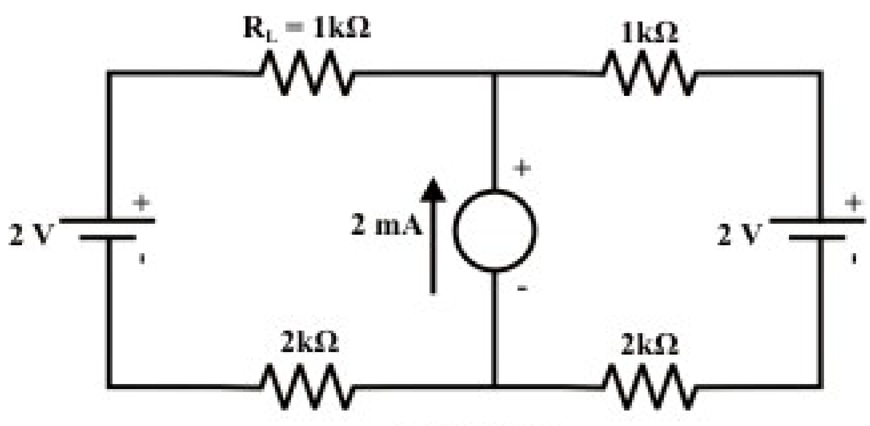


Figure 4.7

(Answer: $1mA;10\,mW$)

T.4.8 Using 'mesh current' method, find the current flowing through $2\,\Omega$ resistor for the circuit shown in Figure 4.8 and hence compute the power consumed by the same $2\,\Omega$ resistor. [10]

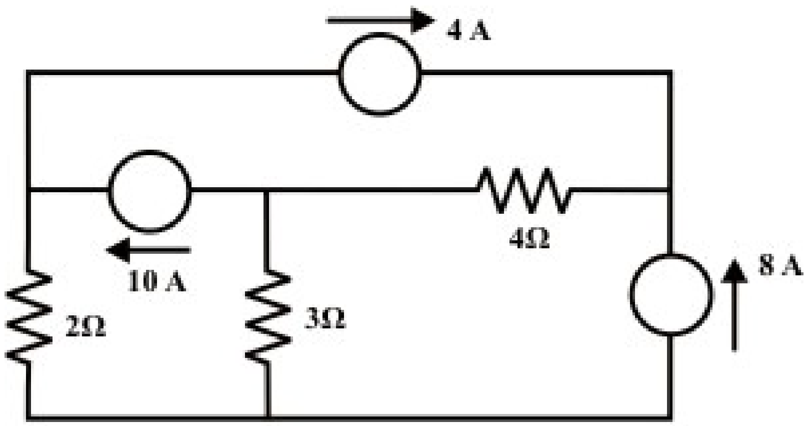


Figure 4.8

(Answer: $6\,A$; $72W$)

Module
2
DC Circuit

Lesson 5

Node-voltage analysis of resistive circuit in the context of dc voltages and currents

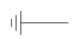
Objectives

- To provide a powerful but simple circuit analysis tool based on Kirchhoff's current law (KCL) only.

L.5.1 Node voltage analysis

In the previous lesson-4, it has been discussed in detail the analysis of a dc network by writing a set of simultaneous algebraic equations (based on KVL only) in which the variables are currents, known as mesh analysis or loop analysis. On the other hand, the node voltage analysis (Nodal analysis) is another form of circuit or network analysis technique, which will solve almost any linear circuit. In a way, this method completely analogous to mesh analysis method, writes KCL equations instead of KVL equations, and solves them simultaneously.

L.5.2 Solution of Electric Circuit Based on Node Voltage Method

In the node voltage method, we identify all the nodes on the circuit. Choosing one of them as the reference voltage (i.e., zero potential) and subsequently assign other node voltages (unknown) with respect to a reference voltage (usually ground voltage taken as zero (0) potential and denoted by (). If the circuit has "n" nodes there are "n-1" node voltages are unknown (since we are always free to assign one node to zero or ground potential). At each of these "n-1" nodes, we can apply KCL equation. The unknown node voltages become the independent variables of the problem and the solution of node voltages can be obtained by solving a set of simultaneous equations.

Let us consider a simple dc network as shown in Figure 5.1 to find the currents through different branches using **"Node voltage"** method.

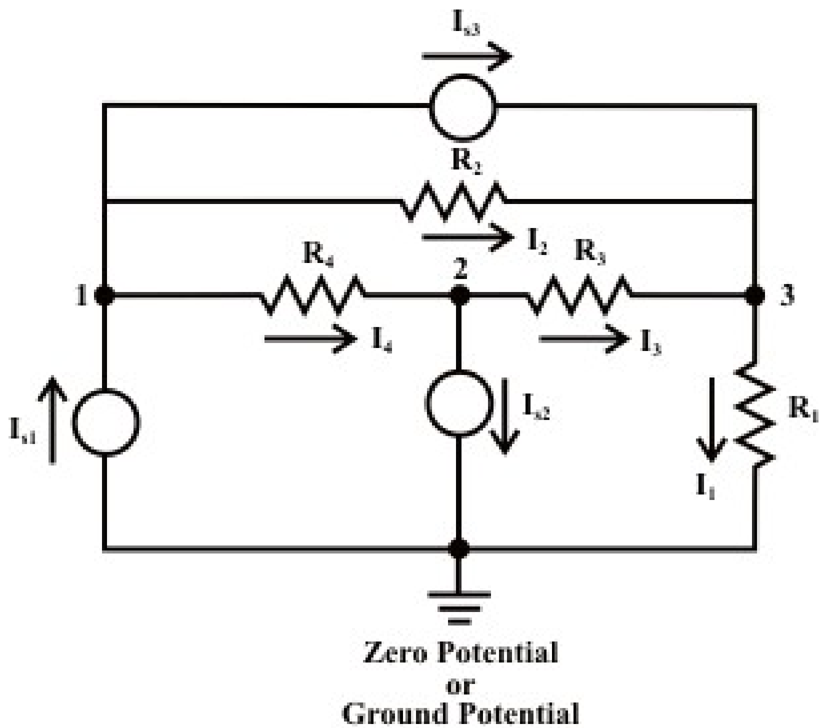


Fig. 5.1

KCL equation at **"Node-1":**

$$I_{s1} - I_{s3} - \left(\frac{V_1 - V_2}{R_4}\right) - \left(\frac{V_1 - V_3}{R_2}\right) = 0 \;\; ; \;\; \rightarrow I_{s1} - I_{s3} - \left(\frac{1}{R_2} + \frac{1}{R_4}\right)V_1 - \left(\frac{1}{R_4}\right)V_2 - \left(\frac{1}{R_2}\right)V_3 = 0$$

$$I_{s1} - I_{s3} = G_{11}V_1 - G_{12}V_2 - G_{13}V_3 \qquad (5.1)$$

where G_{ii} = sum of total conductance (self conductance) connected to Node-1.

KCL equation at **"Node-2":**

$$\left(\frac{V_1 - V_2}{R_4}\right) - \left(\frac{V_2 - V_3}{R_3}\right) - I_{s2} = 0 \;\; ; \;\; \rightarrow -I_{s2} = -\left(\frac{1}{R_4}\right)V_1 + \left(\frac{1}{R_3} + \frac{1}{R_4}\right)V_2 - \left(\frac{1}{R_3}\right)V_3$$

$$-I_{s2} = -G_{21}V_1 + G_{22}V_2 - G_{23}V_3 \qquad (5.2)$$

KCL equation at **"Node-3":**

$$I_{s3} + \left(\frac{V_2 - V_3}{R_3}\right) + \left(\frac{V_1 - V_3}{R_2}\right) - \left(\frac{V_3}{R_1}\right) = 0 \;\; ; \;\; \rightarrow I_{s3} = -\left(\frac{1}{R_2}\right)V_1 - \left(\frac{1}{R_3}\right)V_2 + \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)V_3$$

$$I_{s3} = -G_{31}V_1 - G_{32}V_2 + G_{33}V_3 \qquad (5.3)$$

In general, for the i^{th} Node the KCL equation can be written as

$$\sum I_{ii} = -G_{i1}V_1 - G_{i2}V_2 - \cdots\cdots + G_{ii}V_i - \cdots\cdots - G_{iN}V_N$$

where,

$\sum I_{ii} =$ algebraic sum of all the current sources connected to 'Node-i', $i = 1, 2, \cdots\cdots N$. (Currents entering the node from current source is assigned as +ve sign and the current leaving the node from the current source is assigned as –ve sign).
$G_{ii} =$ the sum of the values of conductance (reciprocal of resistance) connected to the node $'i'$.
$G_{ij} =$ the sum of the values of conductance connected between the nodes $'i'$ *and* $'j'$.
Summarize the steps to analyze a circuit by node voltage method are as follows:

Step-1: Identify all nodes in the circuit. Select one node as the reference node (assign as ground potential or zero potential) and label the remaining nodes as unknown node voltages with respect to the reference node.

Step-2: Assign branch currents in each branch. (The choice of direction is arbitrary).

Step-3: Express the branch currents in terms of node assigned voltages**.**

Step-4: Write the standard form of node equations by inspecting the circuit. (No of node equations = No of nodes (N) – 1).

Step-5: Solve a set of simultaneous algebraic equation for node voltages and ultimately the branch currents.

Remarks:

- Sometimes it is convenient to select the reference node at the bottom of a circuit or the node that has the largest number of branches connected to it.
- One usually makes a choice between a mesh and a node equations based on the least number of required equations.

Example-L-5.1: Find the value of the current I flowing through the battery using 'Node voltage' method.

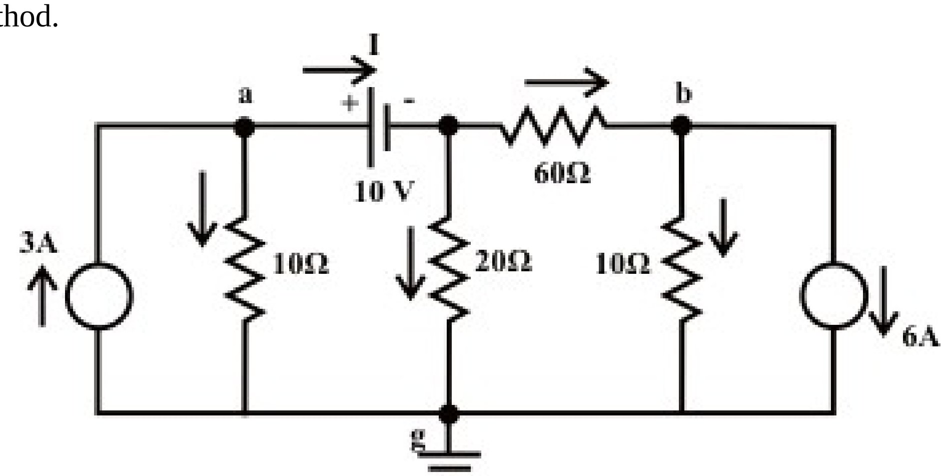


Fig. 5.2

Solution: All nodes are indicated in fig.5.2 and 'Node-g' is selected as reference voltage. If a voltage source is connected directly between the two nodes, the current flowing through the voltage source cannot be determined directly since the source voltage V_S is independent of current. Further to note that the source voltage V_S fixes the voltage between the nodes only. For the present example, the voltage of the central node is known since it is equal to $(V_a - 10)\,volt$.

KCL equation at node-a:

$$3 = \frac{V_a - 0}{10} + I \rightarrow 10I + V_a = 30 \qquad (5.4)$$

KCL equation at node-b:

$$\frac{(V_a - 10) - V_b}{60} = 6 + \frac{V_b - 0}{10} \rightarrow V_a - 7V_b = 370 \qquad (5.5)$$

To solve the equations (5.4)-(5.5), we need one more equation which can be obtained by applying KCL at the central node (note central node voltage is $(V_a - 10)$).

$$I = \frac{V_a - 10}{20} + \frac{(V_a - 10) - V_b}{60} \rightarrow 60I = 4V_a - V_b - 40 \rightarrow I = \frac{(4V_a - V_b - 40)}{60} \qquad (5.6)$$

Substituting the current expression (5.6) in equation (5.4) we get,

$$\frac{(4V_a - V_b - 40)}{6} + V_a = 30 \rightarrow 10V_a - V_b = 220 \qquad (5.7)$$

Equations (5.5) and (5.7) can be solved to find $V_b = -50.43V$ *and* $V_a = 16.99V$.

We can now refer to original circuit (fig.5.2) to find directly the voltage across every element and the current through every element. The value of current flowing through the voltage source can be computed using the equation (5.6) and it is given by $I = 1.307\,A$. Note that the current I (+ve) is entering through the positive terminal of the voltage source and this indicates that the voltage source is absorbing the power, in other words this situation is observed when charging a battery or source.

Example-L-5.2: Find the **current** through 'ab-branch' (I_{ab}) and **voltage (**V_{cg}**)** across the current source using Node-voltage method.

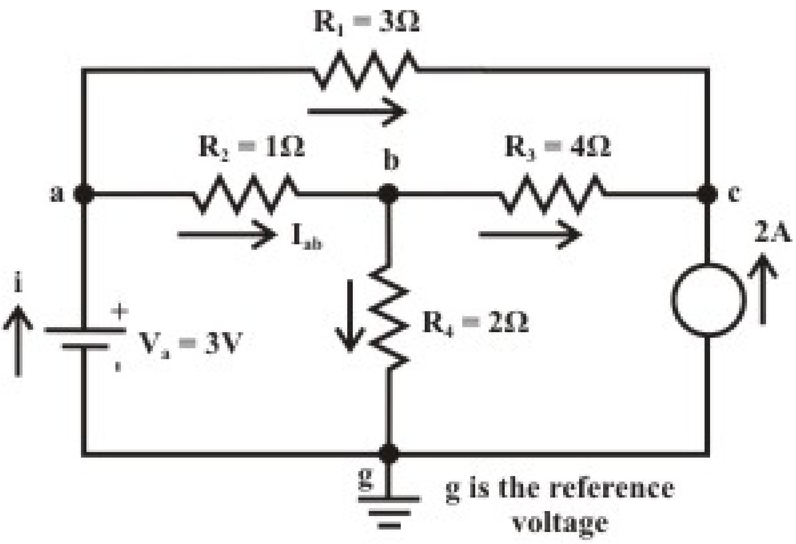


Fig. 5.3

Solution:

KCL at node-a: (note $V_a = 3V$)

$$i = \frac{V_a - V_b}{R_2} + \frac{V_a - V_c}{R_1} \rightarrow i = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)V_a - \frac{1}{R_2}V_b - \frac{1}{R_1}V_c \rightarrow i = 1.33V_a - V_b - \frac{1}{3}V_c \qquad (5.8)$$

KCL at node-b: (note $V_g = 0V$)

$$\frac{V_a - V_b}{R_2} = \frac{V_b - V_c}{R_3} + \frac{V_b - V_g}{R_4} \rightarrow \left(1 + \frac{1}{4} + \frac{1}{2}\right)V_b - V_a - \frac{1}{4}V_c = 0 \qquad (5.9)$$

KCL at node-c:

$$2 + \frac{V_b - V_c}{R_3} + \frac{V_a - V_c}{R_1} = 0 \quad \rightarrow \left(\frac{1}{4} + \frac{1}{3}\right)V_c - \frac{1}{3}V_a - \frac{1}{4}V_b = 2 \qquad (5.10)$$

Using the value of $V_a = 3V$ in equations (5.8)-(5.10) we get the following equations:

$$V_b + \frac{1}{3}V_c = 3.99 - i \qquad (5.11)$$

$$1.75V_b - \frac{1}{4}V_c = 3 \qquad (5.12)$$

$$0.583V_c - \frac{1}{4}V_b = 3 \qquad (5.13)$$

Simultaneous solution of the above three equations, one can get $V_c = 6.26V$, $V_b = 2.61V$ and hence $I_{ab} = \frac{V_a - V_b}{R_2} = \frac{3 - 2.61}{1} = 0.39A$ (current flowing in the direction from ‘a’ to ‘b’).

Example-L-5.3 Determine the current, i shown in fig. 5.4 using node-voltage method --- (a) applying voltage to current source conversion (b) without any source conversion.

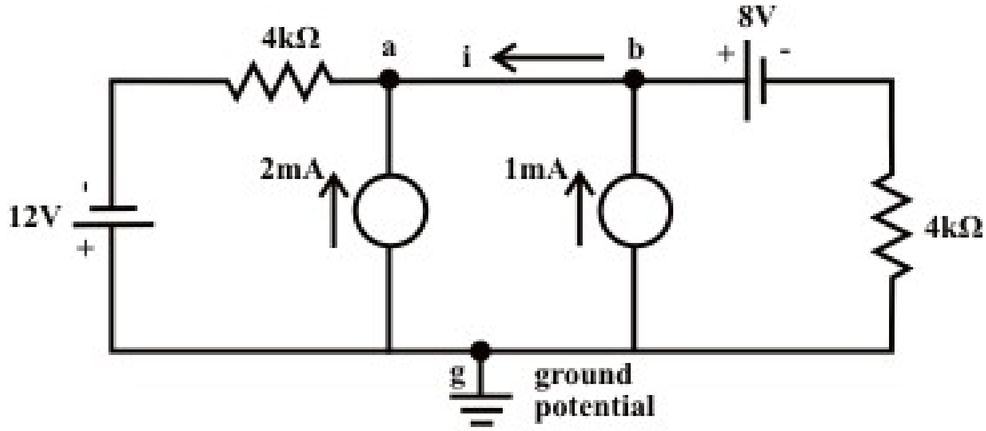


Fig. 5.4

Solution:

Part(a):

In node voltage analysis, sometimes the solution turns out to be very simple while we change all series branches containing voltage sources to their equivalent current sources. On the other hand, we observed in the loop analysis method that the conversion of current source to an equivalent voltage makes the circuit analysis very easy (see example-L4.2) and simple. For this example, both the practical voltage sources (one is left of 'node-a' and other is right of 'node-b') are converted into practical current sources. After transformation, the circuit is redrawn and shown in fig. 5.5(a).

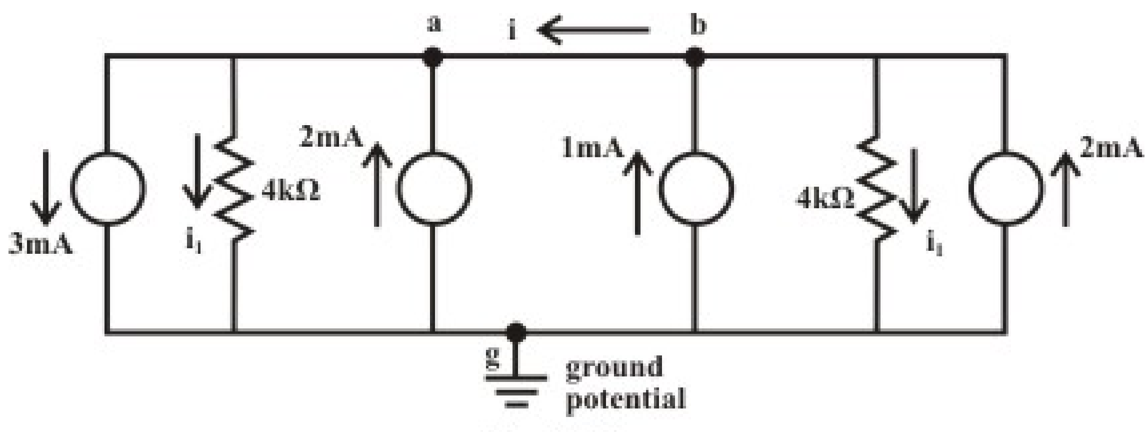


Fig. 5.5(a)

KCL at node 'b':

$$i + i_1 = 2 + 1 = 3 \qquad (5.14)$$

KCL at node 'a':

$$i + 2 = 3 + i_1 \rightarrow i - i_1 = 1 \qquad (5.15)$$

From equations (5.14)-(5.15), one can get $i = 2\,mA$ (current flows from 'b' to 'a') and $i_1 = 1\ mA$.

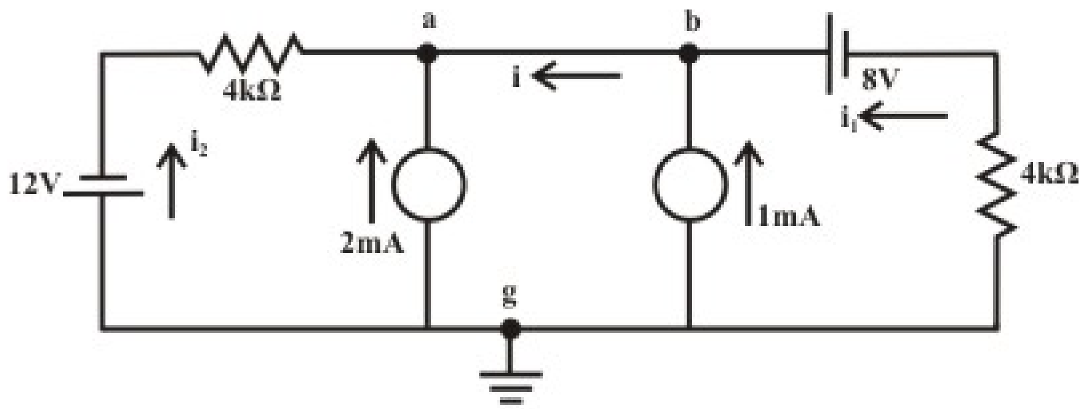


Fig. 5.5(b)

Part(b):
Let us assume i_1 is the current flowing through the $8V$ battery source from 'right to left' and i_2 is the current flowing through the $12V$ battery source from 'bottom to top'(see Fig.5.5(b)).
KCL at node 'b': It is assumed that the current flowing in $4k\Omega$ resistor from bottom to top terminal. This implies that the bottom terminal of $4k\Omega$ resistor is higher potential than the top terminal.(currents are in mA, note $V_a = V_b$)

$$i = 1 + i_1 \rightarrow i = 1 + \frac{0 - (V_a - 8)}{4} \qquad (5.16)$$

KCL at node 'a': (currents are in mA)

$$i + i_2 + 2 = 0 \rightarrow i = -i_2 - 2 \rightarrow i = -\left(\frac{-12 - V_a}{4}\right) - 2 \qquad (5.17)$$

From (6.16) and (5.17), we get $V_a = 4V$ and $i = 2\,mA$ (current flows from 'b' to 'a').

L.5.3 Test Your Understanding [Marks: 50]

T.5.1 Node analysis makes use of Kirchhoff's----------- law just as loop analysis makes use of Kirchhoff's --------- law. [1]

T.5.2 Describe a means of telling how many node voltage equations will be required for a given circuit. [1]

T.5.3 In nodal analysis how are voltage sources handled when (i) a voltage source in a circuit is connected between a non-reference node and the reference node (ii) a voltage source connected between two non-reference nodes in nodal analysis. [4]

T.5.4 A voltage in series with a resistance can be represented by an equivalent circuit that consists of ------------ in parallel with that -------------. [2]

T.5.5 The algebraic sum of the currents ----------- in a node must be equal to the algebraic sum of currents --------- the node. [2]

T.5.6 Apply node voltage analysis to find i_0 and the power dissipated in each resistor in the circuit of Fig.5.6. [10]

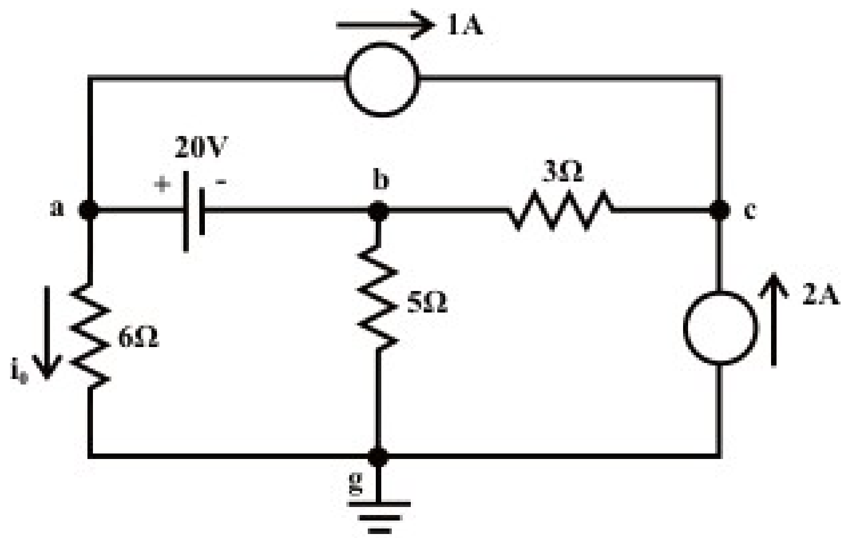


Fig. 5.6

(Ans. $i_0 = 2.73\,A, P_6 = 44.63W,\ P_5 = 3.8W,\ P_3 = 0.333W\,(note \rightarrow V_c = 5.36V,\ V_b = 4.36V)$

T.5.7 For the circuit shown in fig. 5.7, find V_a using the node voltage method. Calculate power delivered or absorbed by the sources. [10]

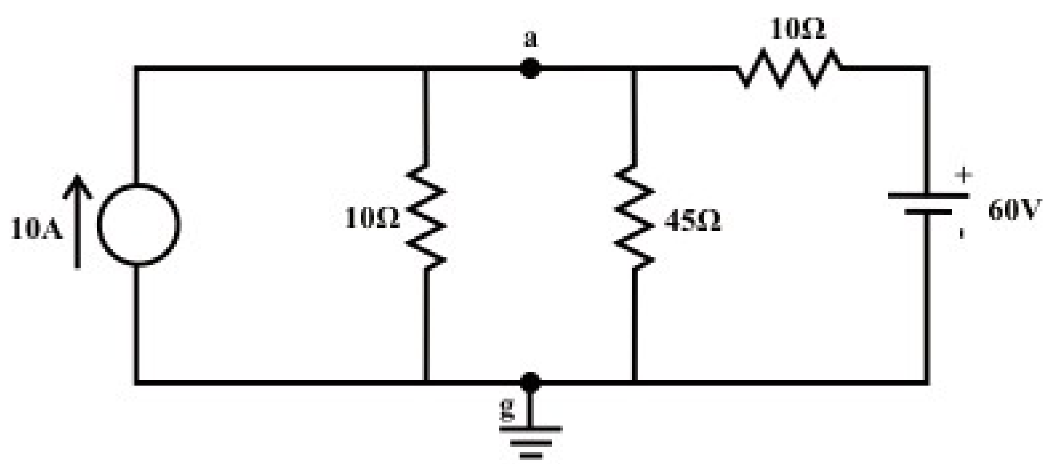


Fig. 5.7

(Answer: $V_a = 72V, P_{(voltage\,source)} = 72W\,(absorbed), P_{(current\,source)} = 201.8W\,(delivered)$)

T.5.8 Using nodal analysis, solve the voltage (V_x) across the $6A$ current source for the circuit of fig. 5.8. Calculate power delivered or absorbed by the sources [10]

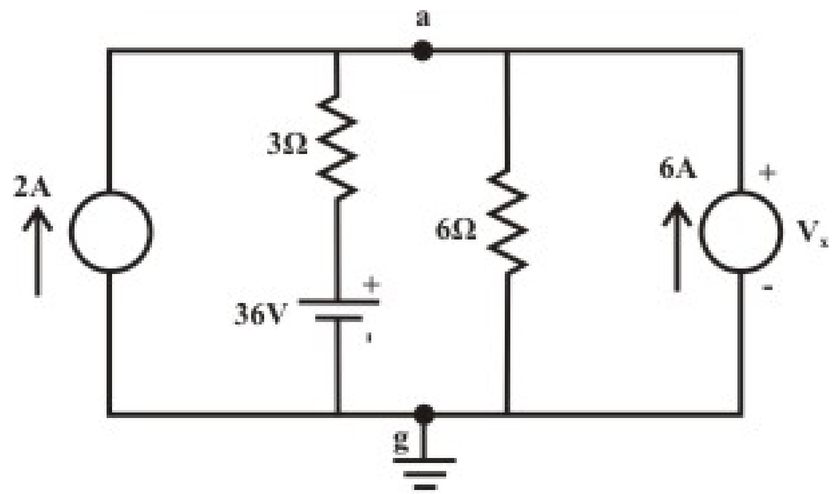


Fig. 5.8

(Answer: $V_a = 60V, P_{(12A\ ideal\ current\ source)} = 720W\,(delivered),$
$P_{(ideal\ voltage\ source)} = 288W\,(absorbed), P_{(6A\ ideal\ current\ source)} = 360W\,(delivered)$)

T.8 Determine the voltage across the 10Ω resistor of fig. 5.9 using nodal analysis. [10]

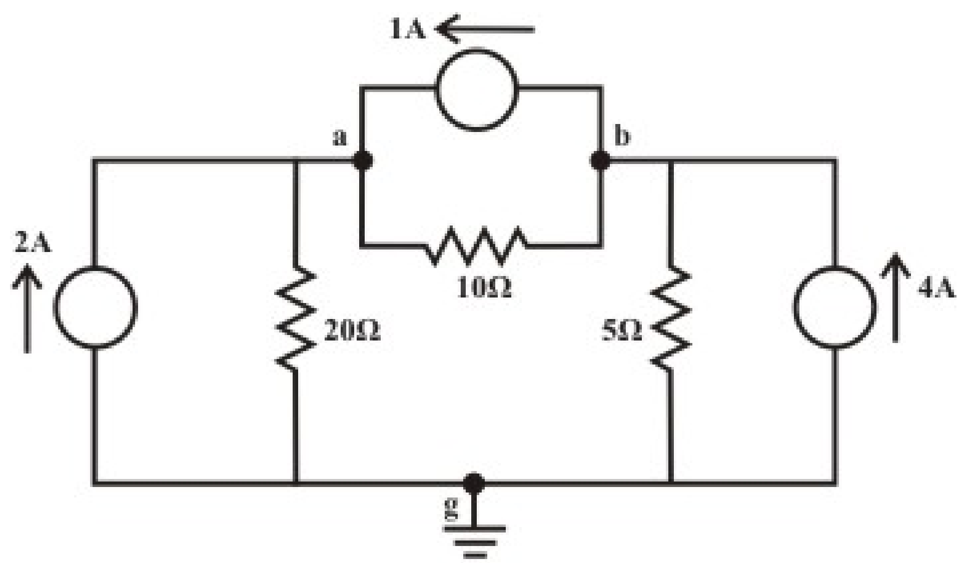


Fig. 5.9

(Answer: $V_{ab} = 34.29V$ (a is higher potential than b)

Module 2

DC Circuit

Lesson 6

Wye (Y) - Delta (Δ) OR Delta (Δ)-Wye (Y) Transformations

Objectives

- A part of a larger circuit that is configured with three terminal network Y (or Δ) to convert into an equivalent Δ (or Y) through transformations.
- Application of these transformations will be studied by solving resistive circuits.

L.6.1 Introduction

There are certain circuit configurations that cannot be simplified by series-parallel combination alone. A simple transformation based on mathematical technique is readily simplifies the electrical circuit configuration. A circuit configuration shown below

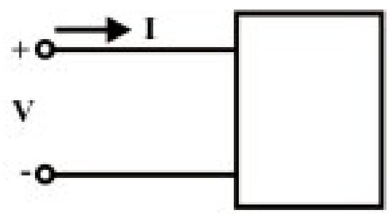


Fig. 6.1(a) One port network

is a general **one-port circuit.** When any voltage source is connected across the terminals, the current entering through any one of the two terminals, equals the current leaving the other terminal. For example, resistance, inductance and capacitance acts as a **one-port.** On the other hand, a **two-port** is a circuit having two pairs of terminals. Each pair behaves as a one-port; current entering in one terminal must be equal to the current living the other terminal.

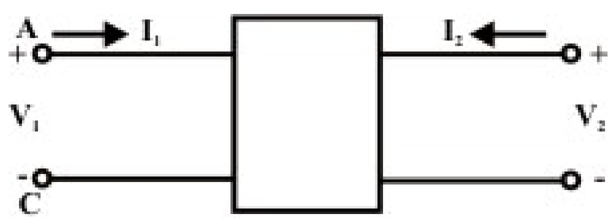


Fig. 6.1(b) Two port network

Fig.6.1.(b) can be described as a four terminal network, for convenience subscript 1 to refer to the variables at the input port (at the left) and the subscript 2 to refer to the variables at the output port (at the right). The most important subclass of two-port networks is the one in which the minus reference terminals of the input and output ports are at the same. This circuit configuration is readially possible to consider the 'π *or* Δ' – network also as a three-terminal network in fig.6.1(c). Another frequently encountered circuit configuration that shown in fig.6.1(d) is approximately refered to as a three-terminal Y connected circuit as well as two-port circuit.

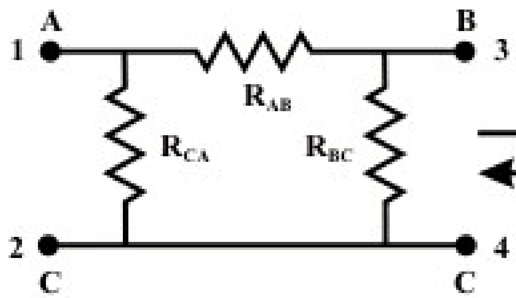


Fig. 6.1 (c)

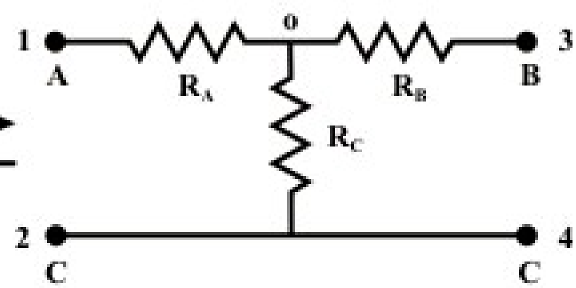


Fig. 6.1 (d)

The name derives from the shape or configuration of the circuit diagrams, which look respectively like the letter Y and the Greek capital letter Δ.

L.6.1.1 Delta (Δ) – Wye (Y) conversion

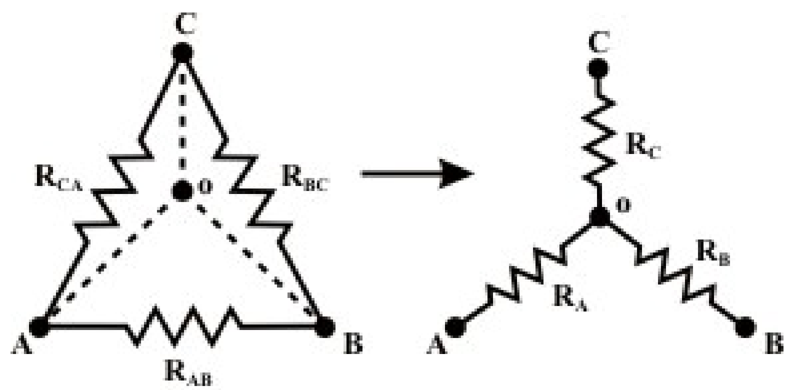


Fig. 6.1 (e)

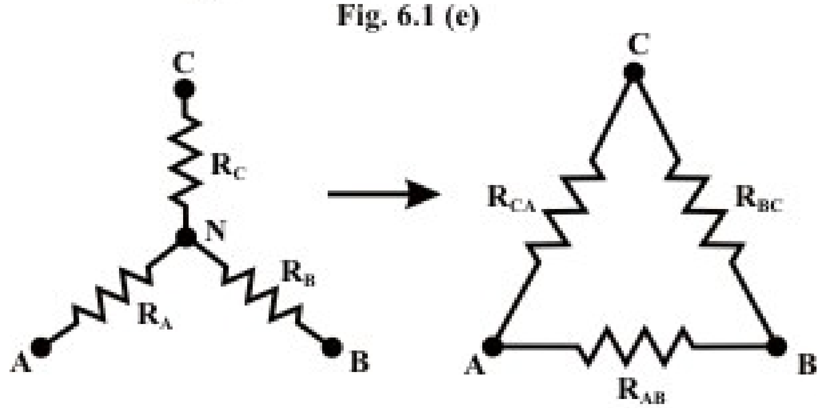


Fig. 6.1 (f)

These configurations may often be handled by the use of a $\Delta - Y$ or $Y - \Delta$ transformation. One of the most basic three-terminal network equivalent is that of three resistors connected in "Delta(Δ)" and in "Wye(Y)". These two circuits identified in fig.L6.1(e) and Fig.L.6.1(f) are sometimes part of a larger circuit and obtained their names from their configurations. These three terminal networks can be redrawn as four-terminal networks as shown in fig.L.6.1(c) and fig.L.6.1(d). We can obtain useful expression for direct transformation or conversion from Δ to Y or Y to Δ by considering that for equivalence the two networks have the same resistance when looked at the similar pairs of terminals.

L.6.2 Conversion from Delta (Δ) to Star or Wye (Y)

Let us consider the network shown in fig.6.1(e) (or fig. $6.1(c)\rightarrow$) and assumed the resistances (R_{AB} , R_{BC} , *and* R_{CA}) in Δ network are known. Our problem is to find the values of R_A, R_B, *and* R_C in Wye (Y) network (see fig.6.1(e)) that will produce the same resistance when measured between similar pairs of terminals. We can write the equivalence resistance between any two terminals in the following form.

Between A & C terminals:

$$R_A + R_C = \frac{R_{CA}\left(R_{AB} + R_{BC}\right)}{R_{AB} + R_{BC} + R_{CA}} \tag{6.1}$$

Between C & B terminals:

$$R_C + R_B = \frac{R_{BA}\left(R_{AB} + R_{CA}\right)}{R_{AB} + R_{BC} + R_{CA}} \tag{6.2}$$

Between B & A terminals:

$$R_B + R_A = \frac{R_{AB}\left(R_{CA} + R_{BC}\right)}{R_{AB} + R_{BC} + R_{CA}} \tag{6.3}$$

By combining above three equations, one can write an expression as given below.

$$R_A + R_B + R_C = \frac{R_{AB}R_{BC} + R_{BC}R_{CA} + R_{CA}R_{AB}}{R_{AB} + R_{BC} + R_{CA}} \tag{6.4}$$

Subtracting equations (6.2), (6.1), and (6.3) from (6.4) equations, we can write the express for unknown resistances of Wye (Y) network as

$$R_A = \frac{R_{AB}R_{CA}}{R_{AB} + R_{BC} + R_{CA}} \tag{6.5}$$

$$R_B = \frac{R_{AB}R_{BC}}{R_{AB} + R_{BC} + R_{CA}} \tag{6.6}$$

$$R_C = \frac{R_{BC}R_{CA}}{R_{AB} + R_{BC} + R_{CA}} \tag{6.7}$$

L.6.2.1 Conversion from Star or Wye (Y) to Delta (Δ)

To convert a **Wye** ($\underline{Y}$) to a **Delta** (Δ), the relationships R_{AB}, R_{BC}, *and* R_3 must be obtained in terms of the **Wye** ($\underline{Y}$) resistances R_A, R_B, *and* R_C (referring to fig.6.1 (f)). Considering the Y connected network, we can write the current expression through R_A resistor as

$$I_A = \frac{(V_A - V_N)}{R_A} \quad \text{(for } Y \text{ network)} \tag{6.8}$$

Appling KCL at 'N' for Y connected network (assume A, B, C terminals having higher potential than the terminal N) we have,

$$\frac{(V_A - V_N)}{R_A} + \frac{(V_B - V_N)}{R_B} + \frac{(V_C - V_N)}{R_C} = 0 \quad \Rightarrow V_N\left(\frac{1}{R_A} + \frac{1}{R_B} + \frac{1}{R_C}\right) = \left(\frac{V_A}{R_A} + \frac{V_B}{R_B} + \frac{V_C}{R_C}\right)$$

or, $$\Rightarrow V_N = \frac{\left(\frac{V_A}{R_A} + \frac{V_B}{R_B} + \frac{V_C}{R_C}\right)}{\left(\frac{1}{R_A} + \frac{1}{R_B} + \frac{1}{R_C}\right)} \tag{6.9}$$

For Δ-network (see fig.6.1.(f)),
Current entering at terminal A = Current leaving the terminal 'A'

$$I_A = \frac{V_{AB}}{R_{AB}} + \frac{V_{AC}}{R_{AC}} \quad \text{(for } \Delta \text{ network)} \tag{6.10}$$

From equations (6.8) and (6.10),

$$\frac{(V_A - V_N)}{R_A} = \frac{V_{AB}}{R_{AB}} + \frac{V_{AC}}{R_{AC}}$$

Using the V_N expression in the above equation, we get

$$\frac{\left(V_A - \frac{\left(\frac{V_A}{R_A} + \frac{V_B}{R_B} + \frac{V_C}{R_C}\right)}{\left(\frac{1}{R_A} + \frac{1}{R_B} + \frac{1}{R_C}\right)}\right)}{R_A} = \frac{V_{AB}}{R_{AB}} + \frac{V_{AC}}{R_{AC}} \quad \Rightarrow \quad \frac{\left(\frac{\left(\frac{V_A - V_B}{R_B} + \frac{V_A - V_C}{R_C}\right)}{\left(\frac{1}{R_A} + \frac{1}{R_B} + \frac{1}{R_C}\right)}\right)}{R_A} = \frac{V_{AB}}{R_{AB}} + \frac{V_{AC}}{R_{AC}}$$

or $$\frac{\left(\frac{\left(\frac{V_{AB}}{R_B} + \frac{V_{AC}}{R_C}\right)}{\left(\frac{1}{R_A} + \frac{1}{R_B} + \frac{1}{R_C}\right)}\right)}{R_A} = \frac{V_{AB}}{R_{AB}} + \frac{V_{AC}}{R_{AC}} \tag{6.11}$$

Equating the coefficients of V_{AB} and V_{AC} in both sides of eq.(6.11), we obtained the following relationship.

$$\frac{1}{R_{AB}} = \frac{1}{R_A R_B\left(\frac{1}{R_A}+\frac{1}{R_B}+\frac{1}{R_C}\right)} \Rightarrow R_{AB} = R_A + R_B + \frac{R_A R_B}{R_C} \tag{6.12}$$

$$\frac{1}{R_{AC}} = \frac{1}{R_A R_C\left(\frac{1}{R_A}+\frac{1}{R_B}+\frac{1}{R_C}\right)} \Rightarrow R_{AC} = R_A + R_C + \frac{R_A R_C}{R_B} \tag{6.13}$$

Similarly, I_B for both the networks (see fig.61(f)) are given by

$$I_B = \frac{\left(V_B - V_N\right)}{R_B} \quad \text{(for } Y \text{ network)}$$

$$I_B = \frac{V_{BC}}{R_{BC}} + \frac{V_{BA}}{R_{BA}} \quad \text{(for } \Delta \text{ network)}$$

Equating the above two equations and using the value of V_N (see eq.(6.9), we get the final expression as

$$\frac{\left(\frac{\left(\frac{V_{BC}}{R_C}+\frac{V_{BA}}{R_A}\right)}{\left(\frac{1}{R_A}+\frac{1}{R_B}+\frac{1}{R_C}\right)}\right)}{R_B} = \frac{V_{BC}}{R_{BC}} + \frac{V_{BA}}{R_{BA}}$$

Equating the coefficient of V_{BC} in both sides of the above equations we obtain the following relation

$$\frac{1}{R_{BC}} = \frac{1}{R_B R_C\left(\frac{1}{R_A}+\frac{1}{R_B}+\frac{1}{R_C}\right)} \Rightarrow R_{BC} = R_B + R_C + \frac{R_B R_C}{R_A} \tag{6.14}$$

When we need to transform a Delta (Δ) network to an equivalent Wye (Y) network, the equations (6.5) to (6.7) are the useful expressions. On the other hand, the equations (6.12) – (6.14) are used for Wye (Y) to Delta (Δ) conversion.

Observations

In order to note the symmetry of the transformation equations, the Wye (Y) and Delta (Δ) networks have been superimposed on each other as shown in fig. 6.2.

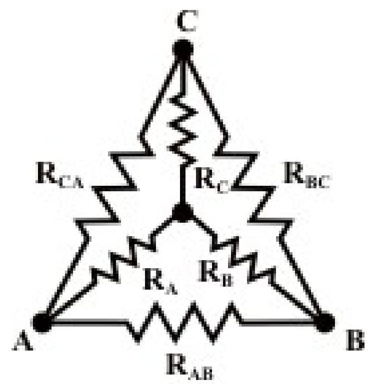


Fig. 6.2

- The equivalent star (Wye) resistance connected to a given terminal is equal to the product of the two Delta (Δ) resistances connected to the same terminal divided by the sum of the Delta (Δ) resistances (see fig. 6.2).
- The equivalent Delta (Δ) resistance between two-terminals is the sum of the two star (Wye) resistances connected to those terminals plus the product of the same two star (Wye) resistances divided by the third star (Wye (Y)) resistance (see fig.6.2).

L.6.3 Application of Star (Y) to Delta (Δ) or Delta (Δ) to Star (Y) Transformation

Example: L.6.1 Find the value of the voltage source (V_S) that delivers 2 Amps current through the circuit as shown in fig.6.3.

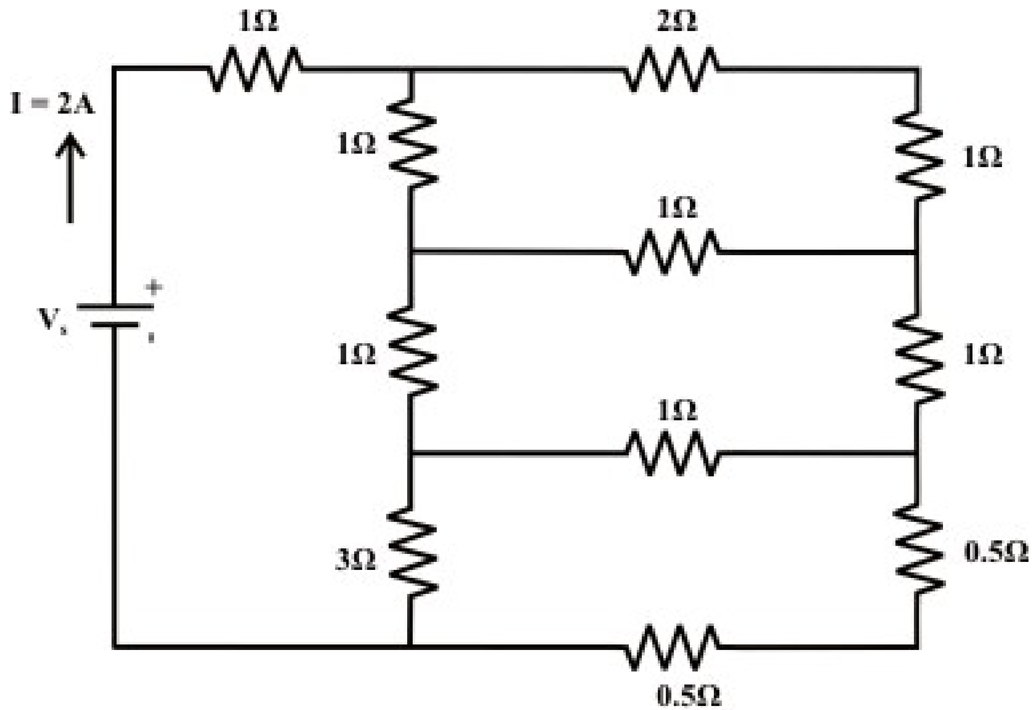
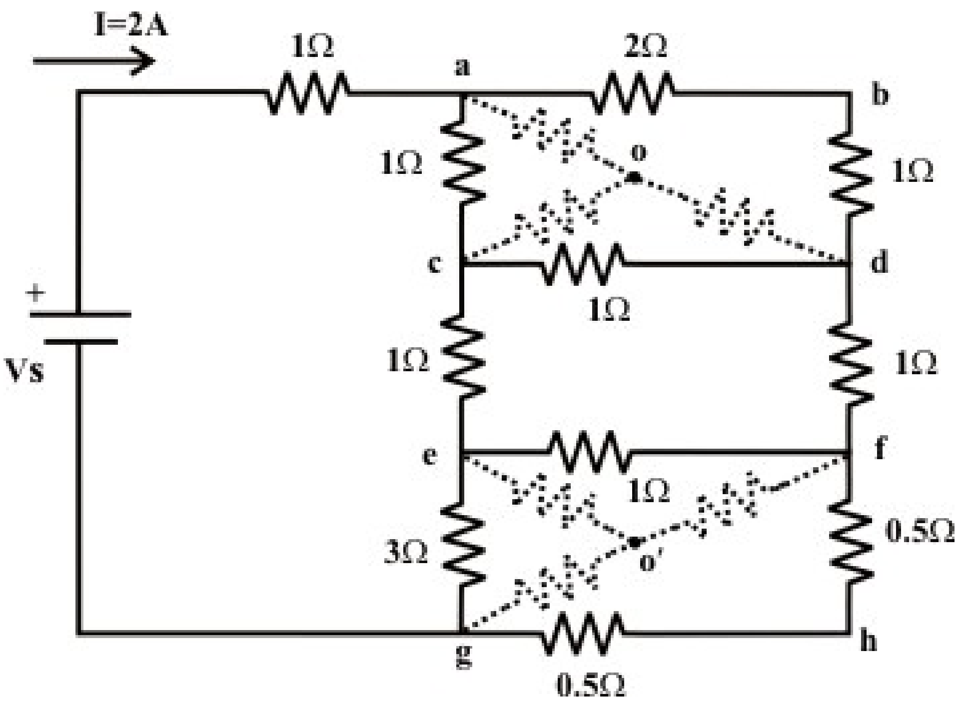


Fig. 6.3

Solution:



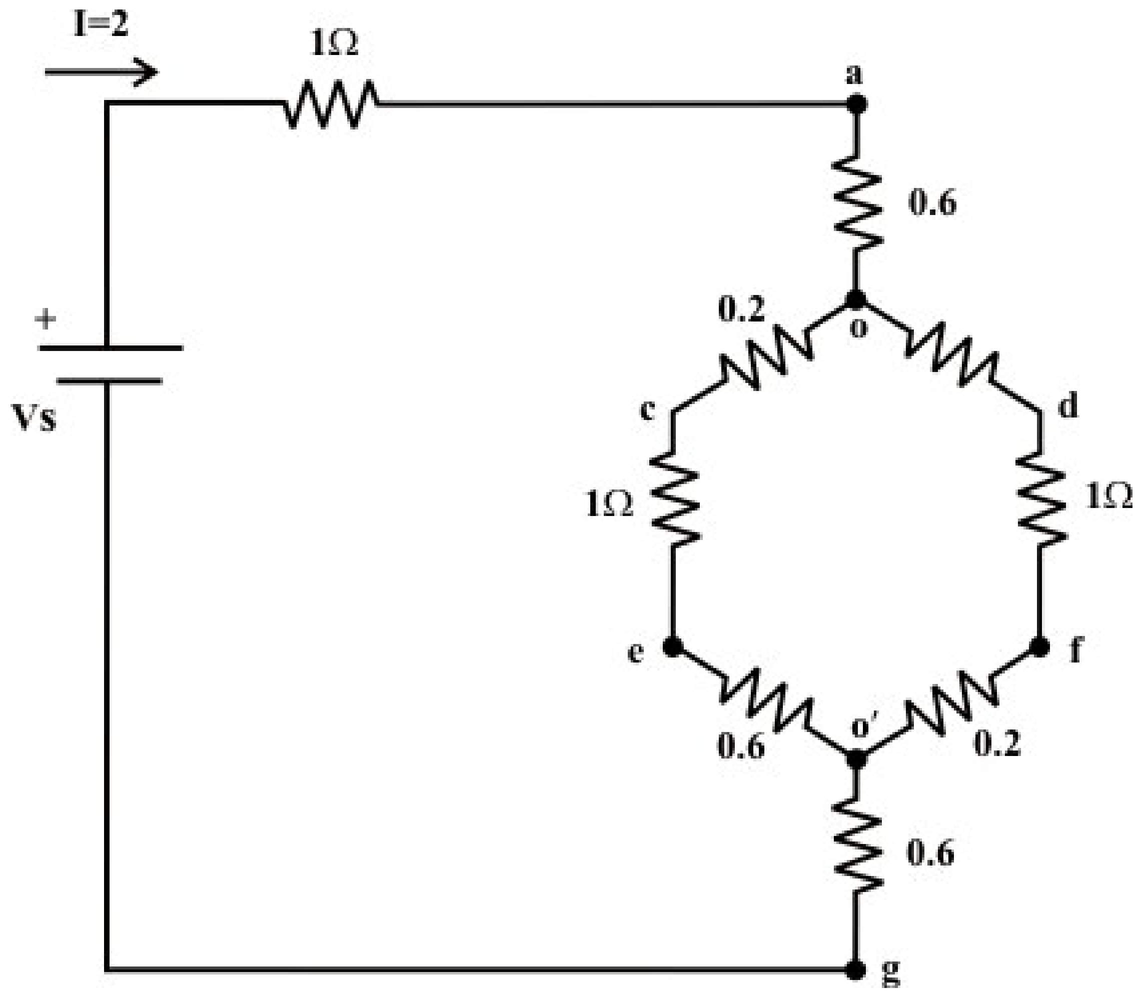
Convert the three terminals Δ-network (a-c-d & e-f-g) into an equivalent Y-connected network. Consider the Δ-connected network 'a-c-d' and the corresponding equivalent Y-connected resistor values are given as

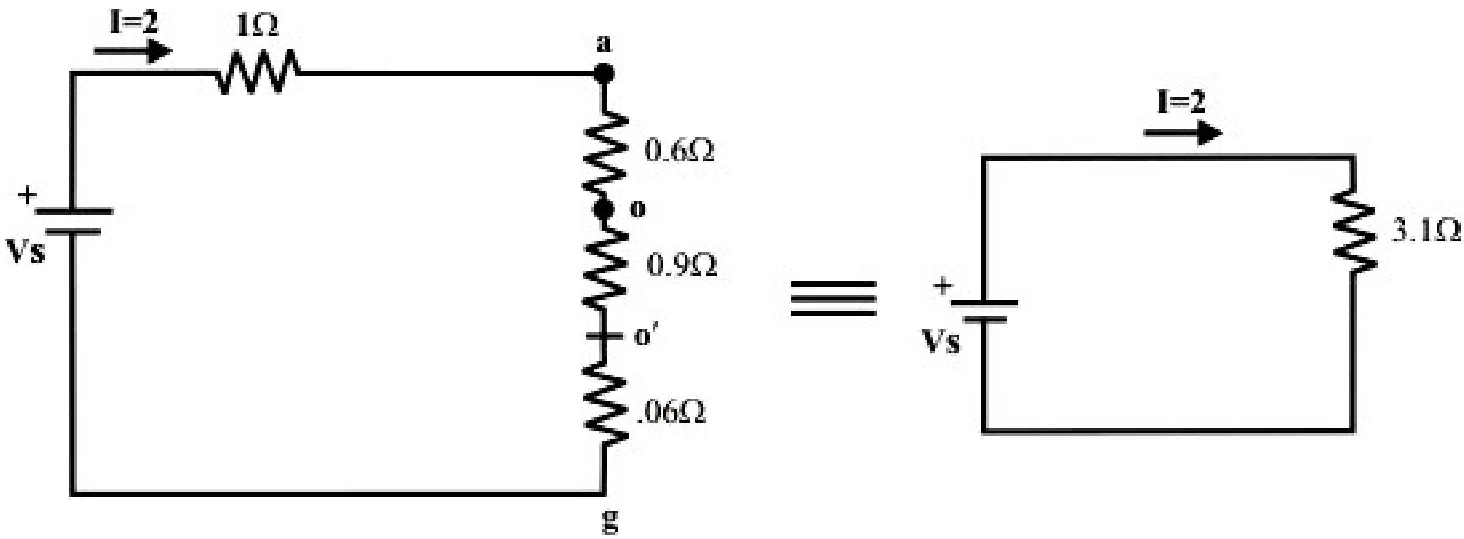
$$R_{ao}=\frac{3\times1}{5}=0.6\,\Omega;\; R_{co}=\frac{1\times1}{5}=0.2\,\Omega;\; R_{do}=\frac{3\times1}{5}=0.6\,\Omega$$

Similarly, for the Δ-connected network 'e-f-g' the equivalent the resistances of Y-connected network are calculated as

$$R_{eo'}=\frac{3\times1}{5}=0.6\Omega;\; R_{go'}=\frac{3\times1}{5}=0.6\,\Omega; R_{fo'}=\frac{1\times1}{5}=0.2\,\Omega$$

Now the original circuit is redrawn after transformation and it is further simplified by applying series-parallel combination formula.





The source V_s that delivers $2A$ current through the circuit can be obtained as $V_s = I \times 3.2 = 2 \times 3.1 = 6.2 Volts$.

Example: L.6.2 Determine the equivalent resistance between the terminals $A \; and \; B$ of network shown in fig.6.4 (a).

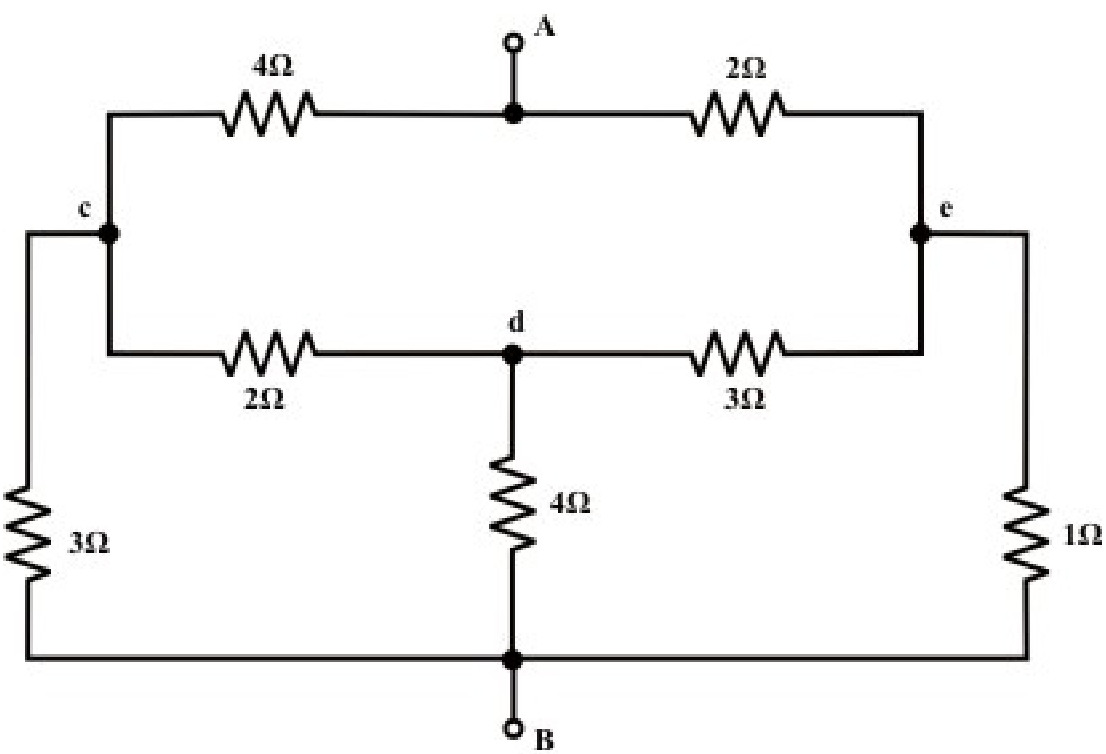


Fig. 6.4 (a)

Solution:

A 'Δ' is substituted for the 'Y' between points c, d, and e as shown in fig.6.4(b); then unknown resistances value for Y *to* Δ transformation are computed below.

$$R_{cB} = 2+4+\frac{2\times4}{3} = 8.66\,\Omega;\; R_{eB} = 3+4+\frac{4\times3}{2} = 13\,\Omega;\; R_{ce} = 2+3+\frac{2\times3}{4} = 6.5\,\Omega$$

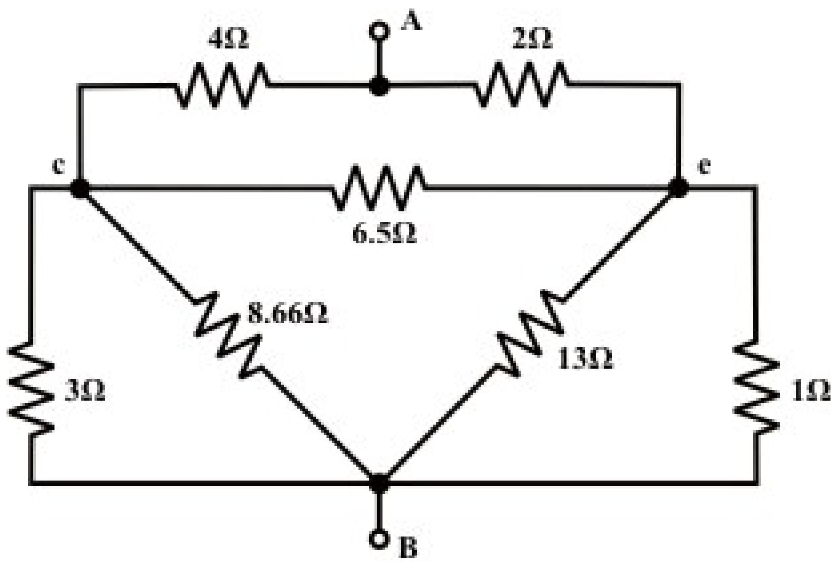


Fig. 6.4 (b)

Next we transform 'Δ'connected 3-terminal resistor to an equivalent 'Y' connected network between points 'A'; 'c' and 'e' (see fig.6.4(b)) and the corresponding Y connected resistances value are obtained using the following expression. Simplified circuit after conversion is shown in fig. 6.4(c).

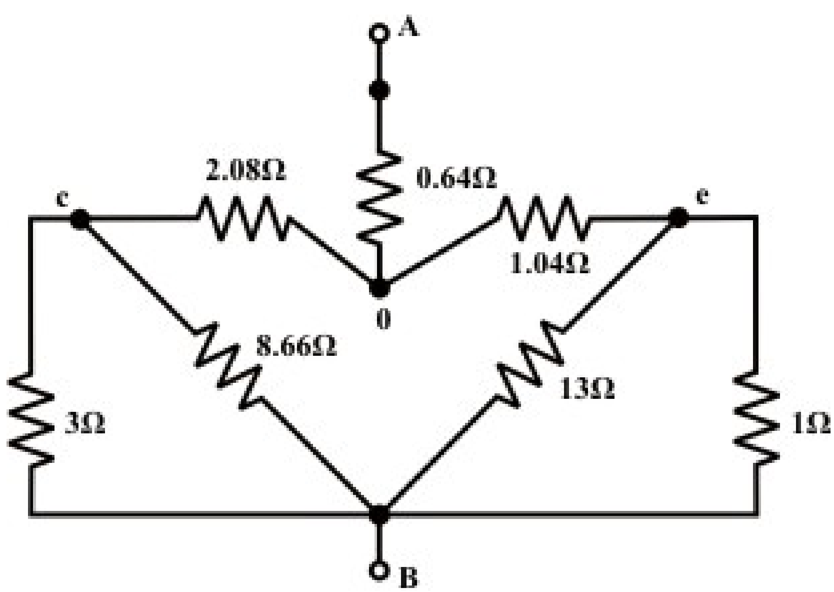


Fig. 6.4 (c)

$R_{Ao} = \frac{4\times 2}{4+2+6.5} = 0.64\,\Omega$; $R_{co} = \frac{4\times 6.5}{4+2+6.5} = 2.08\,\Omega$; $R_{eo} = \frac{6.5\times 2}{4+2+6.5} = 1.04\,\Omega$;

The circuit shown in fig.6.5(c) can further be reduced by considering two pairs of parallel branches $3\,||8.66$ *and* $13||1$ and the corresponding simplified circuit is shown in fig.6.4(d).

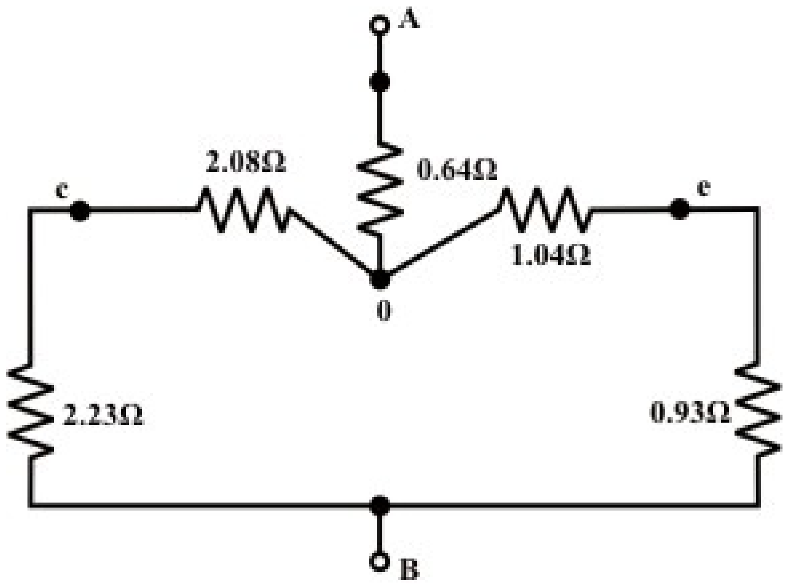
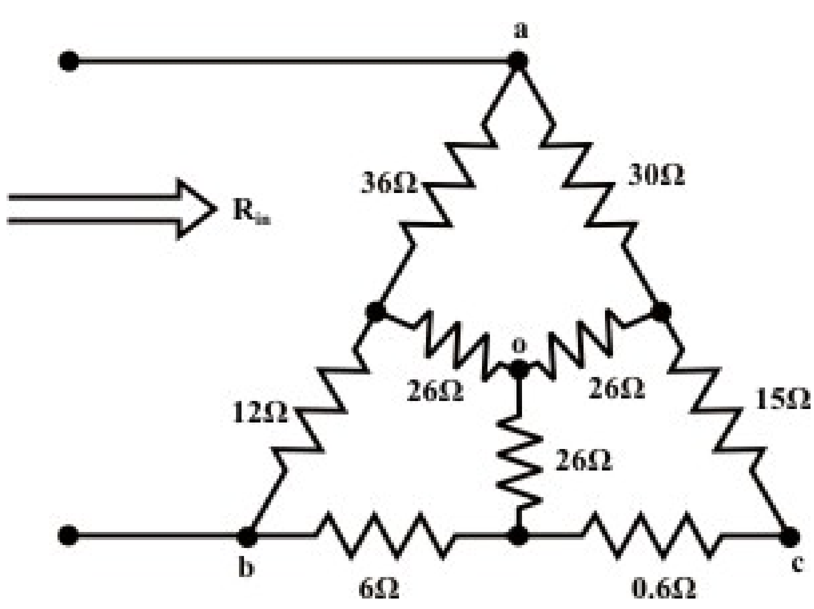


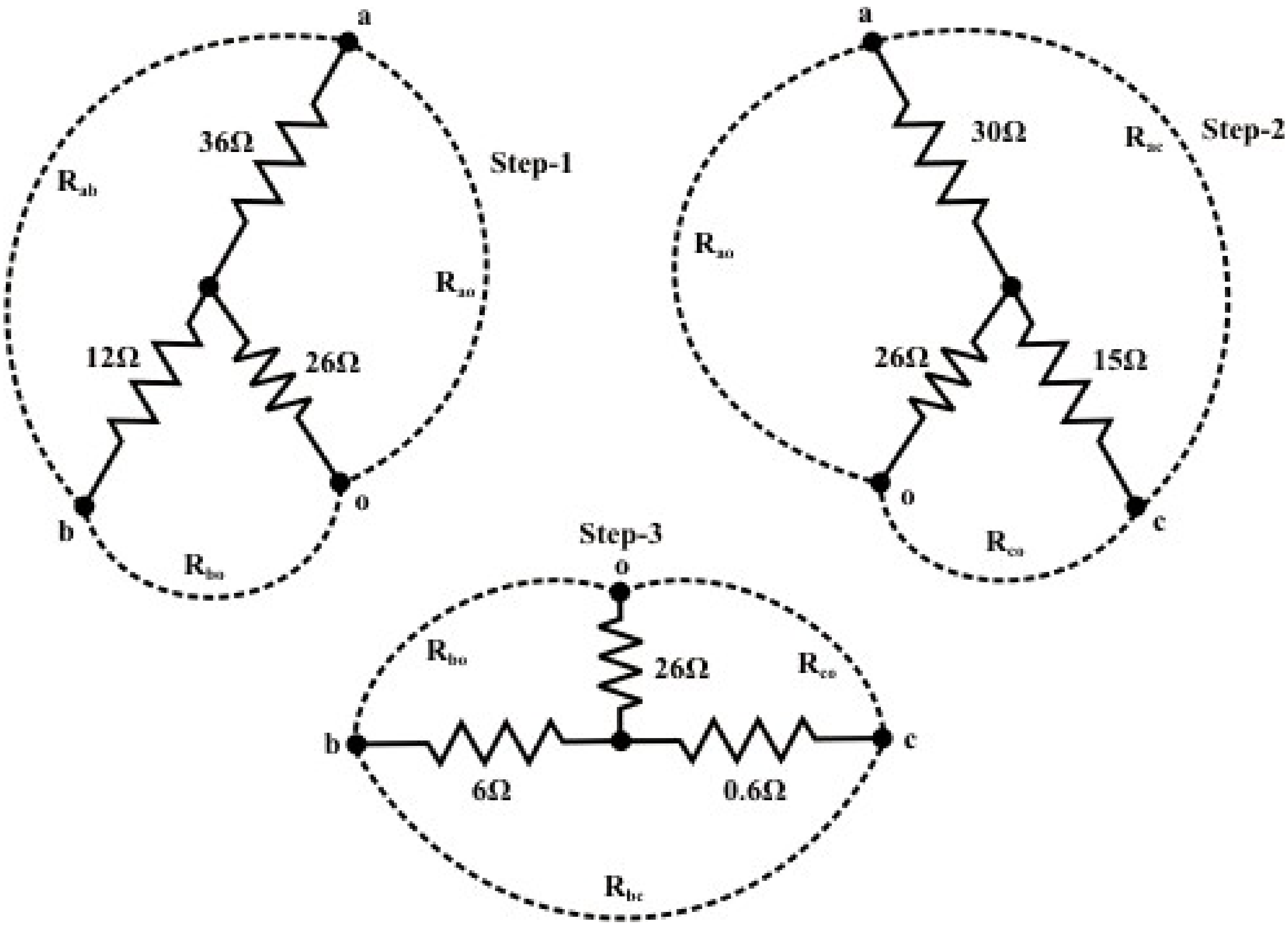
Fig. 6.4 (d)

Now one can find the equivalent resistance between the terminals 'A' and 'B' as $R_{AB} = (2.23+2.08)\,||\,(1.04+0.93)+0.64 = 2.21\,\Omega$.

Example: L.6.3 Find the value of the input resistance R_{in} of the circuit.



Solution:



Y connected network formed with the terminals a-b-o is transformed into Δ connected one and its resistance values are given below.

$R_{ab} = 36 + 12 + \frac{36 \times 12}{26} = 64.61\Omega$; $\quad R_{bo} = 12 + 26 + \frac{26 \times 12}{36} = 46.66\Omega$

$R_{ao} = 26 + 36 + \frac{26 \times 36}{12} = 140\Omega$

Similarly, Y connected networks formed with the terminals 'b-c-o' and 'c-a-o' are transformed to Δ connected networks.

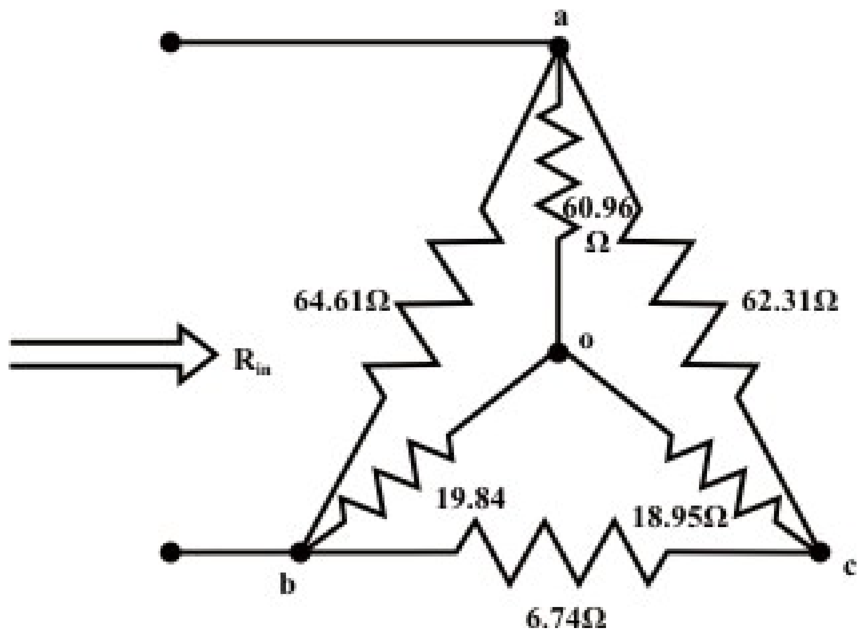
$R_{bc} = 6 + 0.6 + \frac{6 \times 0.6}{26} = 6.738\Omega$; $R_{co} = 0.6 + 26 + \frac{0.6 \times 26}{6} = 29.2\Omega$

$$R_{b0} = 6 + 26 + \frac{6 \times 26}{0.6} = 34.60\Omega$$

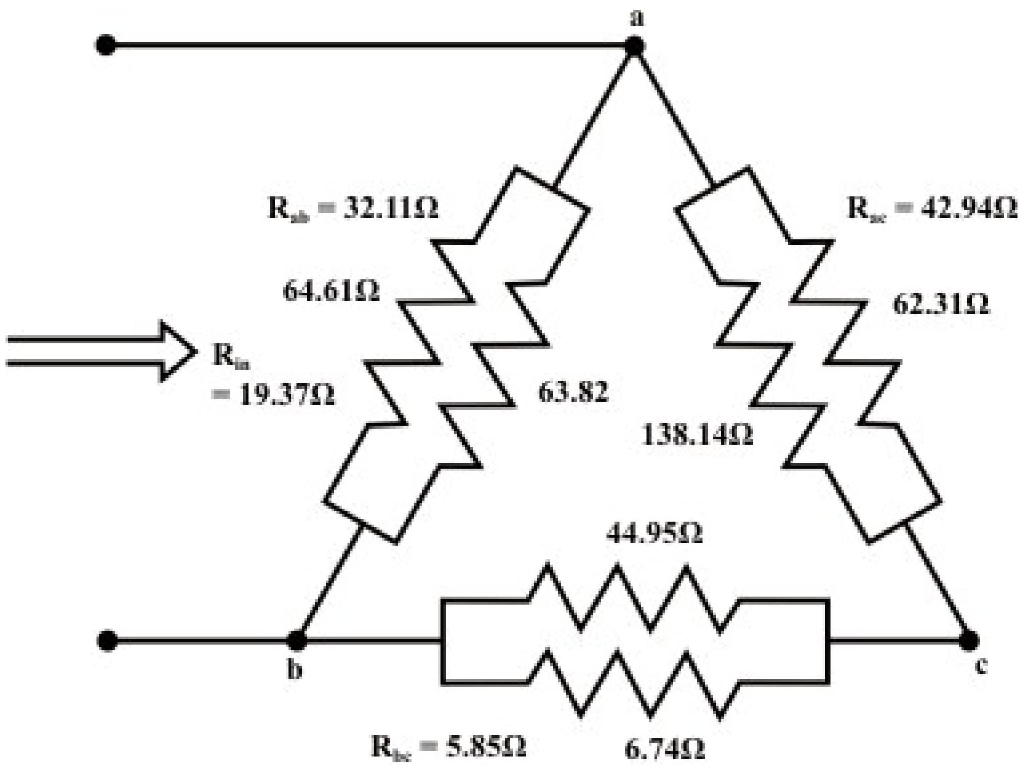
and, $R_{co} = 15 + 26 + \frac{15 \times 26}{30} = 54.00\Omega$; $\quad R_{ao} = 30 + 26 + \frac{30 \times 26}{15} = 108\Omega$

$$R_{ac} = 30 + 15 + \frac{30 \times 15}{26} = 62.31\Omega$$

Note that the two resistances are connected in parallel ($140 \| 108$) between the points 'a' and 'o'. Similarly, between the points 'b' and 'o' two resistances are connected in parallel ($46.66 \| 34.6$) and resistances $54.0\,\Omega$ and $29.2\,\Omega$ are connected in parallel between the points 'c' and 'o'.



Now Y connected network formed with the terminal 'a-b-c' is converted to equivalent Δ connected network.



Now, $$R_{in} = \frac{(R_{ac} + R_{bc})R_{ab}}{R_{ab} + R_{bc} + R_{ca}} = 19.37\Omega$$

Remarks:

- If the $\Delta\ or\ Y$ connected network consists of inductances (assumed no mutual coupling forms between the inductors) then the same formula can be used for $Y\ to\ \Delta\ or\ \Delta\ to\ Y$ conversion (see in detail 3-phase ac circuit analysis in Lesson-19).
- On the other hand, the $\Delta\ or\ Y$ connected network consists of capacitances can be converted to an equivalent $Y\ or\ \Delta$ network provided the capacitance value is replaced by its reciprocal in the conversion formula (see in detail 3-phase ac circuit analysis in Lesson-19).

Example: L.6.4 Find the equivalent inductance R_{eq} of the network (see fig.6.5(a)) at the terminals 'a' & 'b' using $Y-\Delta\ \&\ \Delta-Y$ transformations.

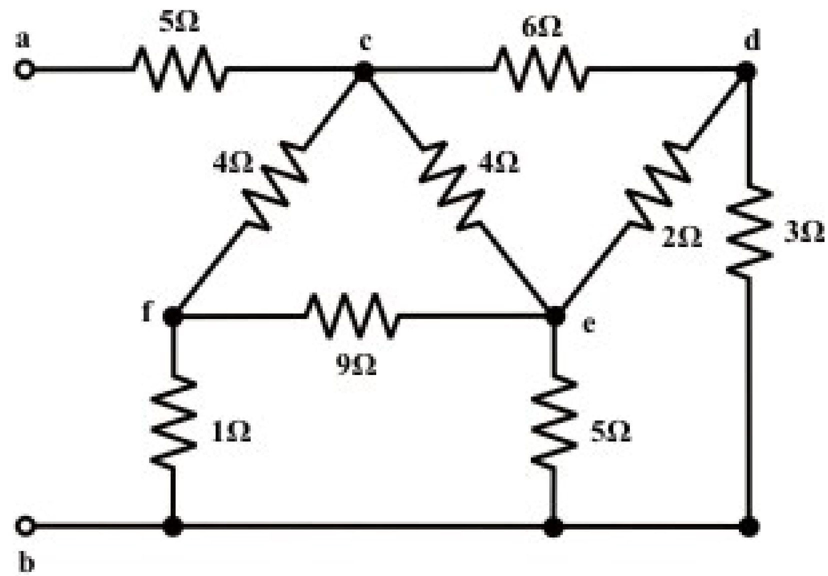


Fig. 6.5(a)

Solution: Convert the three terminals (c-d-e) Δ network (see fig.6.5(a)) comprising with the resistors to an equivalent Y-connected network using the following $\Delta - Y$ conversion formula.

$$R_{co} = \frac{6\times 4}{12} = 2\,\Omega;\; R_{do} = \frac{6\times 2}{12} = 1\,\Omega;\; and\; R_{eo} = \frac{2\times 4}{12} = 0.666\,\Omega$$

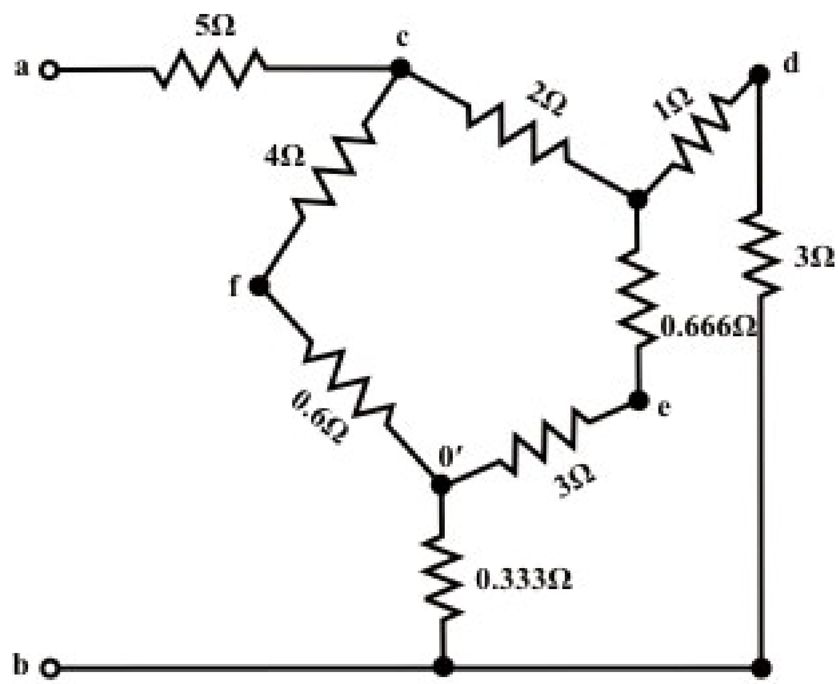


Fig. 6.5(b)

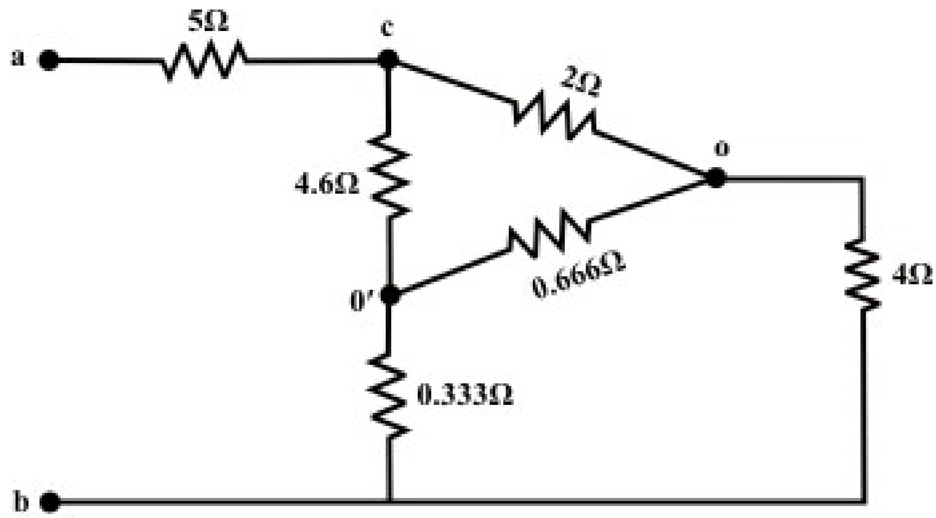


Fig. 6.5(c)

Similarly, the Δ-connected network (f-e-b) is converted to an equivalent Y-connected Network.

$$R_{fo'} = \frac{1\times 9}{15} = 0.6\,\Omega;\; R_{eo'} = \frac{5\times 9}{15} = 3\,\Omega;\; and\;\; R_{bo'} = \frac{1\times 5}{15} = 0.333\Omega$$

After the $\Delta - Y$ conversions, the circuit is redrawn and shown in fig.6.5(b). Next the series-parallel combinations of resistances reduces the network configuration in more simplified form and it is shown in fig.6.5(c). This circuit (see fig.6.5(c)) can further be simplified by transforming Y connected network comprising with the three resistors ($2\,\Omega$, $4\,\Omega$, and $3.666\,\Omega$) to a Δ-connected network and the corresponding network parameters are given below:

$$R_{co'} = 2 + 3.666 + \frac{2\times 3.666}{4} = 7.5\,\Omega;\; R_{cb} = 2 + 4 + \frac{2\times 4}{3.666} = 8.18\,\Omega;$$

$$and\;\; R_{bo'} = 4 + 3.666 + \frac{4\times 3.666}{2} = 15\,\Omega$$

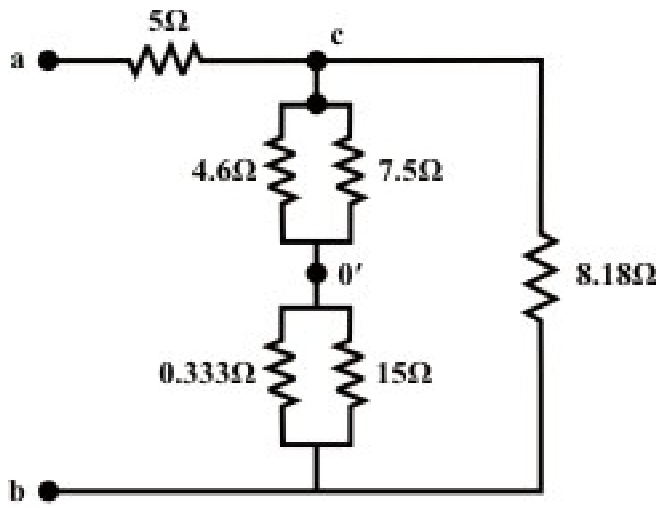


Fig. 6.5(d)

Simplified form of the circuit is drawn and shown in fig.6.5(d) and one can easily find out the equivalent resistance R_{eq} between the terminals 'a' and 'b' using the series-parallel formula. From fig.6.5(d), one can write the expression for the total equivalent resistance R_{eq} at the terminals 'a' and 'b' as

$$R_{eq} = 5 + \left[\left(4.6 \,\|\, 7.5\right) + \left(0.333 \,\|\, 15\right)\right] \| \, 8.18$$
$$= 5 + \left[2.85 + 0.272\right] \| \, 8.18 = 5 + \left(3.122 \,\|\, 8.18\right)$$
$$= 7.26\,\Omega$$

L.6.3 Test Your Understanding [Marks: 40]

T.1 Apply $Y - \Delta$ *or* $\Delta - Y$ transformations only to find the value of the Current I that drives the circuit as shown in fig.6.6. [8]

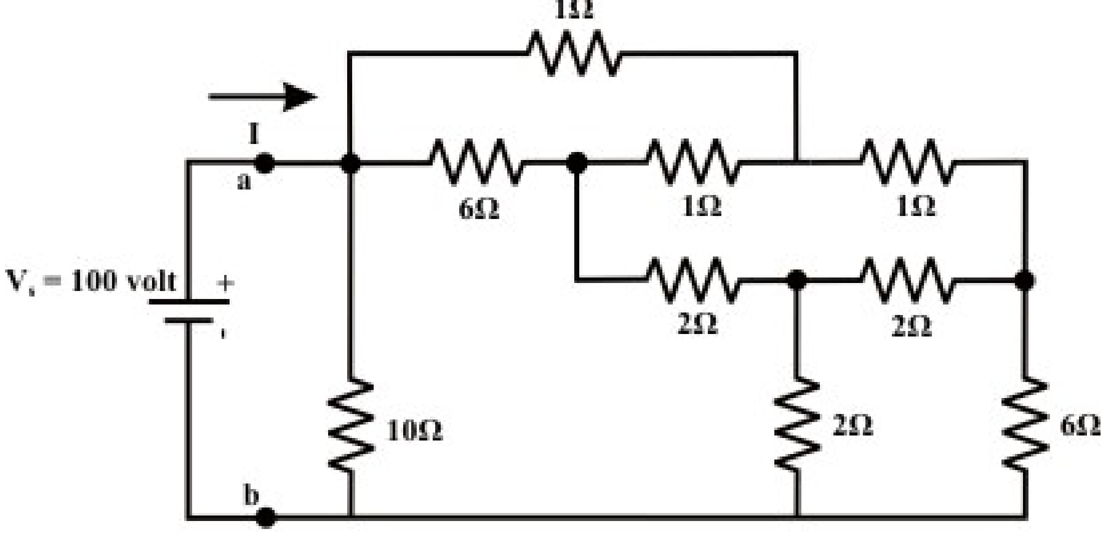


Fig. 6.6

(ans: 10.13Ω)

T.2 Find the current I through 4Ω resistor using $Y-\Delta$ or $\Delta-Y$ transformation technique only for the circuit shown in fig.6.7. [10]

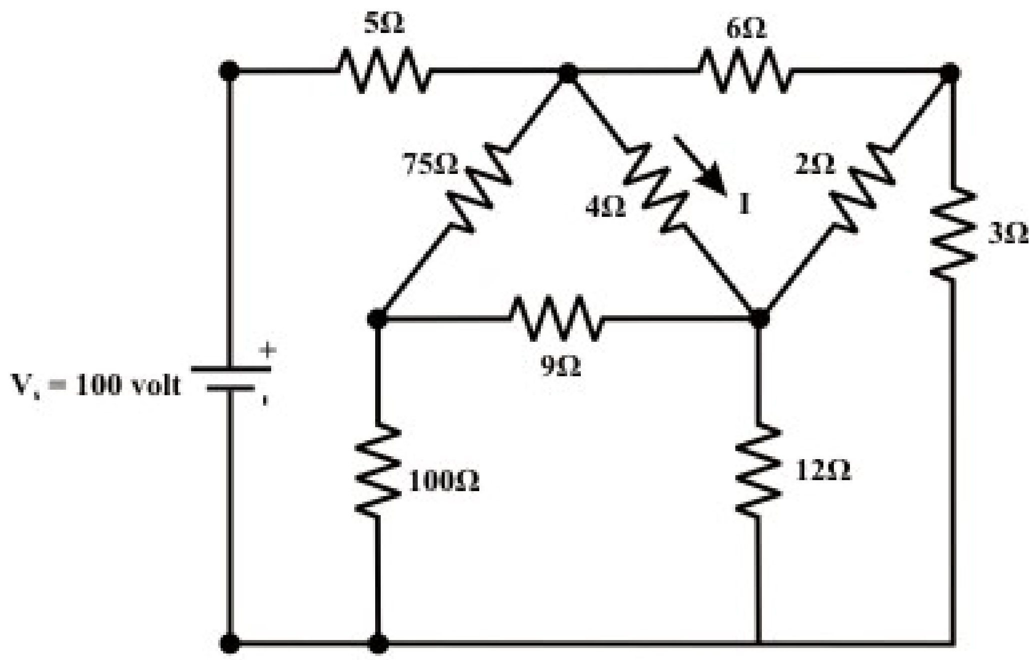


Fig. 6.7

(ans: $7.06\,A$)

T.3 For the circuit shown in fig.6.8, find R_{eq} without performing any conversion. [4]

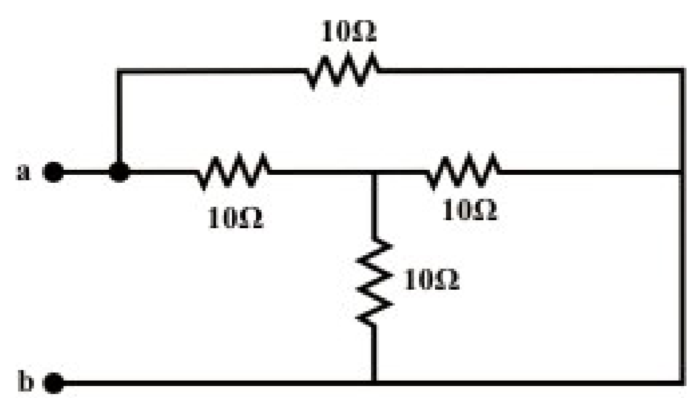


Fig. 6.8

(Ans. $6\ \Omega$)

T.4 For the circuit shown in fig.6.9, calculate the equivalent inductance R_{eq} for each circuit and justify your answer conceptually. [6]

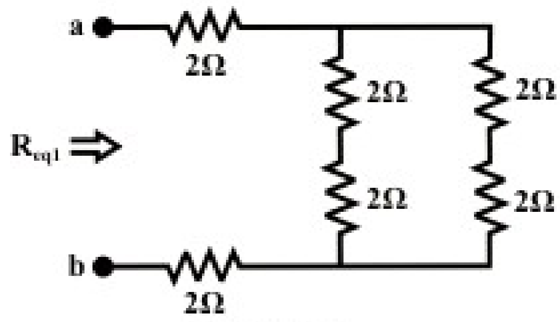


Fig. 6.9(a)

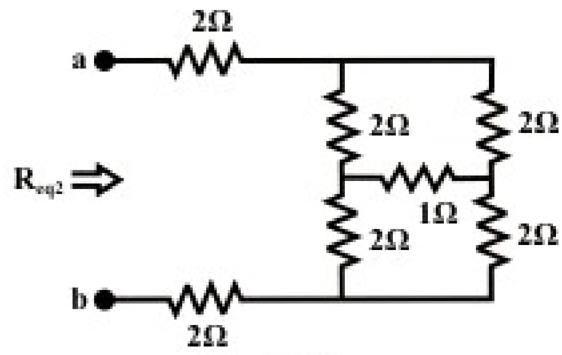


Fig. 6.9(b)

(ans. $R_{eq_1} = R_{eq_2}$)

T.5 Find the value of R_{eq} for the circuit of fig.6.10 when the switch is open and when the switch is closed. [4]

(Ans. $R_{eq} = 8.75\Omega$; $R_{eq} = 7.5\Omega$)

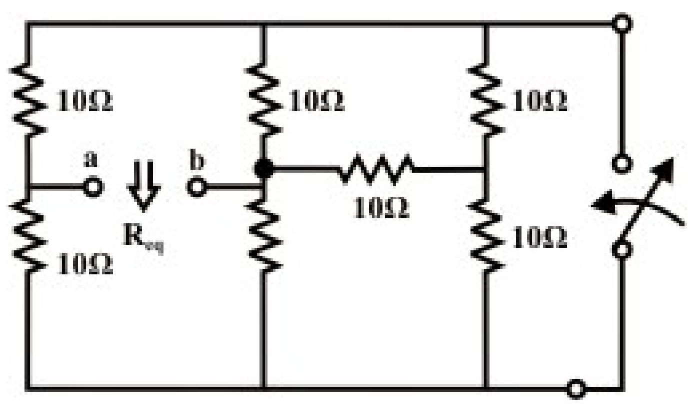


Fig. 6.10

T.6 For the circuit shown in fig.6.11, find the value of the resistance 'R' so that the equivalent capacitance between the terminals 'a' and b' is $20.57\,\Omega$. [6]

(Ans. $30\,\Omega$)

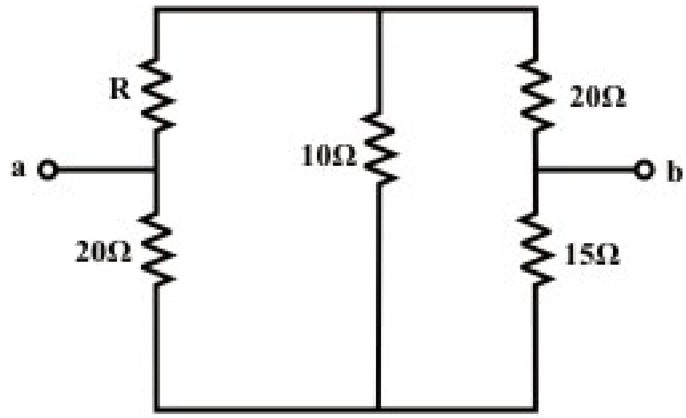


Fig. 6.11

T.7 $Y-\Delta$ *or* $\Delta-Y$ conversion is often useful in reducing the ------------ of a resistor network ---------- to the beginning nodal or mesh analysis. [1]

T.8 Is it possible to find the current through a branch or to find a voltage across the branch using $Y-\Delta/\Delta-Y$ conversions only? If so, justify your answer. [1]

Module 2

DC Circuit

Lesson 7

Superposition Theorem in the context of dc voltage and current sources acting in a resistive network

Objectives

- Statement of superposition theorem and its application to a resistive d.c network containing more than one source in order to find a current through a branch or to find a voltage across the branch.

L.7.1 Introduction

If the circuit has more than one independent (voltage and/or current) sources, one way to determine the value of variable (voltage across the resistance or current through a resistance) is to use nodal or mesh current methods as discussed in detailed in lessons 4 and 5. Alternative method for any linear network, to determine the effect of each independent source (whether voltage or current) to the value of variable (voltage across the resistance or current through a resistance) and then the total effects simple added. This approach is known as the superposition. In lesson-3, it has been discussed the properties of a linear circuit that satisfy (i) homogeneity property [response of output due to input= $\alpha\, u(t)$ equals to α times the response of output due to input= $u(t)$, $S(\alpha\, u(t))$ = $\alpha\, S(u(t))$ for all α; and $u(t)$= input to the system] (ii) additive property [that is the response of $u_1(t)+u_2(t)$ equals the sum of the response of $u_1(t)$ and the response of $u_2(t)$, $S(u_1(t)+u_2(t))$ = $S(u_1(t))+S(u_2(t))$]. Both additive and multiplicative properties of a linear circuit help us to analysis a complicated network. The principle of superposition can be stated based on these two properties of linear circuits.

L.7.1.1 Statement of superposition theorem

In any linear bilateral network containing two or more independent sources (voltage or current sources or combination of voltage and current sources), the resultant current / voltage in any branch is the algebraic sum of currents / voltages caused by each independent sources acting along, with all other independent sources being replaced meanwhile by their respective internal resistances.

Superposition theorem can be explained through a simple resistive network as shown in fig.7.1 and it has two independent practical voltage sources and one practical current source.

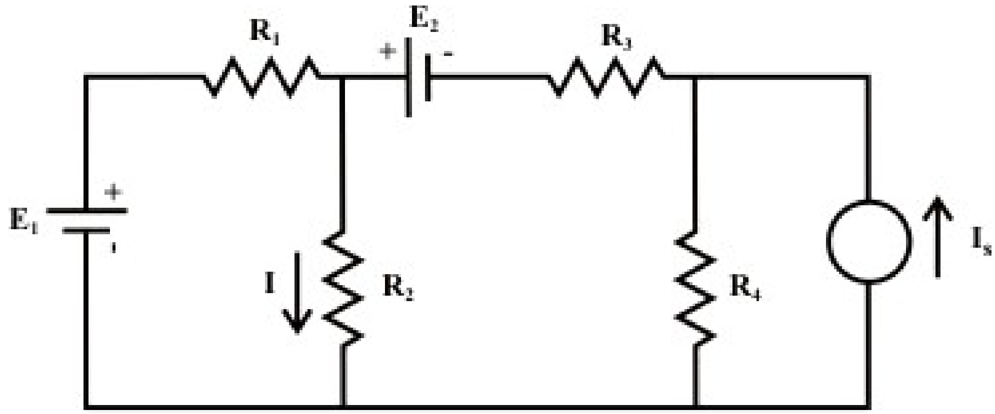


Fig. 7.1

One may consider the resistances R_1 *and* R_3 are the internal resistances of the voltage sources whereas the resistance R_4 is considered as internal resistance of the current source. The problem is to determine the response I in the in the resistor R_2. The current I can be obtained from

$$I = I'|_{due\ to\ E_1\ (alone)} + I''|_{due\ to\ E_2\ (alone)} + I'''|_{due\ to\ I_s\ (alone)}$$

according to the application of the superposition theorem. It may be noted that each independent source is considered at a time while all other sources are turned off or killed. To kill a voltage source means the voltage source is replaced by its internal resistance (i.e. R_1 or R_3; in other words E_1 *or* E_2 should be replaced temporarily by a short circuit) whereas to kill a current source means to replace the current source by its internal resistance (i.e. R_4; in other words I_s should be replaced temporarily by an open circuit).

Remarks: Superposition theorem is most often used when it is necessary to determine the individual contribution of each source to a particular response.

L.7.1.2 Procedure for using the superposition theorem

Step-1: Retain one source at a time in the circuit and replace all other sources with their internal resistances.

Step-2: Determine the output (current or voltage) due to the single source acting alone using the techniques discussed in lessons 3 and 4.

Step-3: Repeat steps 1 and 2 for each of the other independent sources.

Step-4: Find the total contribution by adding algebraically all the contributions due to the independent sources.

L.7.2 Application of superposition theorem

Example- L.7.1 Consider the network shown in fig. 7.2(a). Calculate I_{ab} *and* V_{cg} using superposition theorem.

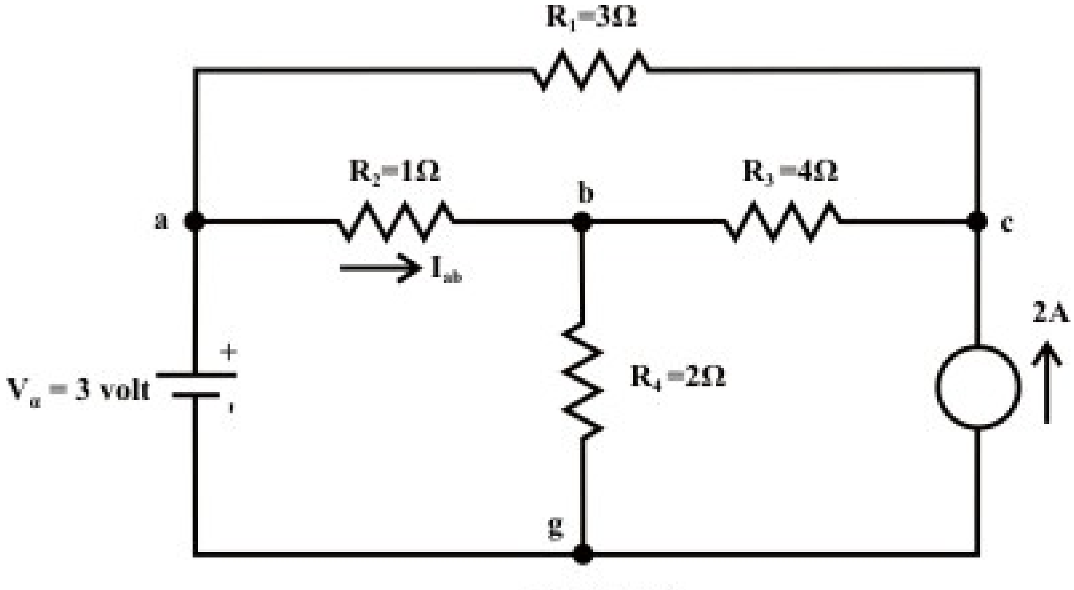


Fig. 7.2(a)

Solution: Voltage Source Only (retain one source at a time):

First consider the voltage source V_a that acts only in the circuit and the current source is replaced by its internal resistance (in this case internal resistance is infinite (∞)). The corresponding circuit diagram is shown in fig.7.2(b) and calculate the current flowing through the 'a-b' branch.

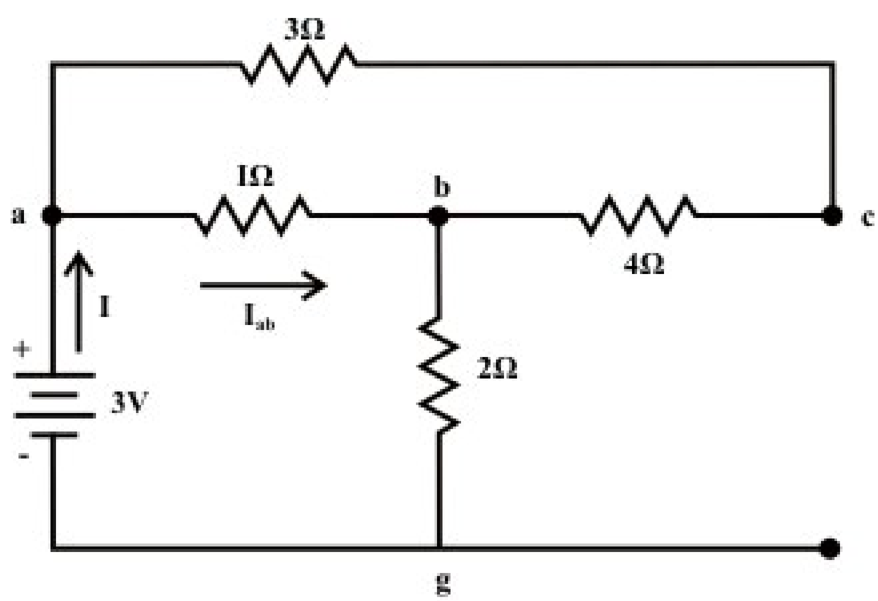


Fig. 7.2(b)

$$R_{eq} = [(R_{ac} + R_{cb}) \| R_{ab}] + R_{bg} = \frac{7}{8} + 2 = \frac{23}{8} \Omega$$

$I = \frac{3}{\frac{23}{8}} A = 1.043A;$ Now current through a to b, is given by

$$I_{ab} = \frac{7}{8} \times \frac{24}{23} = 0.913\text{A} \text{ (a to b)}$$

$$I_{acb} = 1.043 - 0.913 = 0.13A$$

Voltage across c-g terminal :

$V_{cg} = V_{bg} + V_{cb} = 2 \times 1.043 + 4 \times 0.13 = 2.61 volts$ (Note: we are moving opposite to the direction of current flow and this indicates there is rise in potential). Note 'c' is higher potential than 'g'.

Current source only (retain one source at a time):
Now consider the current source $I_s = 2\,A$ only and the voltage source V_a is replaced by its internal resistance which is zero in the present case. The corresponding the simplified circuit diagram is shown below (see fig.7.2(c)& fig.7.2(d)).

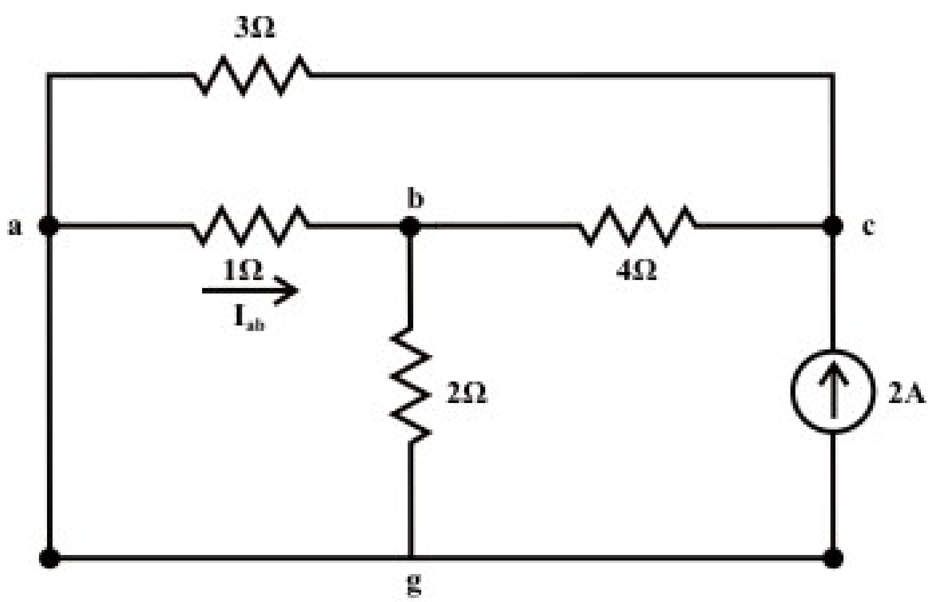


Fig. 7.2(c)

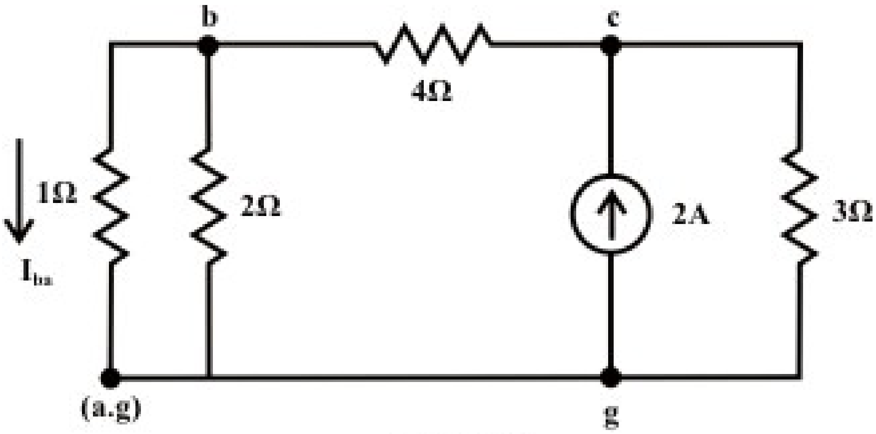


Fig. 7.2(d)

Current in the following branches:

$$3\Omega\, resistor = \frac{(14/3)\times 2}{(14/3)+3} = 1.217A; \qquad 4\Omega\, resistor = 2 - 1.217 = 0.783A$$

$$1\Omega\ resistor = \left(\frac{2}{3}\right)\times 0.783 = 0.522\, A\ (b\ to\, a)$$

Voltage across 3Ω resistor (c & g terminals) $V_{cg} = 1.217\times 3 = 3.651\, volts$

The total current flowing through 1Ω resistor (due to the both sources) from a to b = 0.913 (due to voltage source only; current flowing from 'a' to 'b') – 0.522 (due to current source only; current flowing from 'b'to 'a') $= 0.391A$.

Total voltage across the current source $V_{cg} = 2.61 volt$ (due to voltage source ; 'c' is higher potential than 'g') + 3.651 $volt$ (due to current source only; 'c' is higher potential than 'g') $= 6.26 volt$.

Example L.7.2 For the circuit shown in fig.7.3(a), the value of V_{s1} *and* I_s are fixed. When $V_{s2} = 0$, the current $I = 4A$. Find the value of I when $V_{s2} = 32V$.

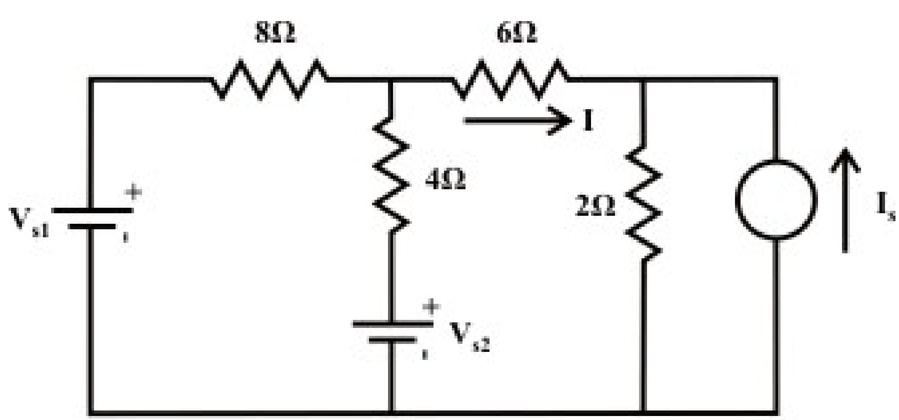


Fig. 7.3(a)

Solution: Let us assume that the current flowing 6Ω resistors due to the voltage and current sources are given by (assume circuit linearity)

$$I = \alpha V_{s1} + \beta V_{s2} + \eta I_s = I'_{(due\ to\ V_{s1})} + I''_{(due\ to\ V_{s2})} + I'''_{(due\ to\ I_s)} \quad (7.1)$$

where the parameters α, β, *and* η represent the positive constant numbers. The parameters α *and* β are the total conductance of the circuit when each voltage source acting alone in the circuit and the remaining sources are replaced by their internal resistances. On the other hand, the parameter η represents the total resistance of the circuit when the current source acting alone in the circuit and the remaining voltage sources are replaced by their internal resistances. The expression (7.1) for current I is basically written from the concept of superposition theorem.
From the first condition of the problem statement one can write an expression as (when the voltage source V_{s1} and the current source I_s acting jointly in the circuit and the other voltage source V_{s2} is not present in the circuit.)

$$4 = I = \alpha V_{s1} + \eta I_s = I'_{(due\ to\ V_{s1})} + I'''_{(due\ to\ I_s)} \text{ (Note both the sources are fixed)} \quad (7.2)$$

Let us assume the current following through the 6Ω resistor when all the sources acting in the circuit with $V_{s2} = 32V$ is given by the expression (7.1). Now, one can determine the current following through 6Ω resistor when the voltage source $V_{s2} = 32V$ acting alone in the circuit and the other sources are replaced by their internal resistances. For the circuit shown in fig.7.3 (b), the current delivered by the voltage source to the 6Ω resistor is given by

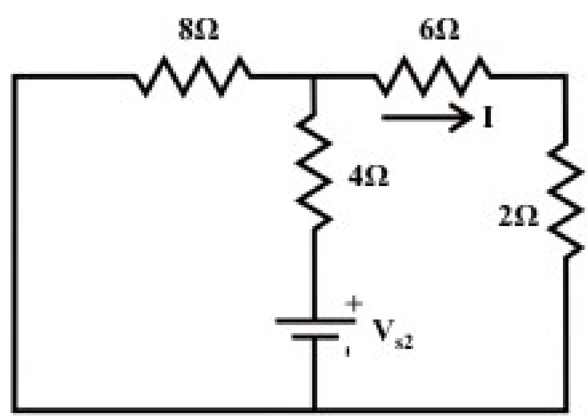


Fig. 7.3(b)

$$I_1 = \frac{V_{S2}}{R_{eq}} = \frac{32}{(8 \| 8) + 4} = 4\,A \quad (7.3)$$

The current following through the 6Ω due to the voltage source $V_{S2} = 32V$ only is $2\,A$ (flowing from left to right; i,e. in the direction as indicated in the figure 7.3(b)). Using equation (7.1), the total current I flowing the 6Ω resistor can be obtained as

$$I = \alpha V_{s1} + \beta V_{s2} + \eta I_s = I'_{(due\ to\ V_{s1})} + I''_{(due\ to\ V_{s2})} + I'''_{(due\ to\ I_s)} = \left[I'_{(due\ to\ V_{s1})} + I'''_{(due\ to\ I_s)} \right] + I''_{(due\ to\ V_{s2})}$$

$= 4\,A + 2A = 6A$ (note: $I'_{(due\ to\ V_{s1})} + I'''_{(due\ to\ I_s)} = 4\,A$ (see eq. 7.2)

Example L.7.3: Calculate the current I_{ab} flowing through the resistor $3\,\Omega$ as shown in fig.7.4(a), using the superposition theorem.

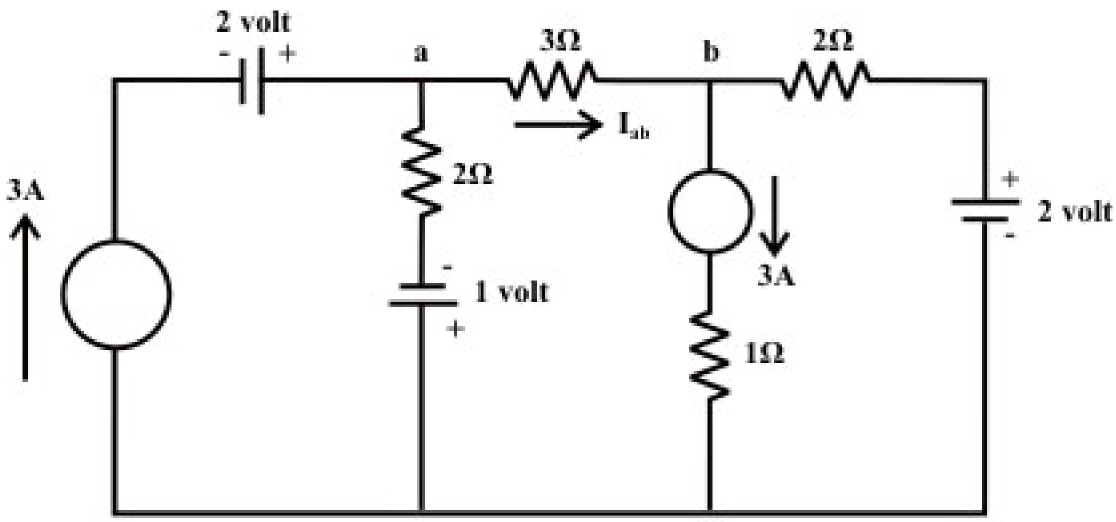


Fig. 7.4 (a)

Solution: Assume that the current source $3\,A$ (left to the $1\,volt$ source) is acting alone in the circuit and the internal resistances replace the other sources. The current flowing through $3\,\Omega$ resistor can be obtained from fig.7.4(b)

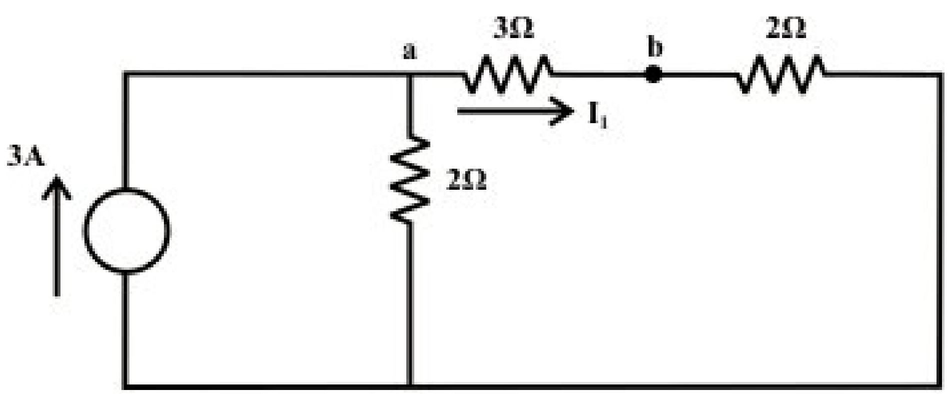


Fig. 7.4 (b)

and it is given by

$$I_{1(due\ to\ 3A\ current\ source)} = 3 \times \frac{2}{7} = \frac{6}{7} A(a\ to\ b) \quad (7.4)$$

Current flowing through 3Ω resistor **due to** $2V$ **source (only)** can be obtained from fig.7.4(c)

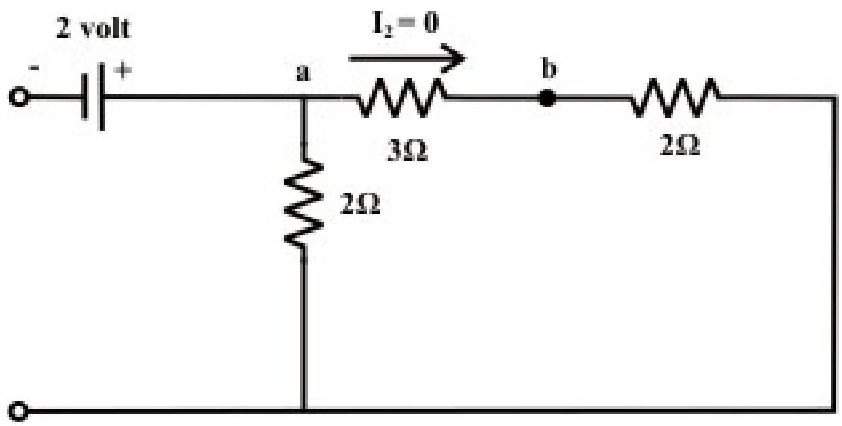


Fig. 7.4 (c)

and it is seen from no current is flowing.

$$I_{2(due\ to\ 2V\ \ voltage\ source)} = 0\,A(a\ to\ b) \qquad (7.5)$$

Current through 3Ω resistor **due to** $1V$ **voltage source only** (see fig.7.3(d)) is given by

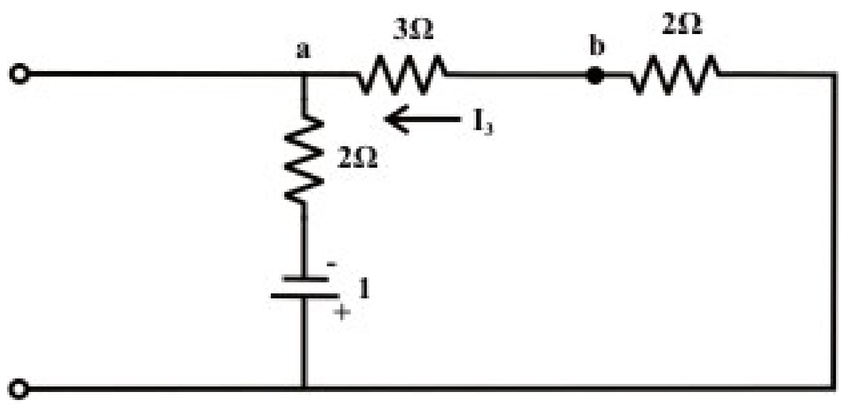


Fig. 7.4 (d)

$$I_{3(due\ to\ 1V\ \ voltage\ source)} = \frac{1}{7}A(b\ to\ a) \qquad (7.6)$$

Current through 3Ω resistor **due to** $3A$ **current source only** (see fig.7.3(e)) is obtained by

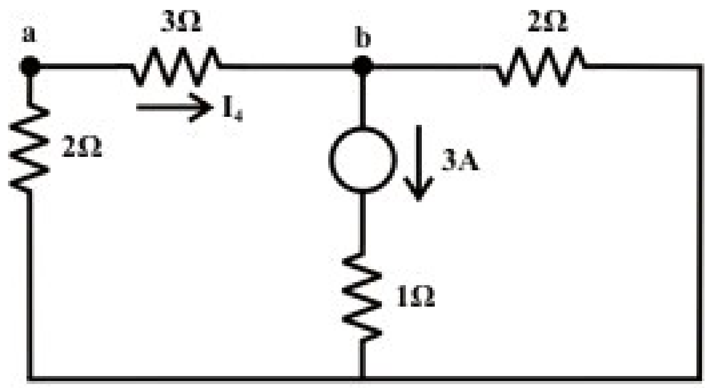


Fig. 7.4 (e)

$$I_{4(due\ to\ 3A\ current\ source)} = 3\times\frac{2}{7} = \frac{6}{7}A(\,a\ to\ b) \qquad (7.7)$$

Current through 3Ω resistor **due to** $2V$ **voltage source only** (see fig.7.3(f)) is given by

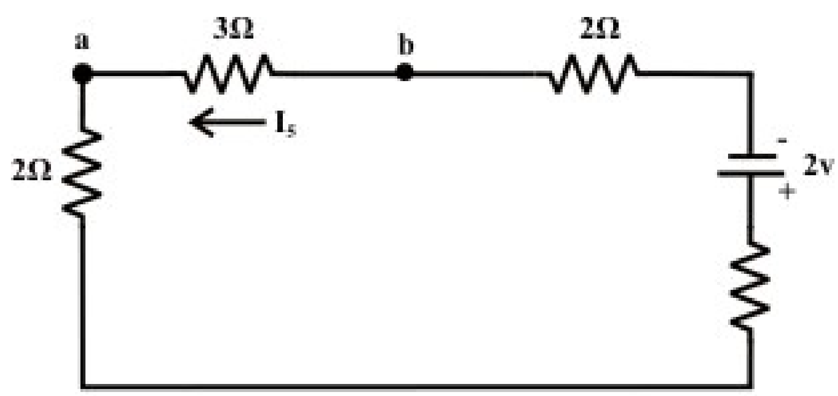


Fig. 7.4 (f)

$$I_{5(due\ to\ 2V\ voltage\ source)} = \frac{2}{7}A(b\ to\ a) \qquad (7.8)$$

Resultant current I_{ab} flowing through 3Ω resistor due to the combination of all sources is obtained by the following expression (the algebraic sum of all currents obtained in eqs. (7.4)-(7.8) with proper direction of currents)

$$I_{ab} = I_{1(due\ to\ 3A\ current\ source)} + I_{2(due\ to\ 2V\ voltage\ source)} + I_{3(due\ to\ 1V\ voltage\ source)} + I_{4(due\ to\ 3A\ current\ source)}$$
$$+ I_{5(due\ to\ 2V\ voltage\ source)}$$
$$= \frac{6}{7} + 0 - \frac{1}{7} + \frac{6}{7} - \frac{2}{7} = \frac{9}{7} = 1.285(\,a\ to\ b)$$

L.7.3 Limitations of superposition Theorem

- Superposition theorem doesn't work for power calculation. Because **power calculations** involve either the product of voltage and current, the square of current or the square of the voltage, they are not linear operations. This statement can be explained with a simple example as given below.

Example: Consider the circuit diagram as shown in fig.7.5.

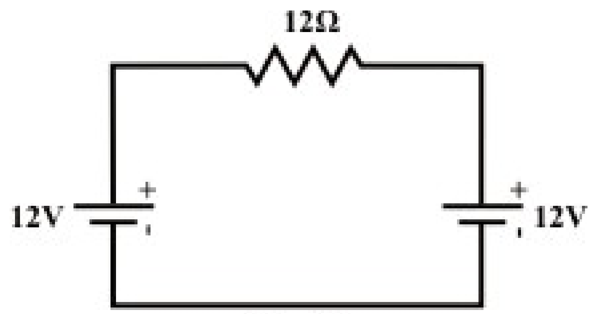


Fig. 7.5

Using superposition theorem one can find the resultant current flowing through 12Ω resistor is zero and consequently power consumed by the resistor is also zero. For power consumed in an any resistive element of a network can not be computed using superposition theorem. Note that the power consumed by each individual source is given by

$P_{W1(due\ to\ 12V\ source(left))} = 12\ watts$; $P_{W2(due\ to\ 12V\ source(right))} = 12\ watts$

The total power consumed by $12\ \Omega = 24\, watts$ (applying superposition theorem). This result is wrong conceptually. In fact, we may use the superposition theorem to find a current in any branch or a voltage across any branch, from which power is then can be calculated.

- Superposition theorem can not be applied for non linear circuit (Diodes or Transistors).
- This method has weaknesses:- In order to calculate load current I_L or the load voltage V_L for the several choices of load resistance R_L of the resistive network , one needs to solve for every source voltage and current, perhaps several times. With the simple circuit, this is fairly easy but in a large circuit this method becomes an painful experience.

L.7.4 Test Your Understanding [Marks: 40]

T.7.1 When using the superposition theorem, to find the current produced independently by one voltage source, the other voltage source(s) must be ----------- and the current source(s) must be --------------. [2]

T.7.2 For a linear circuit with independent sources p_1, p_2, p_3 ……… p_n and if y_i is the response of the circuit to source p_i, with all other independent sources set to zero), then resultant response $y = \cdots\cdots\cdots$. [1]

T.7.3 Use superposition theorem to find the value of the voltage v_a in fig.7.6. [8]

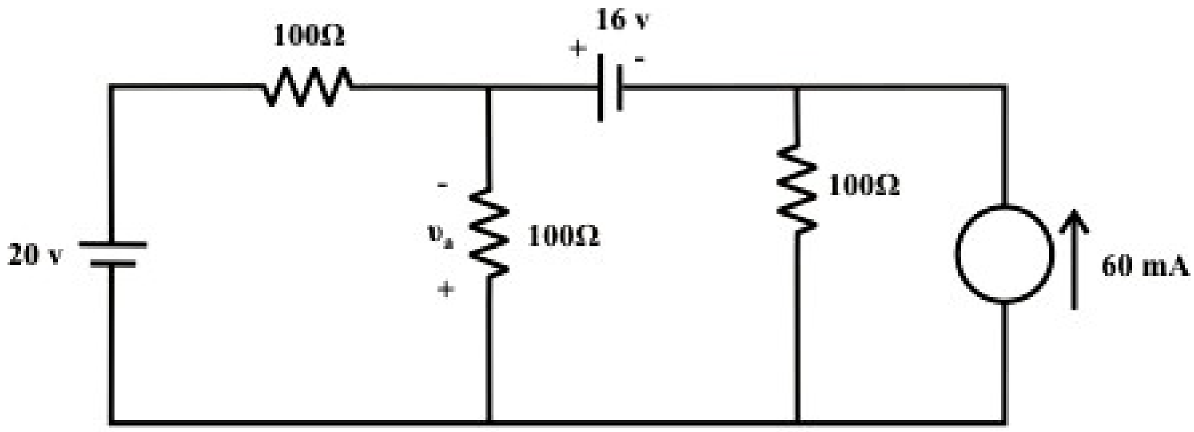


Fig. 7.6

(Ans. $14\,volts$)

T.7.4 For the circuit shown in fig.7.7, calculate the value of source current I_x that yields $I = 0$ if V_A *and* V_C are kept fixed at $7V$ *and* $28V$. [7]

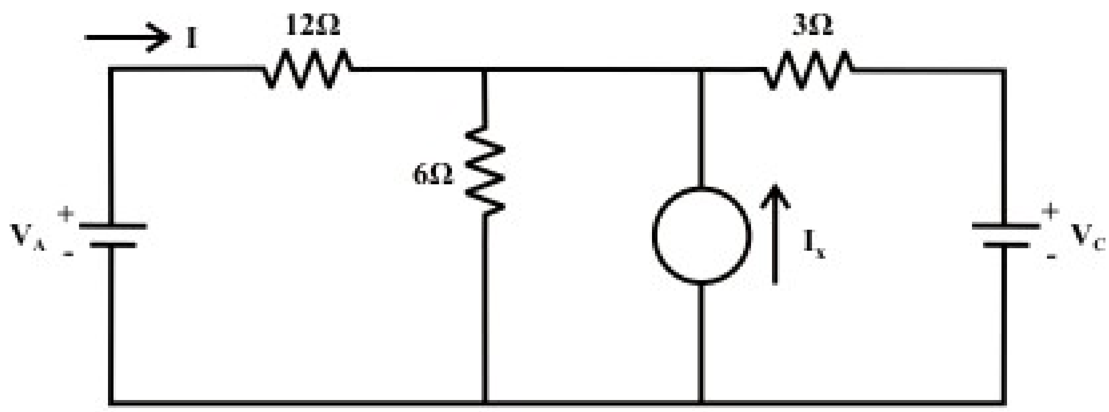


Fig. 7.7

(Ans. $I_x = -5.833\,A$)

T.7.5 For the circuit shown below (see fig.7.8), it follows from linearly that we can write $V_{ab} = \alpha\, I_x + \beta V_A + \eta V_B$, *where* α, β, *and* η *are* constants. Find the values of (i) $(i)\,\alpha$ (ii) β *and* (iii) η. [7]

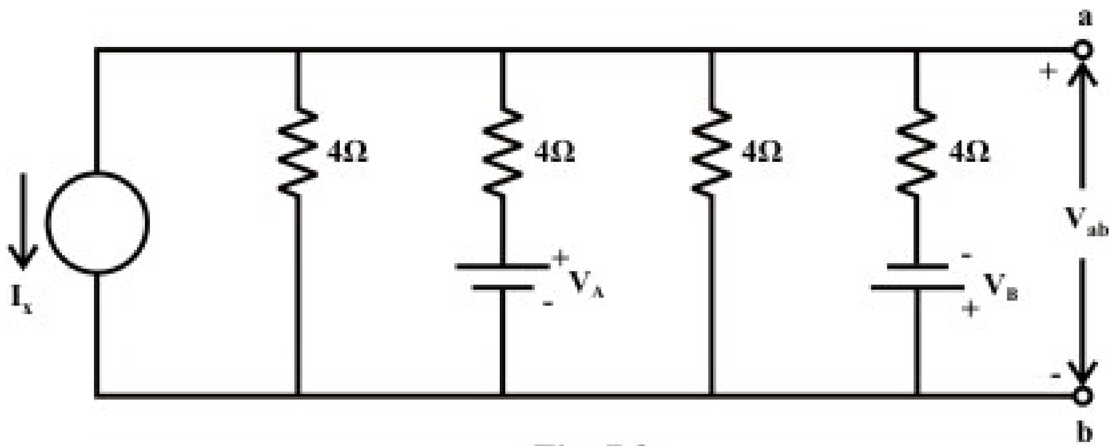


Fig. 7.8

(Ans. $\alpha = -1$; $\beta = 0.063$; *and* $\eta = -0.063$)

T.7.6 Using superposition theorem, find the current i through $5\,\Omega$ resistor as shown in fig.7.9. [8]

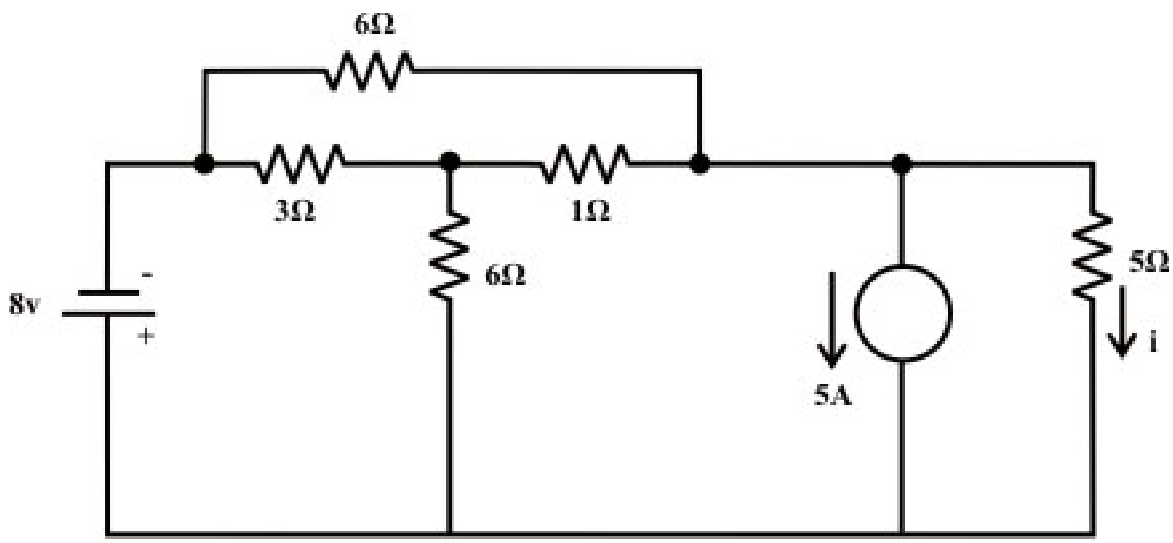


Fig. 7.9

(Ans. $-0.538\,A$)

T.7.7 Consider the circuit of fig.7.10

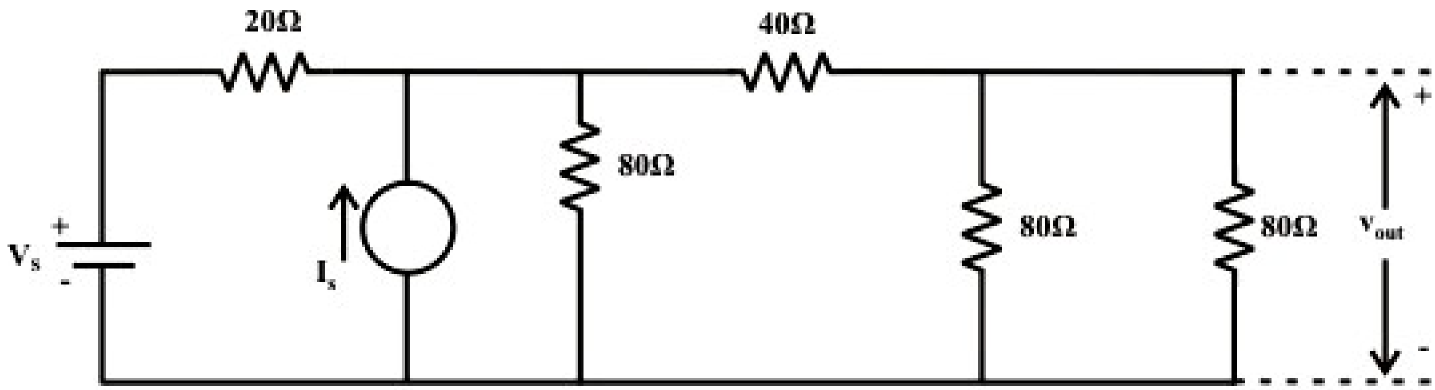


Fig. 7.10

(a) Find the linear relationship between V_{out} and input sources V_s *and* I_s

(b) If $V_s = 10V$ *and* $I_s = 1$, *find* V_{out}

(c) What is the effect of doubling all resistance values on the coefficients of the linear relationship found in part (a)? [7]

(Ans. (a) $V_{out} = 0.3333V_s + 6.666\, I_s$; (b) $V_{out} = 9.999V$ (c) $V_{out} = 0.3333V_s + 13.332 I_s$)

Module
2
DC Circuit

Lesson 8

Thevenin's and Norton's theorems in the context of dc voltage and current sources acting in a resistive network

Objectives

- To understand the basic philosophy behind the Thevenin's theorem and its application to solve dc circuits.
- Explain the advantage of Thevenin's theorem over conventional circuit reduction techniques in situations where load changes.
- Maximum power transfer theorem and power transfer efficiency.
- Use Norton's theorem for analysis of dc circuits and study the advantage of this theorem over conventional circuit reduction techniques in situations where load changes.

L.8.1 Introduction

A simple circuit as shown in fig.8.1 is considered to illustrate the concept of equivalent circuit and it is always possible to view even a very complicated circuit in terms of much simpler equivalent source and load circuits. Subsequently the reduction of computational complexity that involves in solving the current through a branch for different values of load resistance (R_L) is also discussed. In many applications, a network may contain a variable component or element while other elements in the circuit are kept constant. If the solution for current (I) or voltage (V) or power (P) in any component of network is desired, in such cases the whole circuit need to be analyzed each time with the change in component value. In order to avoid such repeated computation, it is desirable to introduce a method that will not have to be repeated for each value of variable component. Such tedious computation burden can be avoided provided the fixed part of such networks could be converted into a very simple equivalent circuit that represents either in the form of practical voltage source known as Thevenin's voltage source ($V_{Th} = magnitude\ of\ voltage\ source$, $R_{Th} = internal\ resistance\ of\ the\ source$) or in the form of practical current source known as Norton's current source ($I_N = magnitude\ of\ current\ source$, $R_N = internal\ resistance\ of\ current\ source$). In true sense, this conversion will considerably simplify the analysis while the load resistance changes. Although the conversion technique accomplishes the same goal, it has certain advantages over the techniques that we have learnt in earlier lessons.

Let us consider the circuit shown in fig. 8.1(a). Our problem is to find a current through R_L using different techniques; the following observations are made.

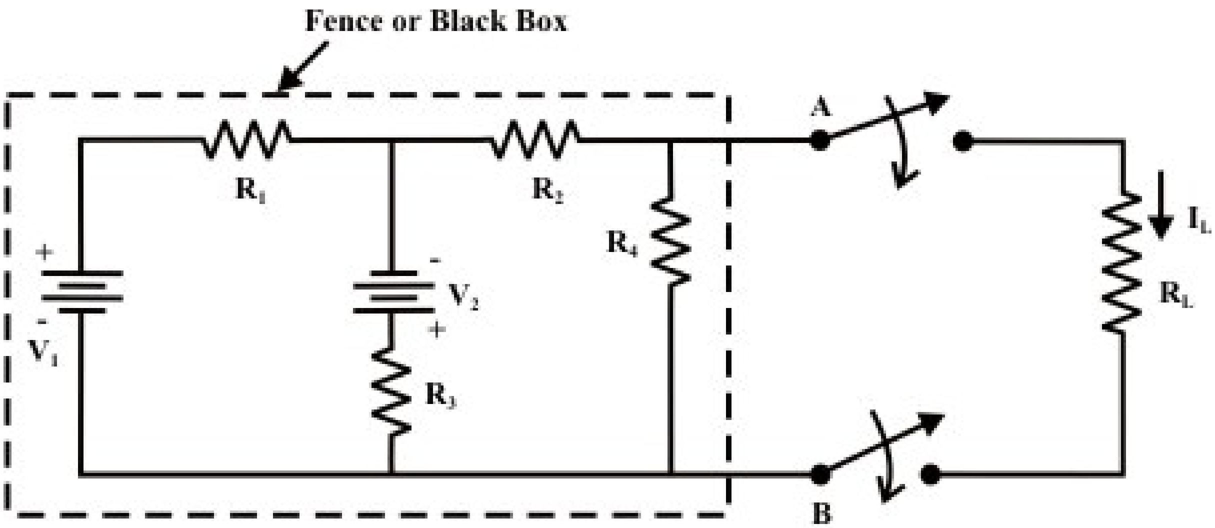


Fig. 8.1(a): A simple dc network

Find

- Mesh current method needs 3 equations to be solved
- Node voltage method requires 2 equations to be solved
- Superposition method requires a complete solution through load resistance (R_L) by considering each independent source at a time and replacing other sources by their internal source resistances.

Suppose, if the value of R_L is changed then the three (mesh current method) or two equations (node voltage method) need to be solved again to find the new current in R_L. Similarly, in case of superposition theorem each time the load resistance R_L is changed, the entire circuit has to be analyzed all over again. Much of the tedious mathematical work can be avoided if the fixed part of circuit (fig. 8.1(a)) or in other words, the circuit contained inside the imaginary fence or black box with two terminals A & B, is replaced by the simple equivalent voltage source (as shown in fig. 8.1(b)) or current source (as shown in fig.8.1(c)).

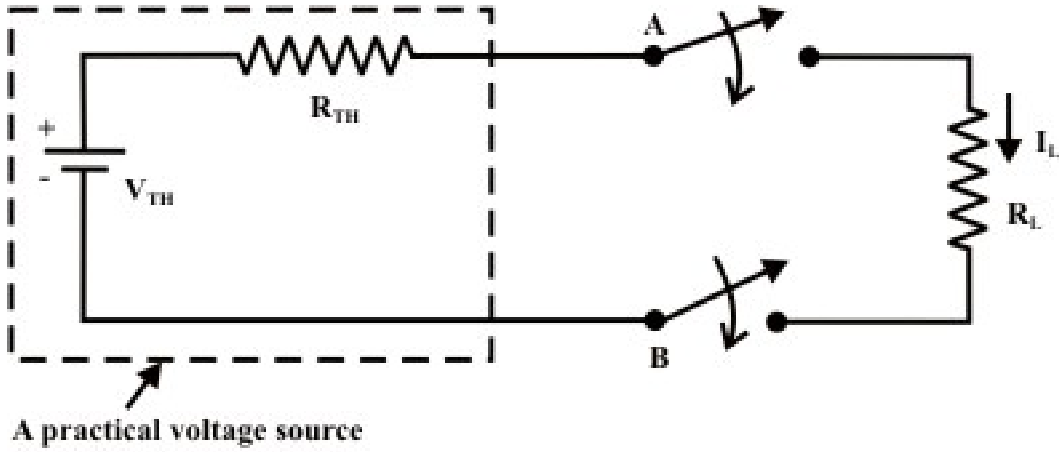


Fig. 8.1(b): circuit 8.1(a) is equivalently replaced by a simple practical voltage source

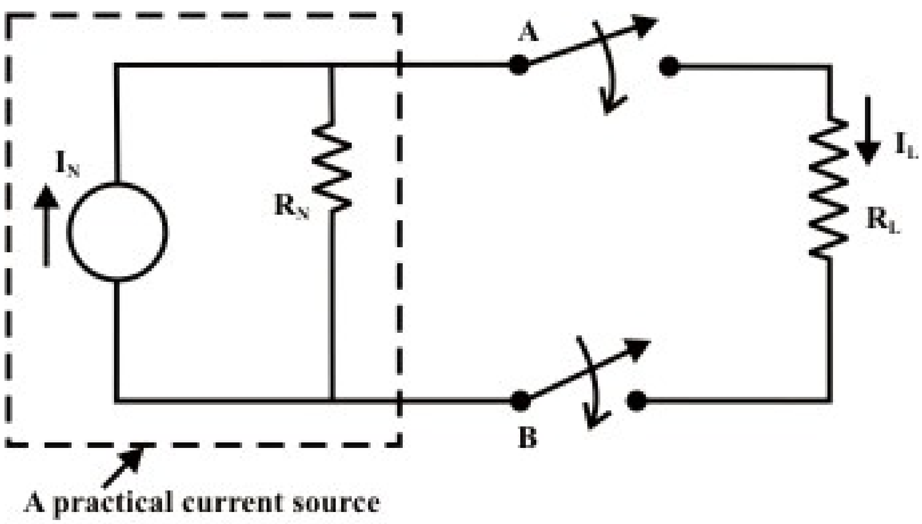


Fig. 8.1(c): Circuit 8.1(a) is equivalently replaced by a simple practical current source

Thevenin's Theorem: Thevenin's theorem states that any two output terminals (A & B, shown in fig. 8.2.(a)) of an active linear network containing independent sources (it includes voltage and current sources) can be replaced by a simple voltage source of magnitude V_{Th} in series with a single resistor R_{Th} (see fig. 8.2(d)) where R_{Th} is the equivalent resistance of the network when looking from the output terminals A & B with all sources (voltage and current) removed and replaced by their internal resistances (see fig. 8.2(c)) and the magnitude of V_{Th} is equal to the open circuit voltage across the A & B terminals. (The proof of the theorem will be given in section- L8. 5).

L.8.2 The procedure for applying Thevenin's theorem

To find a current I_L through the load resistance R_L (as shown in fig. 8.2(a)) using Thevenin's theorem, the following steps are followed:

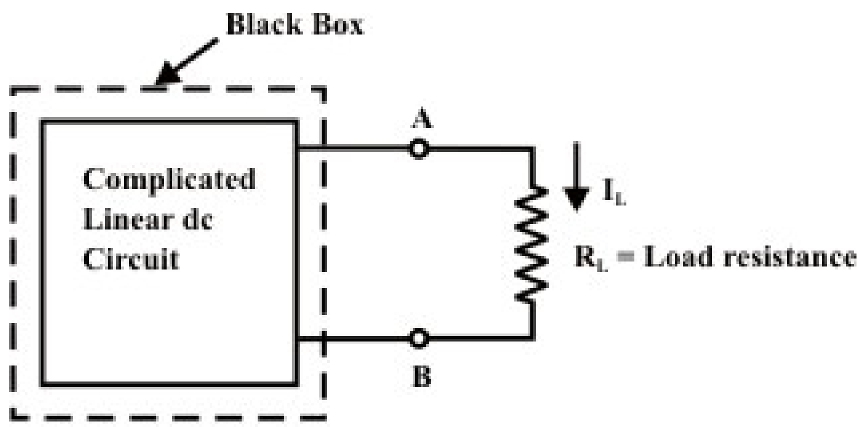


Fig. 8.2(a)

Step-1: Disconnect the load resistance (R_L) from the circuit, as indicated in fig. 8.2(b).

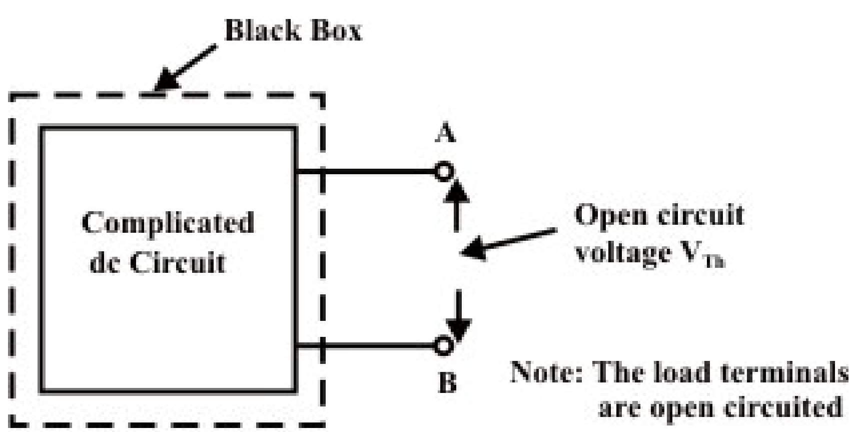


Fig. 8.2(b)

Step-2: Calculate the open-circuit voltage V_{TH} (shown in fig.8.2(b)) at the load terminals ($A \& B$) after disconnecting the load resistance (R_L). In general, one can apply any of the techniques (mesh-current, node-voltage and superposition method) learnt in earlier lessons to compute V_{Th} (experimentally just measure the voltage across the load terminals using a voltmeter).

Step-3: Redraw the circuit (fig. 8.2(b)) with each practical source replaced by its internal resistance as shown in fig.8.2(c). (note, voltage sources should be short-circuited (just remove them and replace with plain wire) and current sources should be open-circuited (just removed).

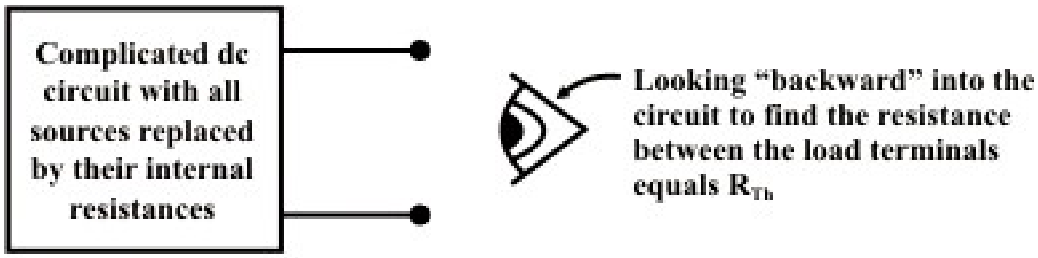


Fig. 8.2(c)

Step-4: Look backward into the resulting circuit from the load terminals $(A \& B)$, as suggested by the eye in fig.L.8.2(c). Calculate the resistance that would exist between the load terminals (or equivalently one can think as if a voltage source is applied across the load terminals and then trace the current distribution through the circuit (fig.8.2 (c)) in order to calculate the resistance across the load terminals.) The resistance R_{Th} is described in the statement of Thevenin's theorem. Once again, calculating this resistance may be a difficult task but one can try to use the standard circuit reduction technique or $Y-\Delta$ *or* $\Delta - Y$ transformation techniques.

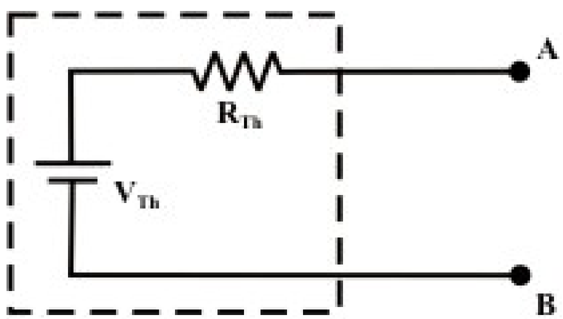


Fig. 8.2(d): Dash-portion of the circuit (Fig. 8.2(a)) is equivalently replaced by a practical voltage source.

Step-5: Place R_{Th} in series with V_{Th} to form the Thevenin's equivalent circuit (replacing the imaginary fencing portion or fixed part of the circuit with an equivalent practical voltage source) as shown in fig. 8.2(d).

Step-6: Reconnect the original load to the Thevenin voltage circuit as shown in fig.8.2(e); the load's voltage, current and power may be calculated by a simple arithmetic operation only.

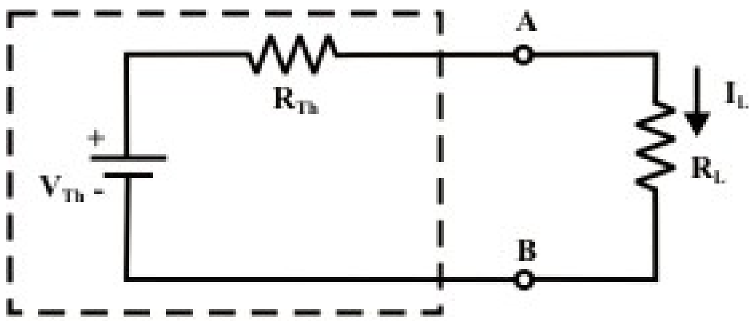


Fig. 8.2(e)

Load current $I_L = \dfrac{V_{Th}}{R_{Th} + R_L}$ (8.1)

Voltage across the load $V_L = \dfrac{V_{Th}}{R_{Th} + R_L} \times R_L = I_L \times R_L$ (8.2)

Power absorbed by the load $P_L = I_L^{\,2} \times R_L$ (8.3)

Remarks: (i) One great advantage of Thevenin's theorem over the normal circuit reduction technique or any other technique is this: once the Thevenin equivalent circuit has been formed, it can be reused in calculating load current (I_L), load voltage (V_L) and load power (P_L) for different loads using the equations (8.1)-(8.3).

(ii) Fortunately, with help of this theorem one can find the choice of load resistance R_L that results in the maximum power transfer to the load. On the other hand, the effort necessary to solve this problem-using node or mesh analysis methods can be quite complex and tedious from computational point of view.

L.8.3 Application of Thevenin's theorem

Example: L.8.1 For the circuit shown in fig.8.3(a), find the current through $R_L = R_2 = 1\Omega$ resistor (I_{a-b} branch) using Thevenin's theorem & hence calculate the voltage across the current source (V_{cg}).

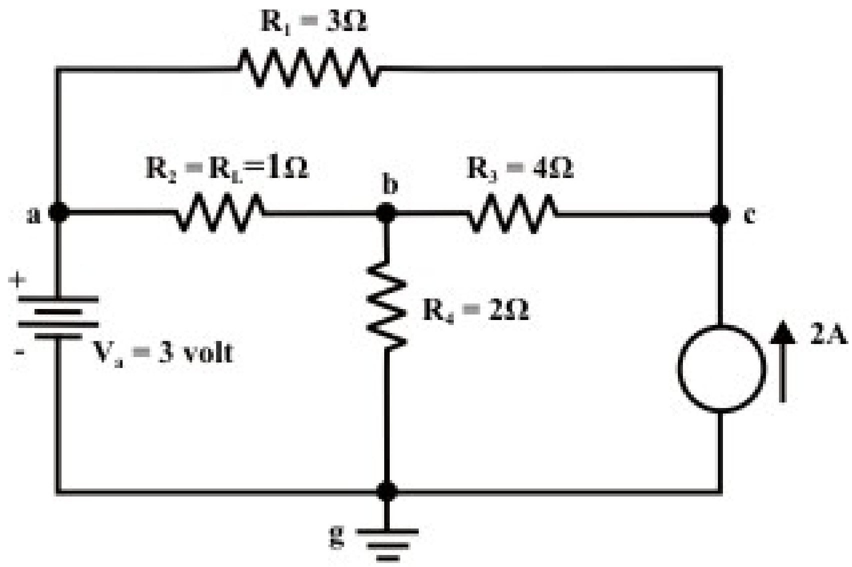


Fig. 8.3(a)

Solution:
Step-1: Disconnect the load resistance R_L and redraw the circuit as shown in fig.8.3(b).

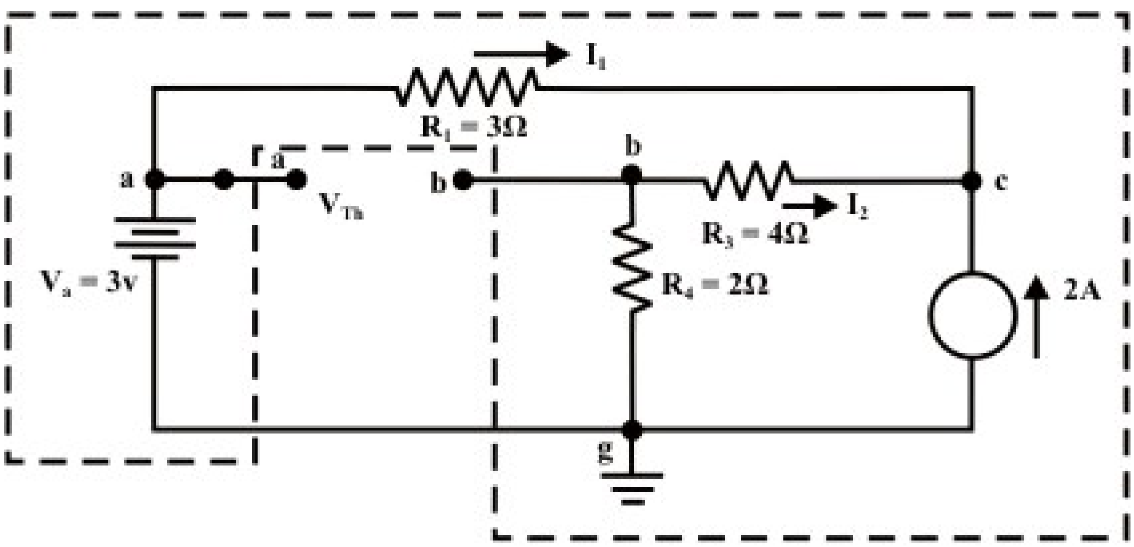


Fig. 8.3(b)

Step-2: Apply any method (say node-voltage method) to calculate V_{Th}.
At node C:

$$2 + I_1 + I_2 = 0$$

$$2 + \frac{(3 - V_c)}{3} + \frac{(0 - V_c)}{6} \Rightarrow V_c = 6\,volt$$

Now, the currents I_1 & I_2 can easily be computed using the following expressions.

$I_1 = \frac{V_a - V_c}{3} = \frac{3-6}{3} = -1A$ (note, current I_1 is flowing from ‘c’ to ‘a’)

$I_2 = \frac{0 - V_c}{6} = \frac{-6}{6} = -1A$ (note, current I_2 is flowing from ‘c’ to ‘g’)

Step-3: Redraw the circuit (fig.8.3(b) indicating the direction of currents in different branches. One can find the Thevenin’s voltage V_{Th} using KVL around the closed path ‘gabg’ (see fig.8.3.(c).

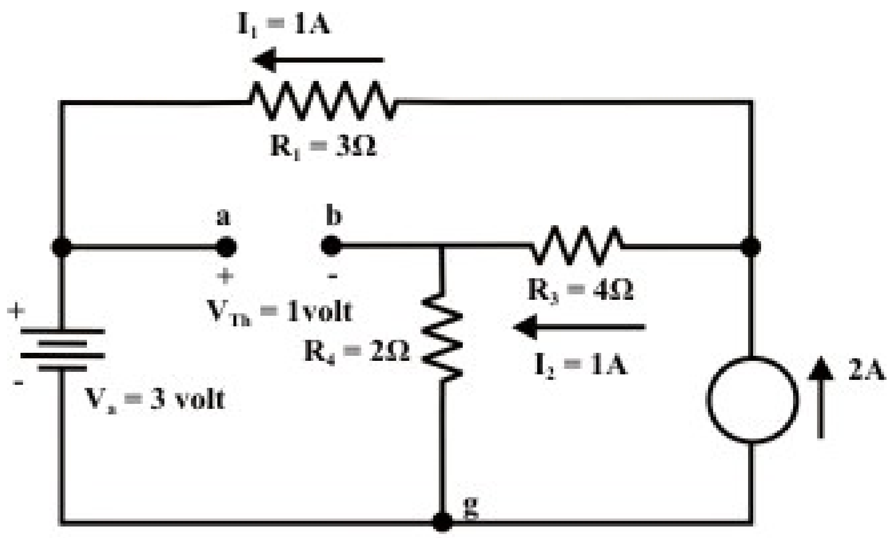


Fig. 8.3(c)

$V_{Th} = V_{ag} - V_{bg} = 3 - 2 = 1 volt$

Step-4: Replace all sources by their internal resistances. In this problem, voltage source has an internal resistance zero (0) (ideal voltage source) and it is short-circuited with a wire. On the other hand, the current source has an infinite internal resistance (ideal current source) and it is open-circuited (just remove the current source). Thevenin’s resistance R_{Th} of the fixed part of the circuit can be computed by looking at the load terminals ‘a’- ‘b’ (see fig.8.3(d)).

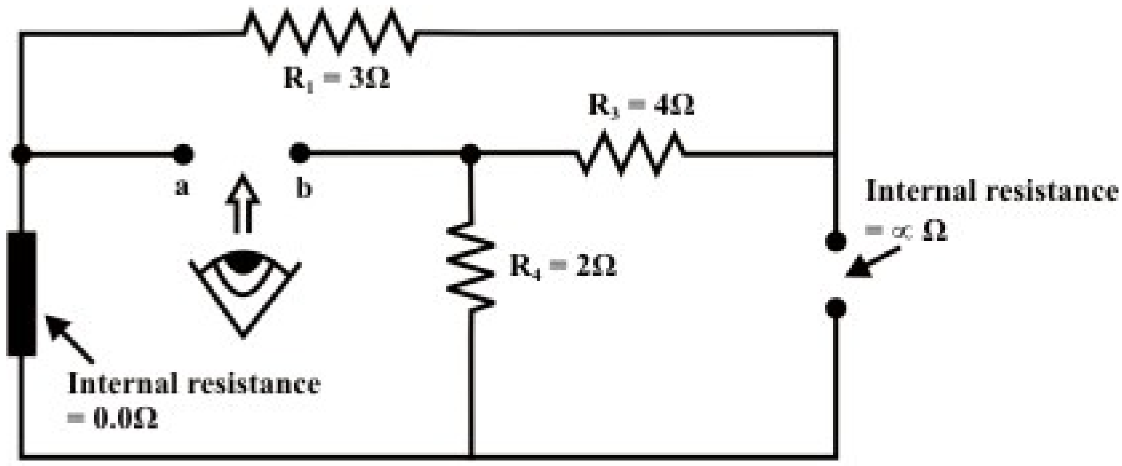


Fig. 8.3(d)

$$R_{Th} = \left(R_1 + R_3\right) || R_4 = \left(3+4\right)||2 = 1.555\Omega$$

Step-5: Place R_{Th} in series with V_{Th} to form the Thevenin's equivalent circuit (a simple practical voltage source). Reconnect the original load resistance $R_L = R_2 = 1\Omega$ to the Thevenin's equivalent circuit (note the polarity of 'a' and 'b' is to be considered carefully) as shown in fig.8.3(e).

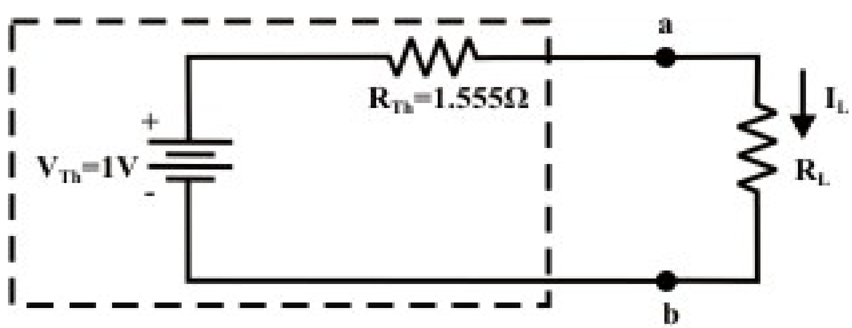


Fig. 8.3(e): Equivalent dc circuit fig. 8.3(b) is replace by a practical voltage source.

$$I_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{1}{1.555 + 1} = 0.39\,A(a\ to\ b)$$

Step-6: The circuit shown in fig.8.3 (a) is redrawn to indicate different branch currents. Referring to fig.8.3 (f), one can calculate the voltage V_{bg} and voltage across the current source (V_{cg}) using the following equations.

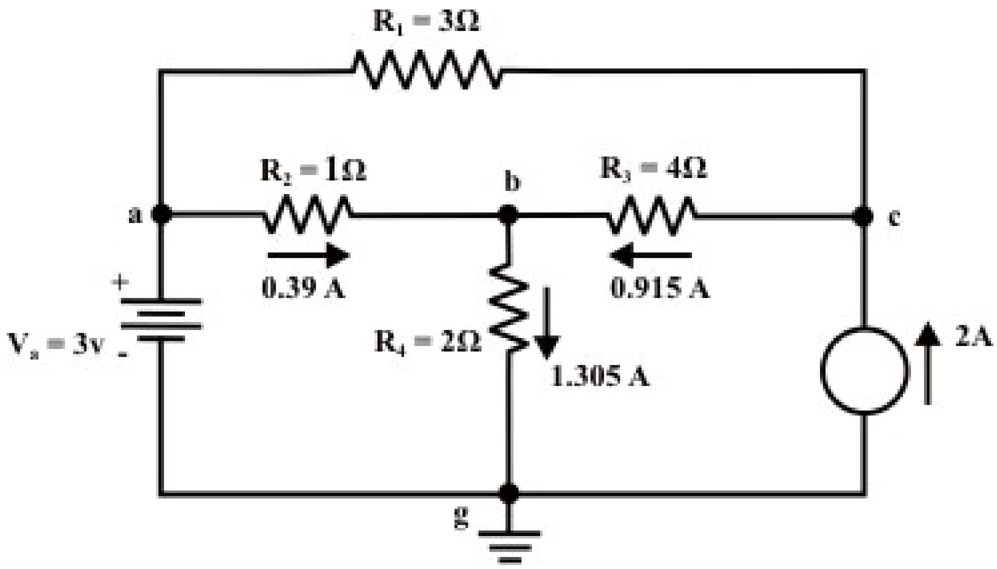


Fig. 8.3(f)

$V_{bg} = V_{ag} - V_{ab} = 3 - 1 \times 0.39 = 2.61 volt.$

$I_{bg} = \frac{2.61}{2} = 1.305\,A;\;\; I_{cb} = 1.305 - 0.39 = 0.915\,A$

$V_{cg} = 4 \times 0.915 + 2 \times 1.305 = 6.27\, volt.$

Example-L.8.2 For the circuit shown in fig.8.4 (a), find the current I_L through 6 Ω resistor using Thevenin's theorem.

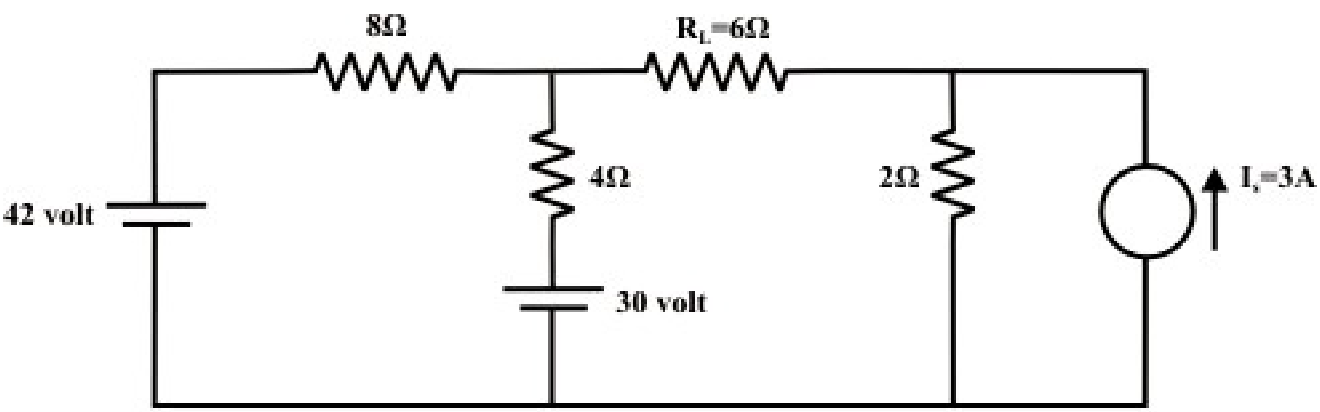


Fig. 8.4(a)

Solution:

Step-1: Disconnect 6Ω from the terminals 'a' and 'b' and the corresponding circuit diagram is shown in fig.L.8.4 (b). Consider point 'g' as ground potential and other voltages are measured with respect to this point.

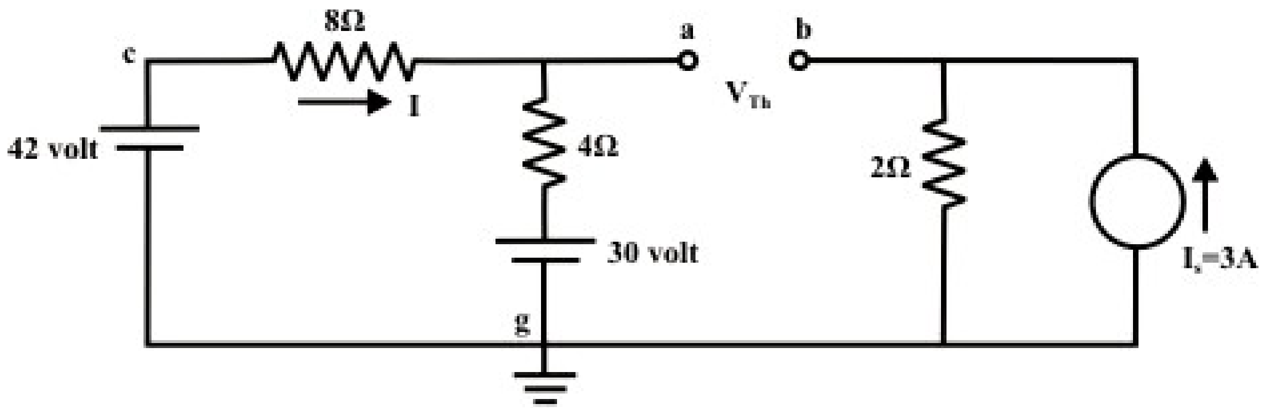


Fig. 8.4(b)

Step-2: Apply any suitable method to find the Thevenin's voltage (V_{Th}) (or potential between the terminals 'a' and 'b'). KVL is applied around the closed path 'gcag' to compute Thevenin's voltage.

$$42 - 8I - 4I - 30 = 0 \;\Rightarrow\; I = 1A$$
$$Now,\; V_{ag} = 30 + 4 = 34\,volt;\; V_{bg} = 2 \times 3 = 6\,volt.$$

$V_{Th} = V_{ab} = V_{ag} - V_{bg} = 34 - 6 = 28\,volt$ (note 'a' is higher potential than 'b')

Step-3: Thevenin's resistance R_{Th} can be found by replacing all sources by their internal resistances (all voltage sources are short-circuited and current sources are just removed or open circuited) as shown in fig.8.4 (c).

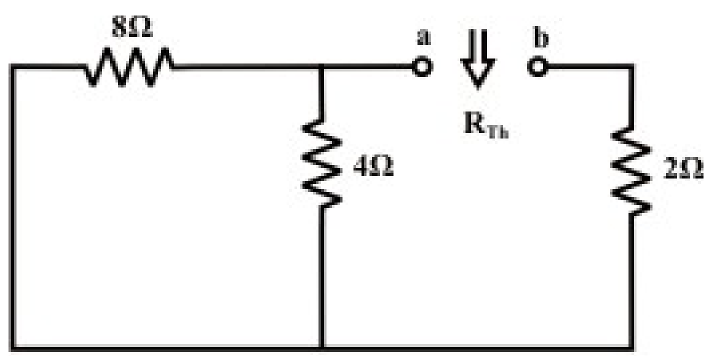


Fig. 8.4(c)

$$R_{Th} = (8 \,||\, 4) + 2 = \frac{8 \times 4}{12} + 2 = \frac{14}{3} = 4.666\,\Omega$$

Step-4: Thevenin's equivalent circuit as shown in fig.8.4 (d) is now equivalently represents the original circuit (fig.L.8.4(a).

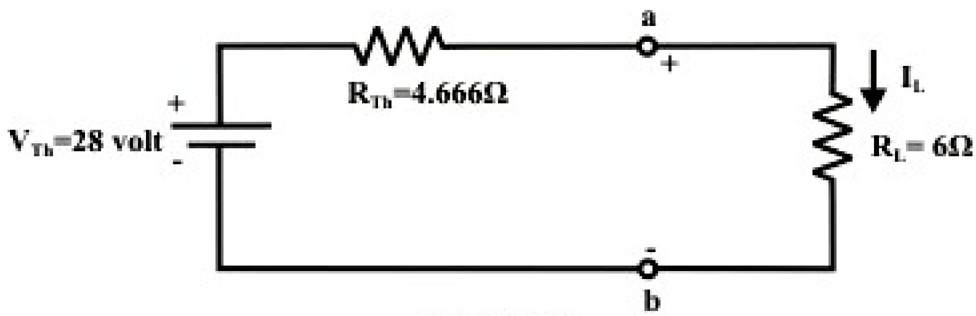


Fig. 8.4(d)

$$I_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{28}{4.666 + 6} = 2.625\,A$$

Example-L.8.3 The box shown in fig.8.5 (a) consists of independent dc sources and resistances. Measurements are taken by connecting an ammeter in series with the resistor R and the results are shown in table.

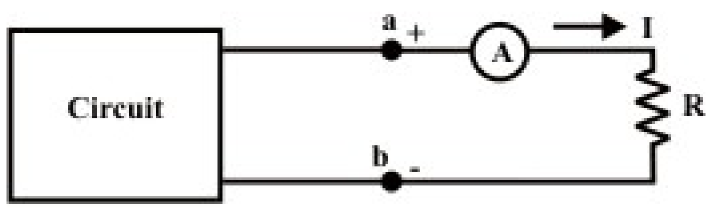


Fig. 8.5(a)

Table

R	I
10Ω	$2\,A$
20Ω	$1.5\,A$
?	$0.6\,A$

Solution: The circuit shown in fig.8.5(a) can be replaced by an equivalent Thevenin's voltage source as shown in fig.8.5(b). The current flowing through the resistor R is expressed as

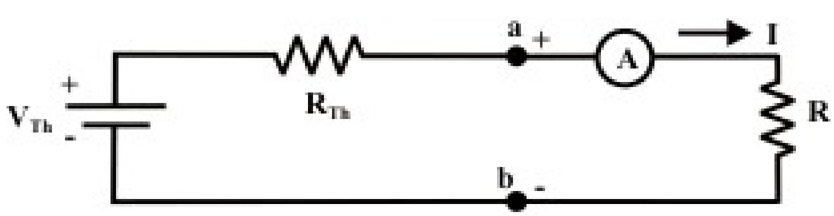


Fig. 8.5(b): Equivalent circuit of fig. 8.5(a)

$$I = \frac{V_{Th}}{R_{Th} + R} \qquad (8.4)$$

The following two equations are written from measurements recorded in table.

$$\frac{V_{Th}}{R_{Th}+10}=2 \Rightarrow \; V_{Th}-2R_{Th}=20 \qquad (8.5)$$

$$\frac{V_{Th}}{R_{Th}+20}=1.5 \Rightarrow \; V_{Th}-1.5R_{Th}=30 \qquad (8.6)$$

Solving equations (8.5) and (8.6) we get,
$V_{Th}=60\ volt;\ R_{Th}=20\Omega$
The choice of R that yields current flowing the resistor is $0.6A$ can be obtained using the equation (8.4).

$$I=\frac{V_{Th}}{R_{Th}+R}=\frac{60}{20+R}=0.6 \;\Rightarrow R=80\Omega.$$

L.8.4 Maximum Power Transfer Theorem

In an electric circuit, the load receives electric energy via the supply sources and converts that energy into a useful form. The maximum allowable power receives by the load is always limited either by the heating effect (incase of resistive load) or by the other power conversion taking place in the load. The Thevenin and Norton models imply that the internal circuits within the source will necessarily dissipate some of power generated by the source. A logical question will arise in mind, how much power can be transferred to the load from the source under the most practical conditions? In other words, what is the value of load resistance that will absorbs the maximum power from the source? This is an important issue in many practical problems and it is discussed with a suitable example.

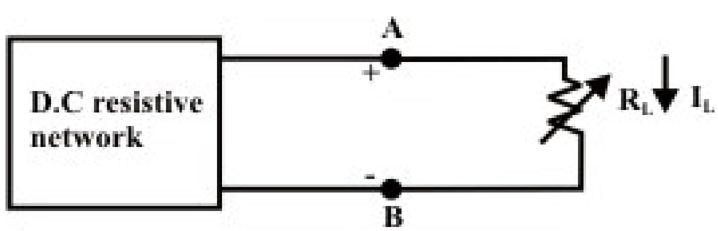


Fig. 8.6(a)

Let us consider an electric network as shown in fig.8.6(a), the problem is to find the choice of the resistance R_L so that the network delivers maximum power to the load or in other words what value of load resistance R_L will absorb the maximum amount of power from the network. This problem can be solved using nodal or mesh current analysis to obtain an expression for the power absorbed by R_L, then the derivative of this expression with respect to R_L will establish the condition under what circumstances the maximum power transfer occurs. The effort required for such an approach can be quite tedious and complex. Fortunately, the network shown in fig.L.8.6(a) can be represented by an equivalent Thevenin's voltage source as shown in fig.L.8.6(b).

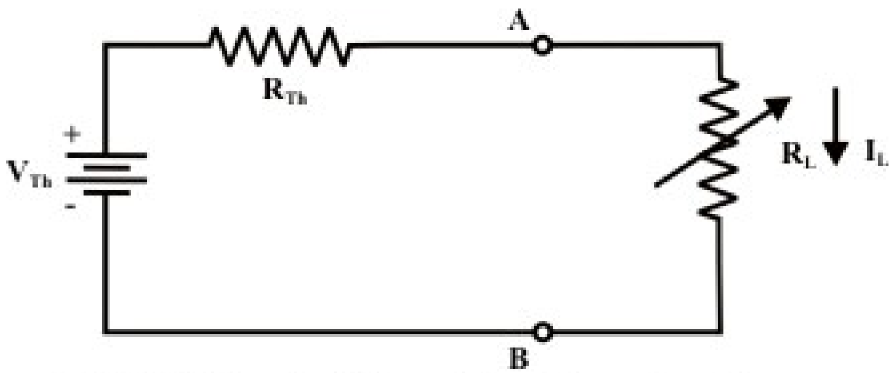


Fig. 8.6(b): The circuit for maximum Power transfer

In fig.8.6(b) a variable load resistance R_L is connected to an equivalent Thevenin circuit of original circuit(fig.8.6(a)). The current for any value of load resistance is

$$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$

Then, the power delivered to the load is

$$P_L = I_L^{\,2} \times R_L = \left[\frac{V_{Th}}{R_{Th} + R_L}\right]^2 \times R_L$$

The load power depends on both R_{Th} *and* R_L; however, R_{Th} is constant for the equivalent Thevenin network. So power delivered by the equivalent Thevenin network to the load resistor is entirely depends on the value of R_L. To find the value of R_L that absorbs a maximum power from the Thevenin circuit, we differentiate P_L with respect to R_L.

$$\frac{dP(R_L)}{dR_L} = V_{Th}^{\,2}\left[\frac{(R_{Th}+R_L)^2 - 2R_L \times (R_{Th}+R_L)}{\left(R_{Th}+R_L\right)^4}\right] = 0 \Rightarrow (R_{Th}+R_L) - 2R_L = 0 \Rightarrow R_L = R_{Th} \qquad (8.7)$$

For maximum power dissipation in the load, the condition given below must be satisfied

$$\left.\frac{d^2P(R_L)}{dR_L^{\,2}}\right|_{R_L = R_{Th}} = -\frac{V_{Th}^{\,2}}{8R_{Th}} < 0$$

This result is known as "Matching the load" or maximum power transfer occurs when the load resistance R_L matches the Thevenin's resistance R_{Th} of a given systems. Also, notice that under the condition of maximum power transfer, the load voltage is, by voltage division, one-half of the Thevenin voltage. The expression for maximum power dissipated to the load resistance is given by

$$P_{\max} = \left[\frac{V_{Th}}{R_{Th} + R_L} \right]^2 \times R_L \Bigg|_{R_L = R_{Th}} = \frac{V_{Th}^{\ 2}}{4\,R_{Th}}$$

The total power delivered by the source
$P_T = I_L^{\ 2}\left(R_{Th} + R_L\right) = 2 \times I_L^{\ 2}\, R_L$
This means that the Thevenin voltage source itself dissipates as much power in its internal resistance R_{Th} as the power absorbed by the load R_L. Efficiency under maximum power transfer condition is given by

$$Efficiency = \frac{I_L^{\ 2}\, R_L}{2\, I_L^{\ 2}\, R_L} \times 100 = 50\,\% \qquad (8.8)$$

For a given circuit, V_{Th} *and* R_{Th} are fixed. By varying the load resistance R_L, the power delivered to the load varies as shown in fig.8.6(c).

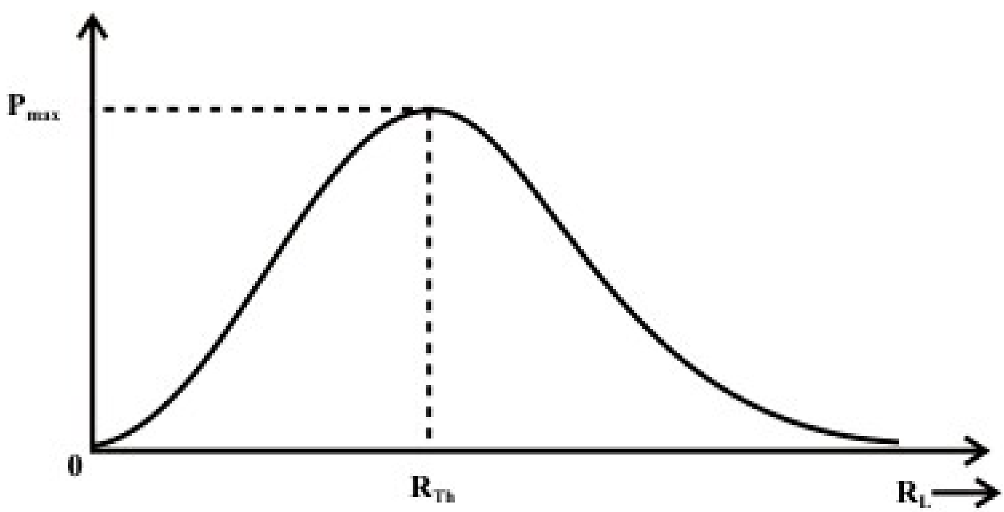


Fig. 8.6(c): Power dissipated to the load as a function of R_L

Remarks: The Thevenin equivalent circuit is useful in finding the maximum power that a linear circuit can deliver to a load.

Example-L.8.4 For the circuit shown in fig.8.7(a), find the value of R_L that absorbs maximum power from the circuit and the corresponding power under this condition.

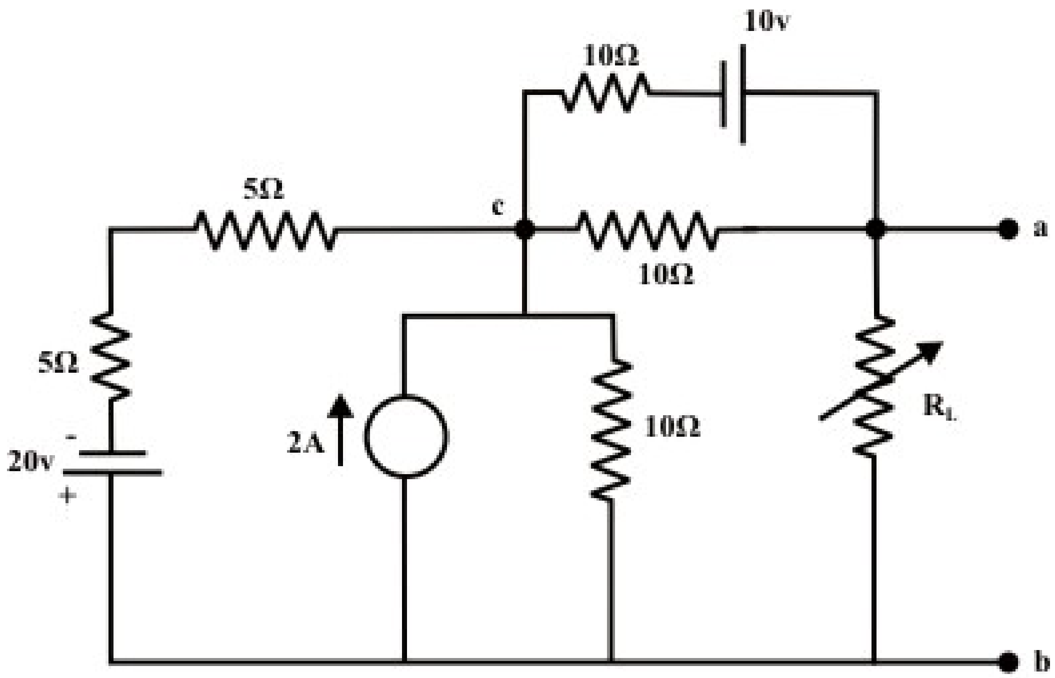


Fig. 8.7(a)

Solution: Load resistance R_L is disconnected from the terminals 'a' and 'b' and the corresponding circuit diagram is drawn (see fig.8.7(b)).

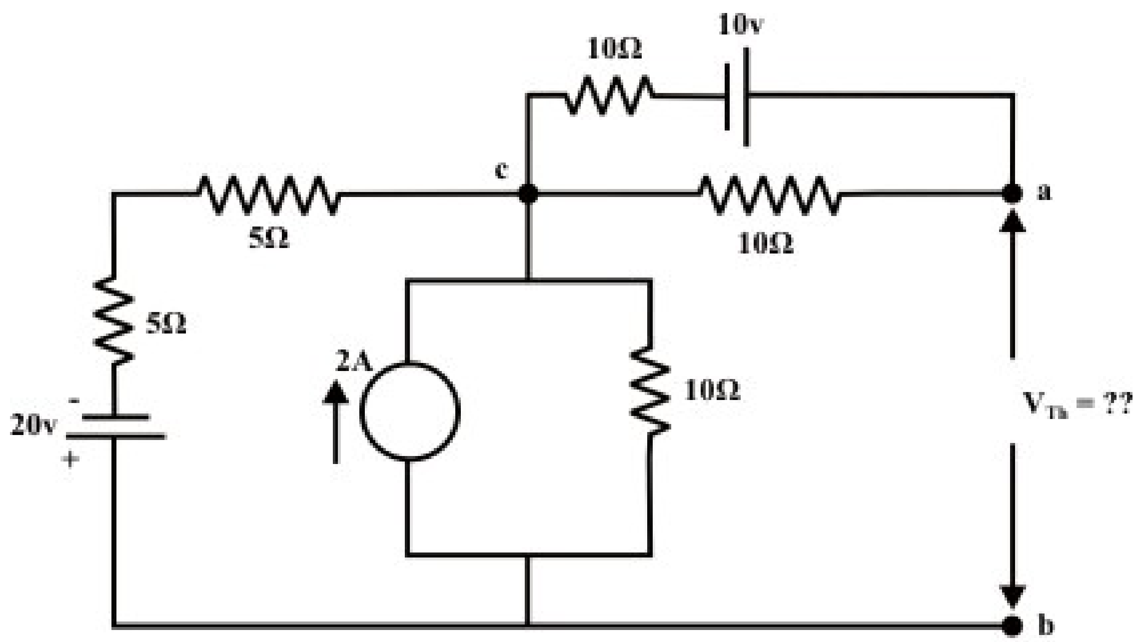


Fig. 8.7(b)

The above circuit is equivalently represented by a Thevenin circuit and the corresponding Thevenin voltage V_{Th} and Thevenin resistance R_{Th} are calculated by following the steps given below:

Now applying 'Super position theorem', one can find V_{Th} (voltage across the 'a' and 'b' terminals, refer fig. 8.7(b)). Note any method (node or mesh analysis) can be applied to find V_{Th}.

Considering only 20v source only

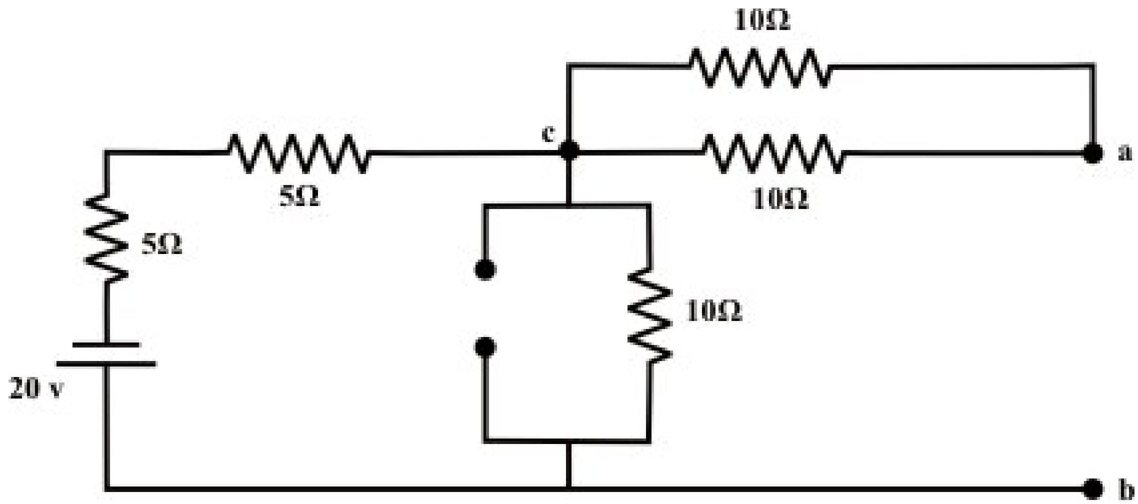


Fig. 8.7(c)

From the above circuit the current through 'b-c' branch $= \frac{20}{20} = 1A$ (from 'b' to 'a') whereas the voltage across the 'b-a' branch $v_{ba} = 1 \times 10 = 10\ volt$. ('b' is higher potential than 'a'). $\therefore v_{ab} = -10\,volt$

Considering only 10v source only

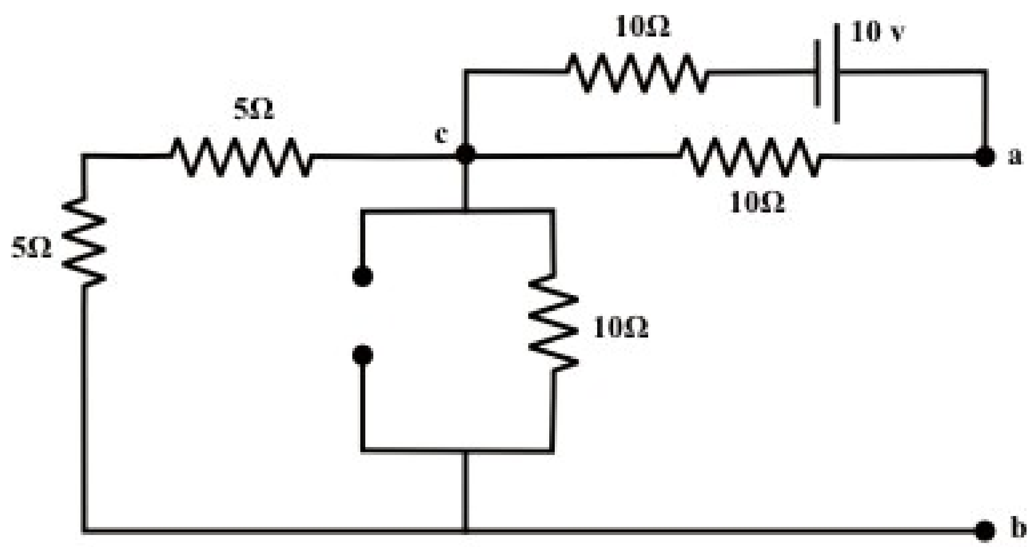


Fig. 8.7(d)

Note: No current is flowing through 'cb'-branch.

$\therefore$ V_{ab} = 5v ('a' is higher potential than 'b')

Consider only $2A$ current source only

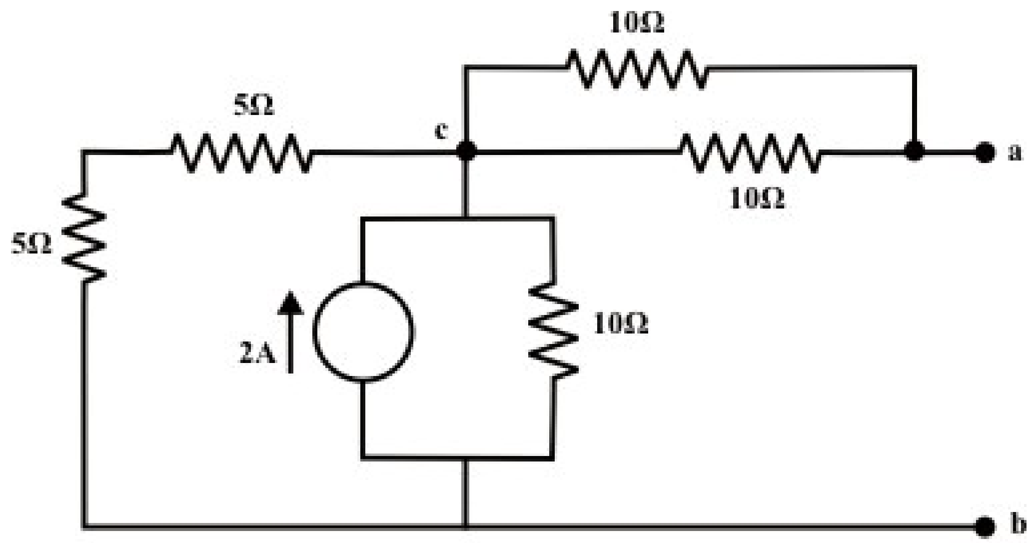


Fig. 8.7(e)

Note that the current flowing the 'c-a' branch is zero

$\therefore$ V_{ab} =10 v ('a' is higher potential than 'b' point).

The voltage across the 'a' and 'b' terminals due to the all sources = V_{Th} = V_{ab} (due to 20v) + V_{ab} (due to 10v) + V_{ab} (due to 2A source) = - 10 + 5 + 10 = 5v (a is higher potential than the point 'b').

To computute R_{Th}:

Replace all voltage and current sources by their internal resistance of the circuit shown in fig.8.7(b).

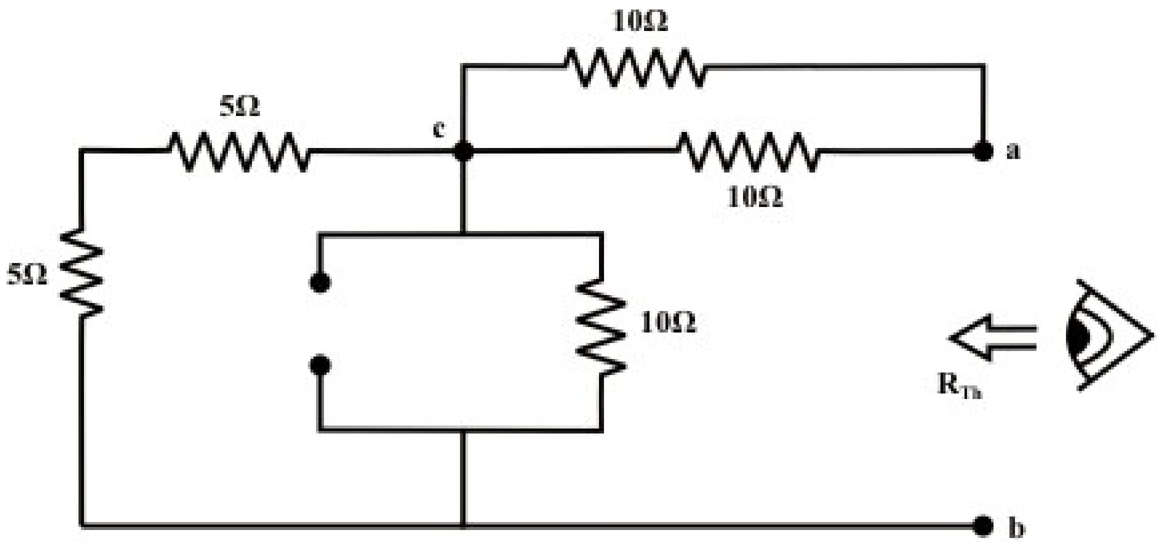
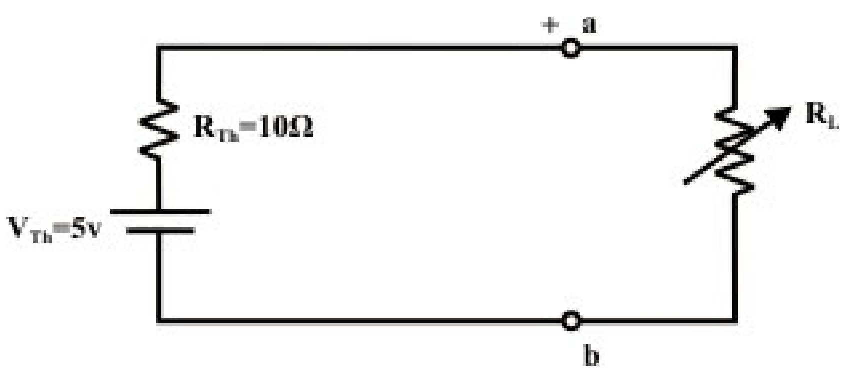


Fig. 8.7(f)

$R_{Th} = R_{ab} = ((5+5) \parallel 10) + (10 \parallel 10)$
$\quad = 5 + 5 = 10\ \Omega$

Thevenin equivalent circuit is drawn below:



The choice of R_L that absorbs maximum power from the circuit is equal to the value of Thevenin resistance R_{Th}

$R_L = R_{Th} = 10\Omega$

Under this condition, the maximum power dissipated to R_L is

$$P_{max} = \frac{1}{4}\frac{V_{Th}^2}{R_{Th}} = \frac{1}{4}\cdot\frac{25}{10} = 0.625 \text{ watts.}$$

L.8.5 Proof of Thevenin Theorem

The basic concept of this theorem and its proof are based on the principle of superposition theorem. Let us consider a linear system in fig.L.8.8(a).

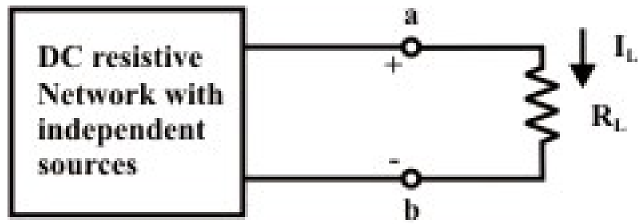


Fig. 8.8(a)

It is assumed that the dc resistive network is excited by the independent voltage and current sources. In general, there will be certain potential difference ($V_{oc} = V_{Th}$) between the terminals 'a' and 'b' when the load resistance R_L is disconnected from the terminals. Fig.8.8(b) shows an additional voltage source E (ideal) is connected in series with the load resistance R_L in such a way (polarity of external voltage source E in opposition the open-circuit voltage V_{oc} across 'a' and 'b' terminals) so that the combined effect of all internal and external sources results zero current through the load resistance R_L .

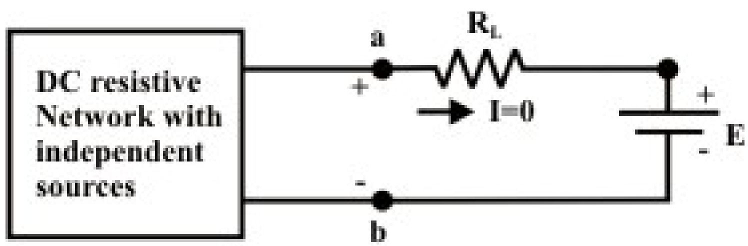


Fig. 8.8(b)

According to the principle of superposition, zero current flowing through R_L can be considered as a algebraic sum (considering direction of currents through R_L) of (i) current through R_L due to the external source E only while all other internal sources are replaced by their internal resistances (all voltage sources are short-circuited and all current sources are open circuited), and (ii) current through R_L due to the combined effect of all internal sources while the external source E is shorted with a wire. For the first case, assume the current $I_1 \left(= \dfrac{E}{R_{Th} + R_L} \right)$ (due to external source E only) is flowing through R_L from right to left direction($\leftarrow$) as shown in fig.8.8(c).

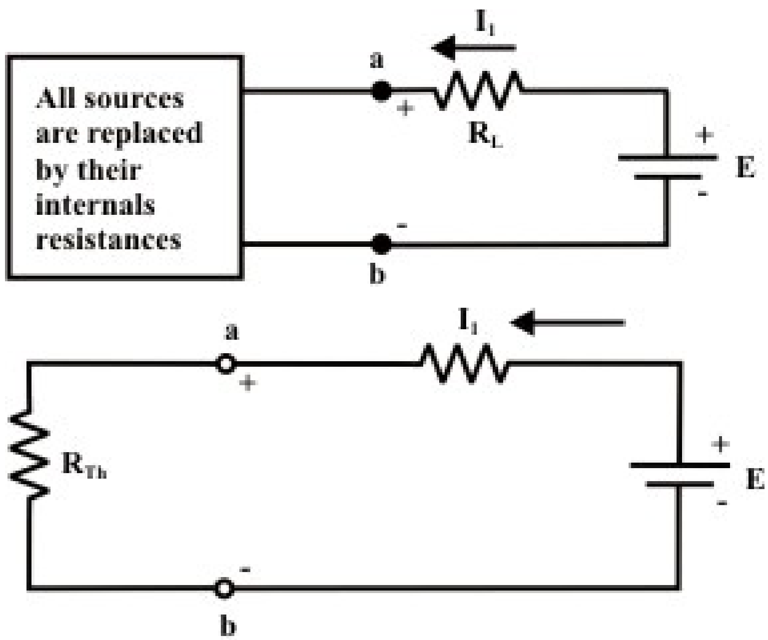


Fig. 8.8(c)

For the second case, the current I_2 (due to combined effect of all internal sources only) is flowing through R_L with same magnitude of I_1 but in opposite direction (left to right). Note that the resultant current I through the resistor R_L is zero due to the combination of internal and external sources (see fig.8.8(b)). This situation will arise provided the voltage (V_{ab}) across the 'a' and 'b' terminals is exactly same (with same polarity) as that of external voltage E and this further implies that the voltage across V_{ab} is nothing but an open-circuit voltage (V_{Th}) while the load resistance R_L is disconnected from the terminals 'a' and 'b'. With the logics as stated above, one can find the current expression $I_2\left(=\dfrac{V_{Th}}{R_{Th}+R_L}\right)$ for the circuit (fig.8.8(b)) when the external source E is short-circuited with a wire. In other words, the original circuit (fig.8.8(a)) can be replaced by an equivalent circuit that delivers the same amount of current I_L through R_L. Fig.8.8(d) shows the equivalent Thevenin circuit of the original network (fig.8.8(a)).

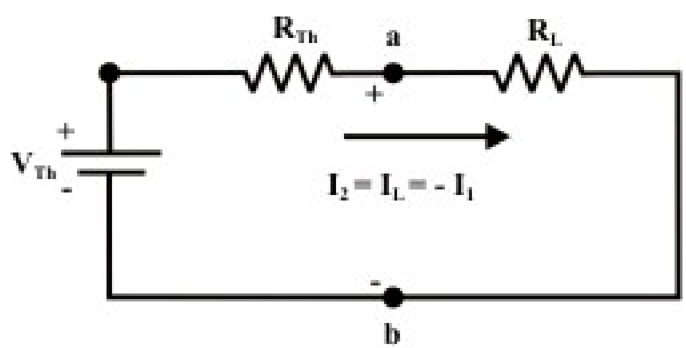


Fig. 8.8(d)

L.8.6 Norton's theorem

Norton's theorem states that any two terminals A & B of a network composed of linear resistances (see fig.8.9(a)) and independent sources (voltage or current, combination of voltage and current sources) may be replaced by an equivalent current source and a parallel resistance. The magnitude of current source is the current measured in the short circuit placed across the terminal pair A & B. The parallel resistance is the equivalent resistance looking into the terminal pair A & B with all independent sources has been replaced by their internal resistances.

Any linear dc circuit, no matter how complicated, can also be replaced by an equivalent circuit consisting of one dc current source in parallel with one resistance. Precisely, Norton's theorem is a dual of Thevenin's theorem. To find a current I_L through the load resistance R_L (as shown in fig.8.9(a)) using Norton's theorem, the following steps are followed:

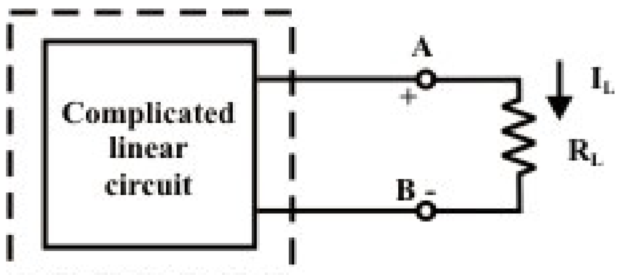


Fig. 8.9(a)

Step-1: Short the output terminal after disconnecting the load resistance (R_L) from the terminals A & B and then calculate the short circuit current I_N (as shown in fig.8.9(b)). In general, one can apply any of the techniques (mesh-current, node-voltage and superposition method) learnt in earlier lessons to compute I_N (experimentally just measure the short-circuit current using an ammeter).

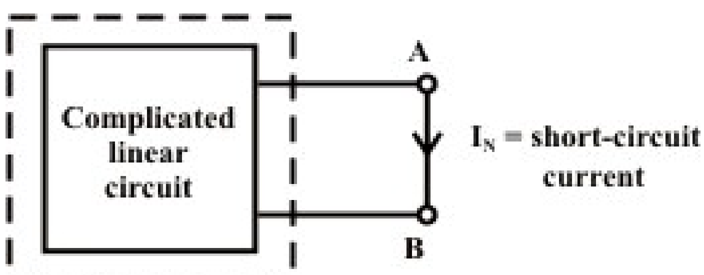


Fig. 8.9(b)

Step-2: Redraw the circuit with each practical sources replaced by its internal resistance while the short–circuit across the output terminals removed (note: voltage sources should be short-circuited (just replace with plain wire) and current sources should be open-circuited (just removed)). Look backward into the resulting circuit from the load terminals ($A \& B$), as suggested by the eye in fig.8.9(c).

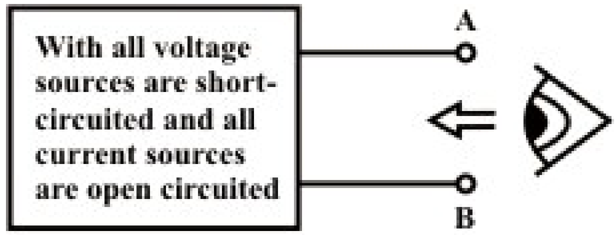


Fig. 8.9(c)

Step-3: Calculate the resistance that would exist between the load terminals A & B (or equivalently one can think as if a voltage source is applied across the load terminals and then trace the current distribution through the circuit (fig.8.9(c)) in order to calculate the resistance across the load terminals). This resistance is denoted as R_N , is shown in fig.8.9 (d). Once again, calculating this resistance may be a difficult task but one can try to use the standard circuit reduction technique or $Y-\Delta$ *or* $\Delta-Y$ transformation techniques. It may be noted that the value of Norton's resistance R_N is truly same as that of Thevenin's resistance R_{Th} in a circuit.

Step-4: Place R_N in parallel with current I_N to form the Norton's equivalent circuit (replacing the imaginary fencing portion or fixed part of the circuit with an equivalent practical current source) as shown in fig.8.8 (d).

Step-5: Reconnect the original load to the Norton current circuit; the load's voltage, current and power may be calculated by a simple arithmetic operation only.

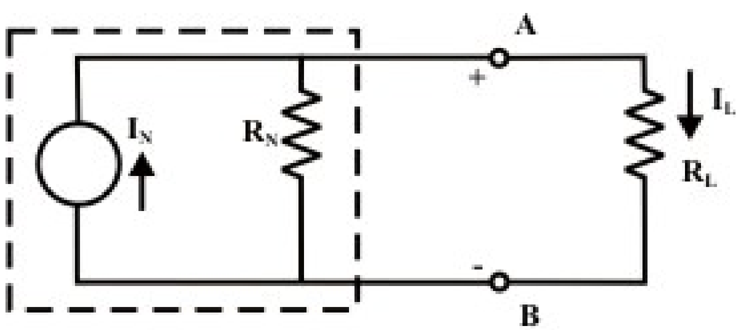


Fig. 8.9(d): Norton's equivalent circuit of the original network

Load current $I_L = \frac{R_N}{R_N + R_L} \times I_N$ (8.9)

Voltage across the load $V_L = I_L \times R_L$ (8.10)

Power absorbed by the load $P_L = I_L^{\,2} \times R_L$ (8.11)

Remarks: (i) Similar to the Thevenin's theorem, Norton's theorem has also a similar advantage over the normal circuit reduction technique or any other technique when it is used to calculate load current (I_L), load voltage (V_L) and load power (P_L) for different loads.

(ii)Fortunately, with help of either Norton's theorem or Thevenin's theorem one can find the choice of load resistance R_L that results in the maximum power transfer to the load.

(iii) Norton's current source may be replaced by an equivalent Thevenin's voltage source as shown in fig.L.8.1(b). The magnitude of voltage source (V_{Th}) and its internal resistances (R_{Th}) are expressed by the following relations

$V_{Th} = I_N \times R_N$; $R_{Th} = R_N$ (with proper polarities at the terminals)

In other words, a source transformation converts a Thevenin equivalent circuit into a Norton equivalent circuit or vice-verse.

L.8.7 Application of Norton's Theorem

Example-L.8.5 For the circuit shown in fig.8.10(a), find the current through $R_L = R_2 = 1\Omega$ resistor (I_{a-b} branch) using Norton's theorem & hence calculate the voltage across the current source (V_{cg}).

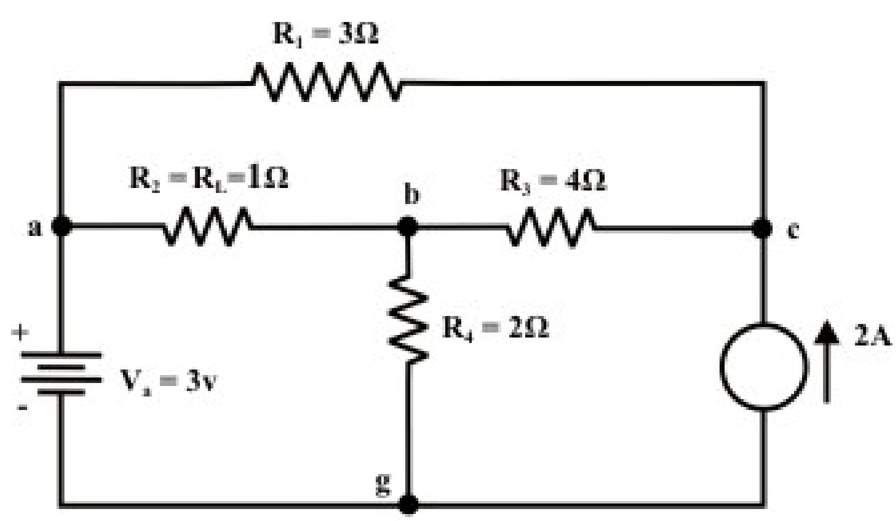


Fig. 8.10(a)

Solution:

Step-1: Remove the resistor through which the current is to be found and short the terminals 'a' and 'b' (see fig.8.10(b)).

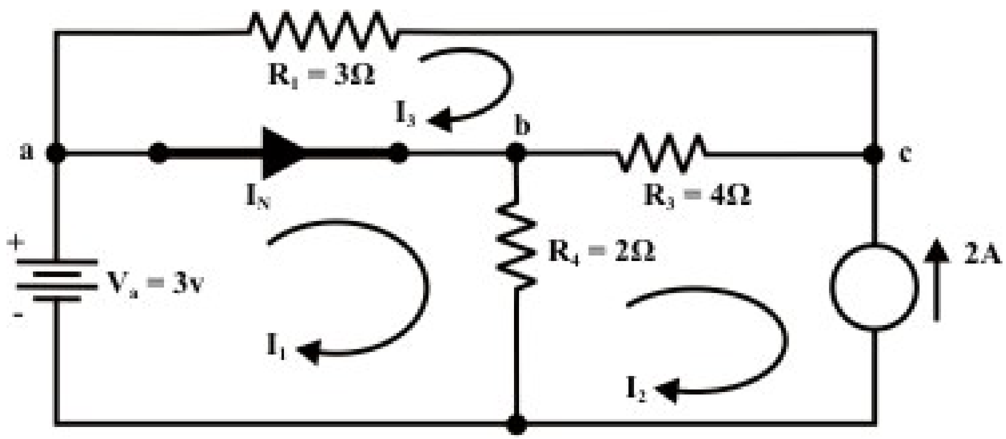


Fig. 8.10(b)

Step-2: Any method can be adopted to compute the current flowing through the a-b branch. Here, we apply 'mesh – current' method.

Loop-1

$3 - R_4(I_1 - I_2) = 0$, where $I_2 = -2A$

$R_4 I_1 = 3 + R_4 I_2 = 3 - 2 \times 2 = -1 \quad \therefore I_1 = -0.5A$

Loop-3

$$-R_1 I_3 - R_3(I_3 - I_2) = 0$$
$$-3I_3 - 4(I_3 + 2) = 0$$
$$-7I_3 - 8 = 0$$
$$I_3 = -\frac{8}{7} =$$
$$\therefore I_N = (I_1 - I_3) = \left(-0.5 + \frac{8}{7}\right) = \frac{-7 + 16}{14}$$
$$= \frac{9}{14} \text{A (current is flowing from 'a' to 'b')}$$

Step-3: To compute R_N, all sources are replaced with their internal resistances. The equivalent resistance between 'a' and 'b' terminals is same as the value of Thevenin's resistance of the circuit shown in fig.8.3(d).

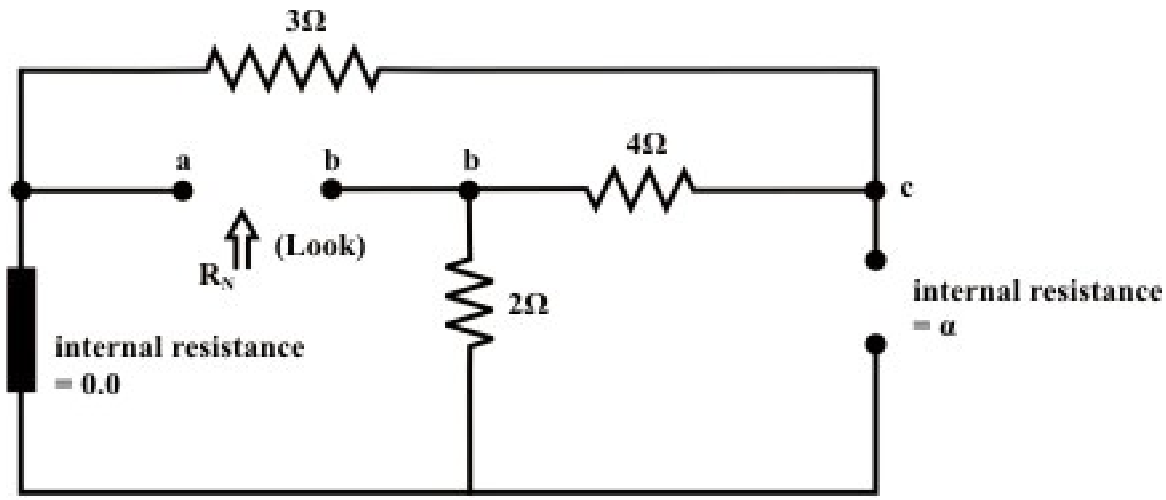


Fig. 8.10(c)

Step-4: Replace the original circuit with an equivalent Norton's circuit as shown in fig.8.10(d).

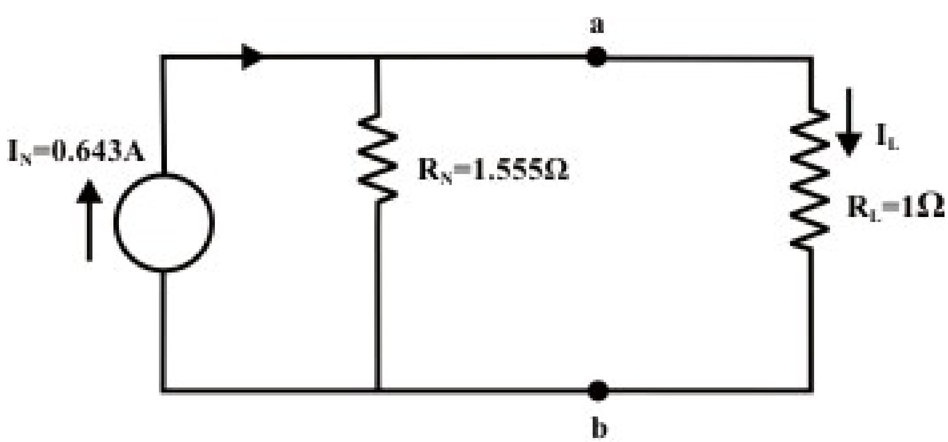
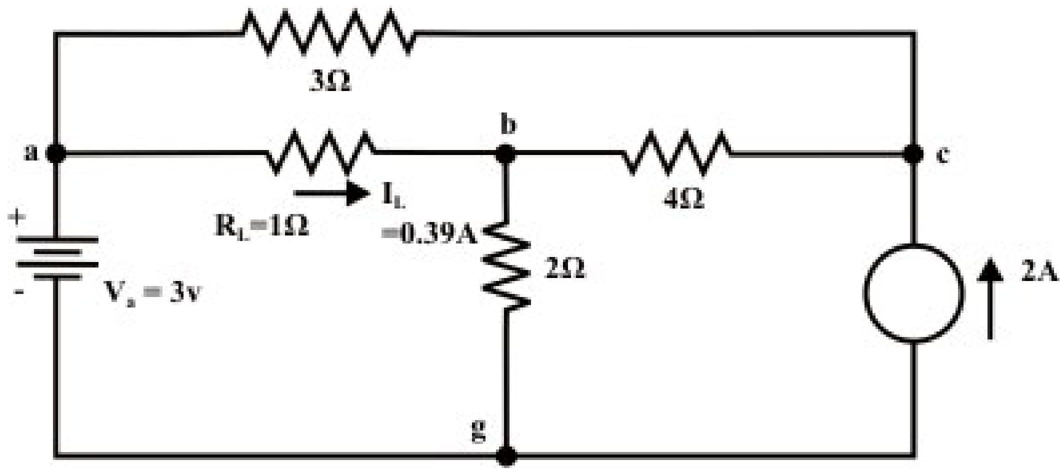


Fig. 8.10(d)

$$I_L = \frac{R_N}{R_N + R_L} \times I_N = \frac{1.555}{1.555 + 1} \times 0.643 = 0.39\text{A (a to b)}$$

In order to calculate the voltage across the current source the following procedures are adopted. Redraw the original circuit indicating the current direction in the load.



$$V_{bg} = 3 - 1 \times 0.39 = 2.61\text{volt}$$

$$I_{bg} = \frac{2.61}{2} = 1.305\text{A}$$

$$I_{cb} = 1.305 - 0.39 = 0.915\text{A ('c' to 'b')}$$

$$\therefore\ V_{cg} = 2 \times 1.305 + 4 \times .915 = 6.26\text{volt ('c' is higher potential than 'g')}$$

Example-L.8.6 For the circuit shown in fig.8.11(a), the following measurements are taken and they are given in table.

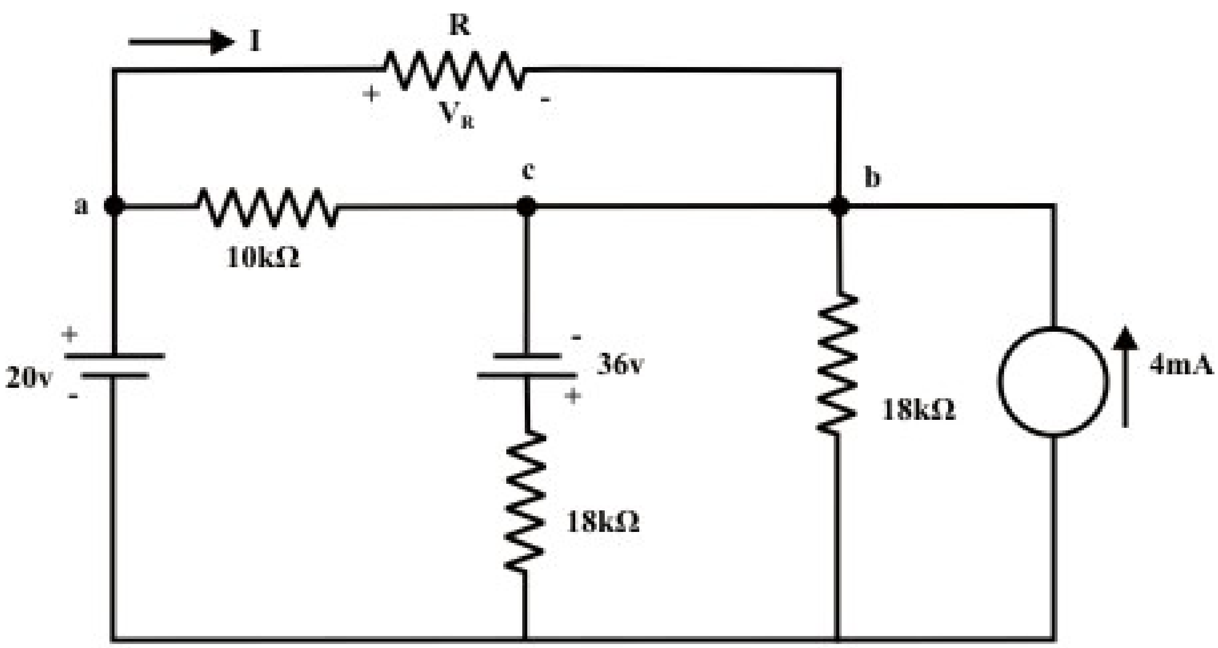


Fig. 8.11(a)

Table

R	I	V_R
Open	$0\ ma$	$1.053\,V$
Short	$0.222\,ma$	$0\,V$
?	$0.108\ ma$	**?**
$25\,k\Omega$	**?**	**?**

Find the current following through the resistor when $R = 25\,k\Omega$ and voltage drop across the resistor.

Solution: First measurement implies the Thevenin's voltage (V_{Th}) across the terminals 'a' and 'b' $= 1.053\,V$.

Second measurement implies the Norton's current (I_N) through the shorted terminals 'a' and 'b' $= 0.222\,ma$.

With the above two measurements one can find out the Thevenin's resistance $R_{Th}\,(= R_N)$ using the following relation

$$R_{Th} = \frac{V_{Th}}{I_N} = \frac{1.053}{0.222 \times 10^{-3}} = 4.74\,k\Omega$$

Thevenin equivalent circuit between the terminals 'a' and 'b' of the original circuit is shown in fig.8.11(b).

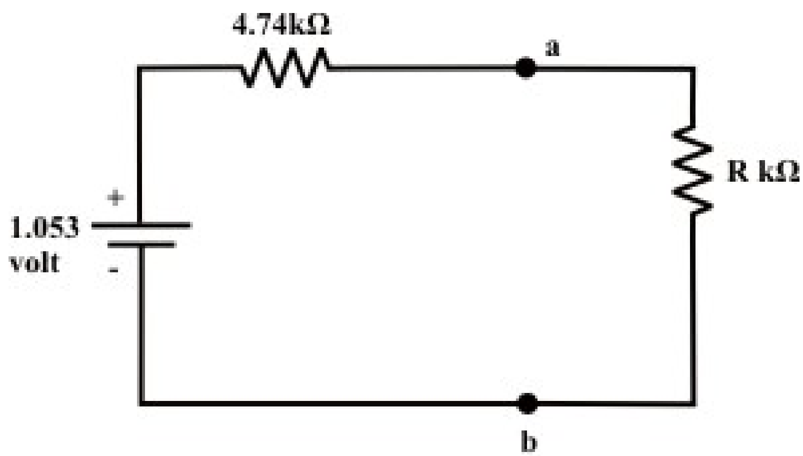


Fig. 8.11(b): Therenin equivalent circuit between the terminal 'a' and 'b'

Third measurement shows that the current in resistor R is given by $= \frac{1.053}{4.74 + R} = 0.108\,(mA) \Rightarrow R = 5\,k\,\Omega$. The voltage across the $5k\,\Omega$ resistor is $5 \times 0.108 = 0.54\,volt$

From the fourth measurement data, the current through $25k\Omega$ resistor is = $I = \frac{V_{Th}}{R_{Th}+R} = \frac{1.053}{(4.74+25)\times 10^3} = 0.0354\ ma$ and the corresponding voltage across the resistor $V_R = I \times R = 0.0354 \times 25 = 0.889V$.

Example-L.8.7 Applying Norton's theorem, calculate the value of R that results in maximum power transfer to the 6.2Ω resistor in fig.8.12(a). Find the maximum power dissipated by the resistor 6.2Ω under that situation.

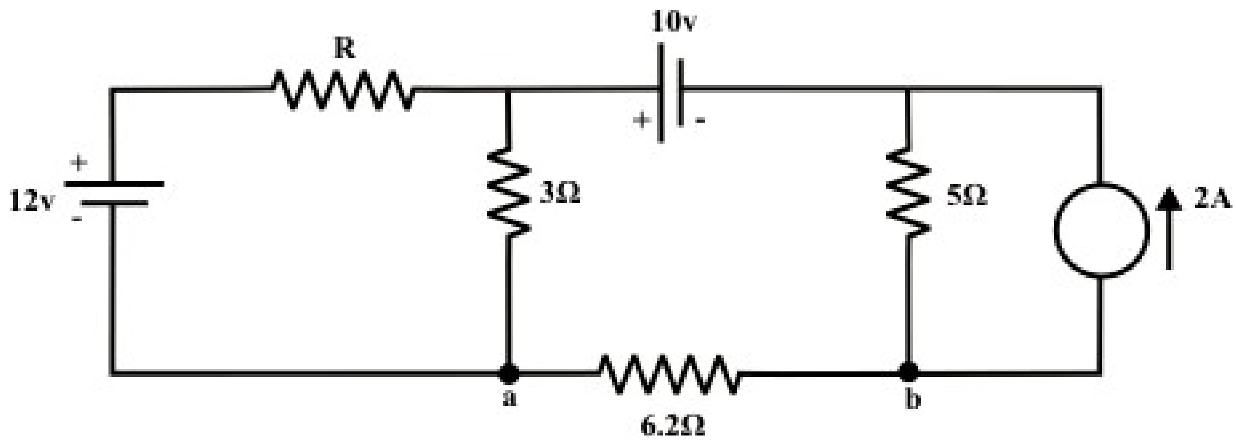


Fig. 8.12(a)

Solution:

Step-1: Short the terminals 'a' and 'b' after disconnecting the 6.2Ω resistor. The Norton's current I_N for the circuit shown in fig.8.12(b) is computed by using 'mesh-current' method.

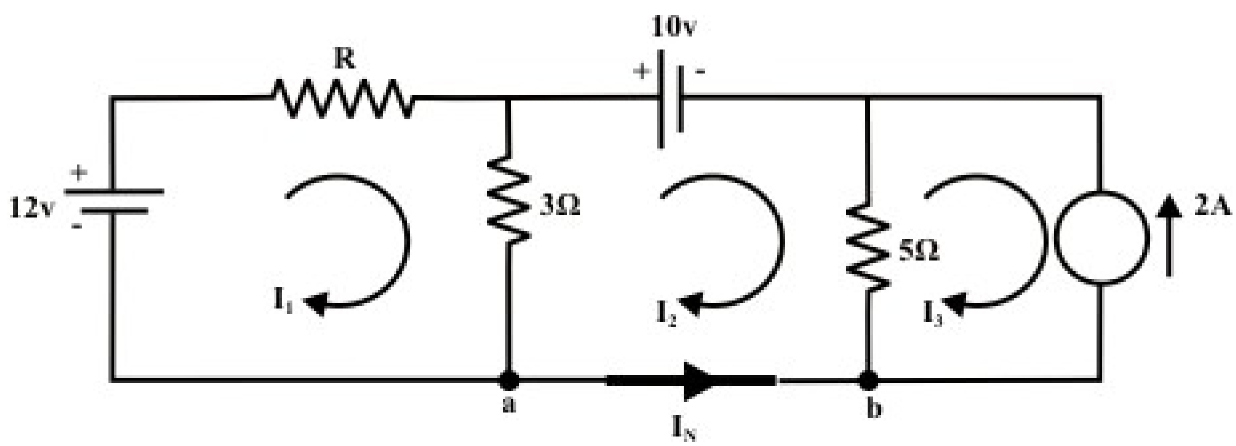


Fig. 8.12(b)

Loop-1:

$$12 - I_1 R - 3(I_1 - I_2) = 0 \tag{8.12}$$

Loop-2:

$$-10 - 5(I_2 - I_3) - 3(I_2 - I_1) = 0\ , \quad note \;\; I_3 = -2A \tag{8.14}$$

Solving equations (8.12) and (8.14), we get

$I_1 = \dfrac{36}{15+8R}$; $I_2 = -\dfrac{24+20R}{15+8R}$ (-ve sign implies that the current is flowing from 'b' to 'a') and Norton's current $I_N = -I_2 = \dfrac{24+20R}{15+8R}$

Norton's resistance R_N is computed by replacing all sources by their internal resistances while the short-circuit across the output terminal 'a' and 'b' is removed. From the circuit diagram fig.8.12(c), the Norton's resistance is obtained between the terminals 'a' and 'b'.

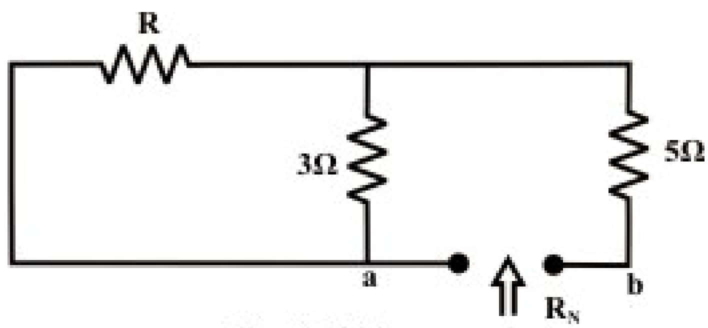


Fig. 8.12(c)

$$R_N = (R \| 3) + 5 = \frac{3R}{3+R} + 5 \tag{8.15}$$

Note that the maximum power will dissipate in load resistance when load resistance = Norton's resistance $R_N = R_L = 6.2\Omega$. To satisfy this condition the value of the resistance R can be obtained from equation (8.15), we get $R = 2\Omega$.. The circuit shown in fig.8.12(a) is now replaced by an equivalent Norton's current source (as shown in fig.L.8.12(d)) and the maximum power delivered by the given network to the load $R_L = 6.2\Omega$ is thus given by

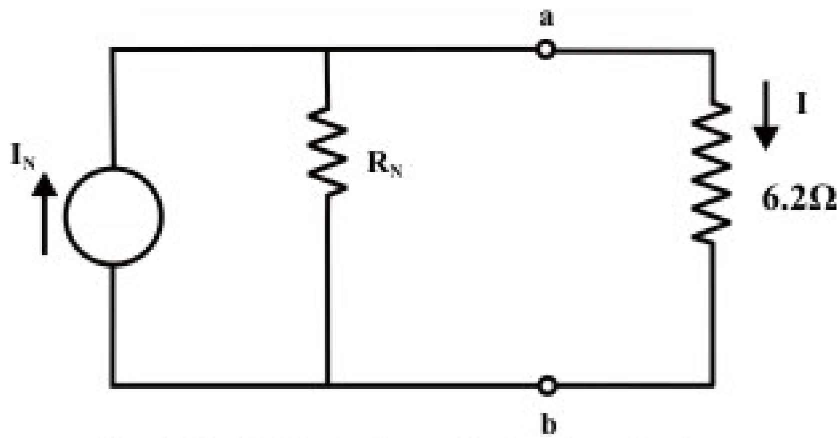


Fig. 8.12(d): Norton's equivalent circuit of original circuit

$$P_{max} = \frac{1}{4} \times I_N^{\;2} R_L = \frac{1}{4} \times \left(\frac{24 + 20R}{15 + 8R} \right)^2 \times R_L = 6.61 \, watts$$

L.8.8 Test Your Understanding [Marks: 60]

T.1 When a complicated dc circuit is replaced by a Thevenin equivalent circuit, it consists of one ------- in series with one --------- . [2]

T.2 When a complicated dc circuit is replaced by a Norton equivalent circuit, it consists of ------ in ----- with one -------. [2]

T.3 The dual of a voltage source is a -----------. [1]

T.4 When a Thevenin theorem is applied to a network containing a current source; the current source is eliminated by --------- it. [1]

T.5 When applying Norton's theorem, the Norton current is determined with the output terminals --------------, but the Norton resistance is found with the output terminals ---------.and subsequently all the independent sources are replaced -----------. [3]

T.6 For a complicated circuit, the Thevenin resistance is found by the ratio of -------- voltage and ------------ current. [2]

T.7 A network delivers maximum power to the load when its -------- is equal to the -------- of circuit at the output terminals. [2]

T.8 The maximum power transfer condition is meaningful in ------------ and --------- systems. [2]

T.9 Under maximum power transfer conditions, the efficiency of the system is only --------- %. [1]

T.10 For the circuit in fig.8.13, find the voltage across the load resistance $R_L = 3\Omega$ using Thevenin theorem. Draw the Thevenin equivalent circuit between the terminals 'a' and 'b' when the load resistance R_L is disconnected. Calculate the maximum power delivered by the circuit to the load $R_L = 3\Omega$. [6]

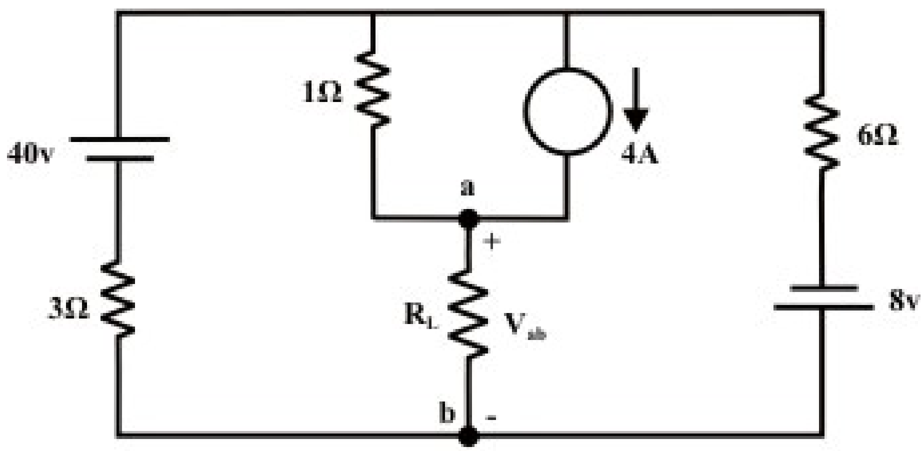


Fig. 8.13

(Ans. $V_{ab} = 18$ volt, $P_{\max} = 108\,W$)

T.11 Solve the problem given in T.10 applying Norton's theorem. [6]
(Ans. $I_N = 12\,A$, $R_N = 3\Omega$)

T.12 For the circuit in fig.8.14, calculate the value of R that results in maximum power transfer to the 10Ω resistor connected between (i) 'a' and 'b' terminals (ii) 'a' and 'c' terminals. Indicate the current direction through (a) a-b branch (b) a-c branch and their magnitudes. [6+6]

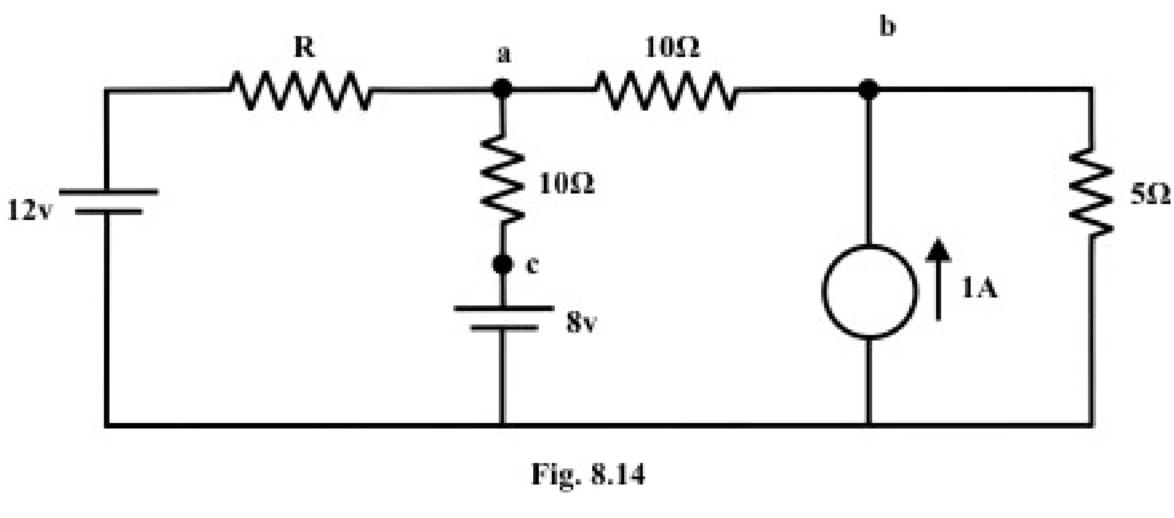


Fig. 8.14

(Ans. (i) $R = 10\Omega$, $I_{ab} = 250\,mA\ (a \rightarrow b)$ (ii) $R = 30\Omega$, $I_{ac} = 33.3\ mA\ (a \rightarrow c)$)

T.13 The box shown in fig.8.15 consists of a dc sources and resistors. Measurements are made at the terminals ‘a’ and ‘b’ and the results are shown in the table. Find the choice of ‘R’ that delivers maximum power to it and subsequently predict the reading of the ammeter under this situation. [6]

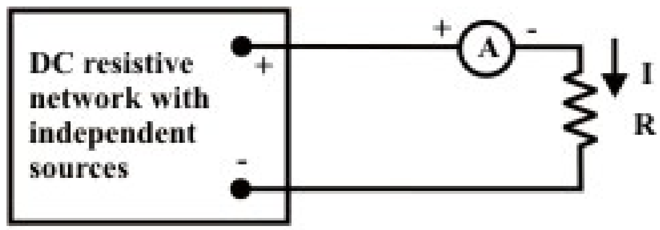


Fig. 8.15

Table

R	I
10Ω	$2A$
80Ω	$0.6A$

(Answer: $R = 20\Omega,\ P_{\max} = 45\, watts$)

T.14 For the circuit shown in fig.8.16, find the value of current I_L through the resistor $R_L = 6\Omega$ using Norton’s equivalent circuit and also write the Norton’s equivalent circuit parameters between the terminals $'A'$ *and* $'B'$. [7]

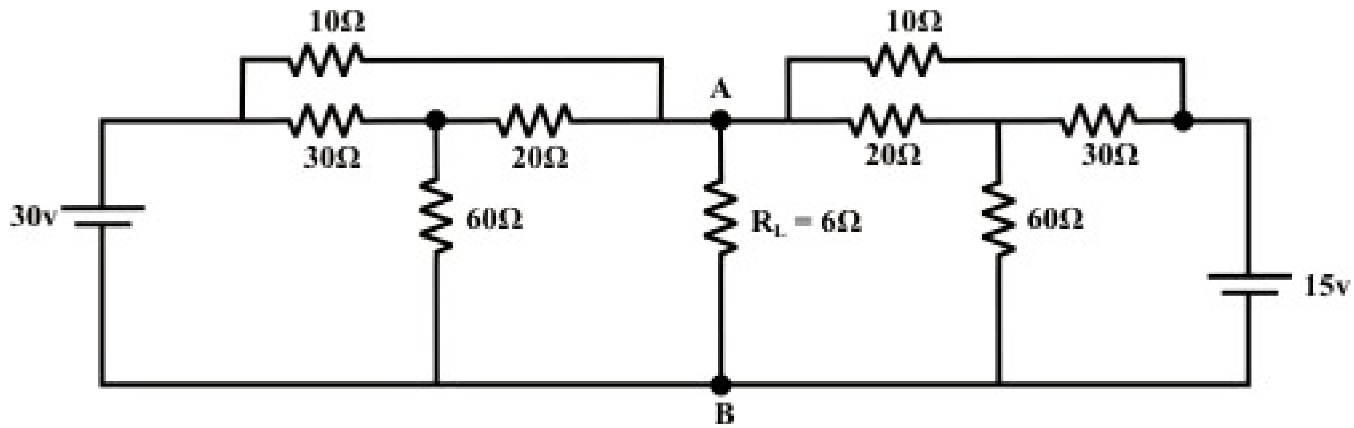


Fig. 8.16

(Ans. $I_L = 2.1\ A,\ I_N = 5.25A;\ R_N = 4\Omega$)

T.15 Find the values of design parameters $R_{1,}\ R_2$ *and* R_3 such that system shown in fig.17(a) satisfies the relation between the current I_L and the voltage V_L as described in fig.8.17(b). Assume the source voltage $V_s = 12\,volt$ and the value of resistance R_2 is the geometric mean of resistances R_1 & R_3. [7]

(Ans. $R_1 = R_2 = R_3 = 0.5k\Omega$)

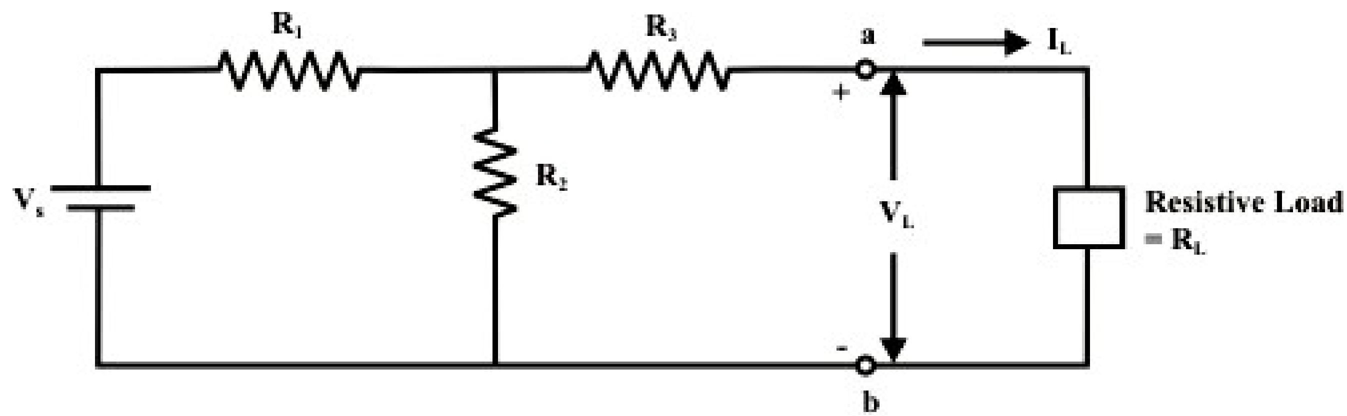


Fig. 8.17(a)

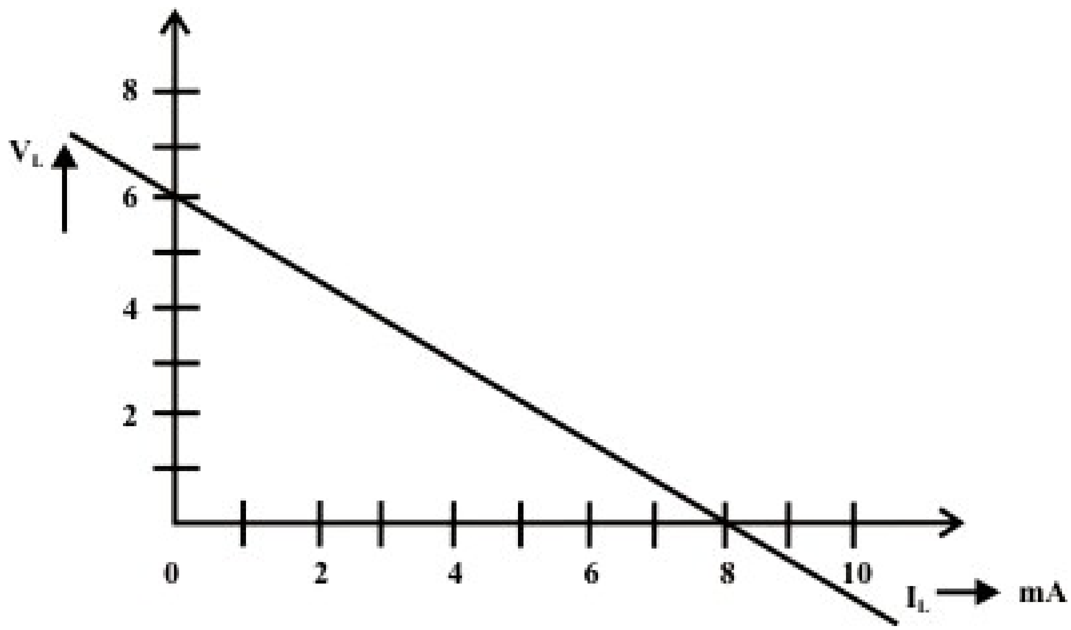


Fig. 8.17(b): Volt-Amp. characteristics at the terminals 'a' and 'b'