## 7.5 Analysis of girder bridges

As discussed above, bridge decks are required to support both static and moving loads. Each element of a bridge must be designed for the most severe conditions that can possibly be developed in that member. Live loads should be placed in such a way that they will produce the most severe conditions. The critical positions of live loads will not be the same for every member. A useful method for determining the most severe condition of loading is by using "influence lines".

An influence line represents some internal force such as shear force, bending moment etc. at a particular section or in a given member of girder, as a unit load moves over the span. The ordinate of influence line represents the value of that function when the unit load is at that particular point on the structure. Influence lines provide a systematic procedure for determining how the force (or a moment or shear) in a given part of a structure varies as the applied load moves about on the structure. Influence lines of responses of statically determinate structures consist only of straight lines whereas this is not true of indeterminate structures. It may be noted that a shear or bending moment diagram shows the variation of shear or moment across an entire structure for loads fixed in one position. On the other hand an influence line for shear or moment shows the variation of that response at one particular section in the structure caused by the movement of a unit load from one end of the structure to the other. In the following section, influence lines only for statically determinate structures are discussed.

### 7.5.1 Influence lines for beams and plate girders

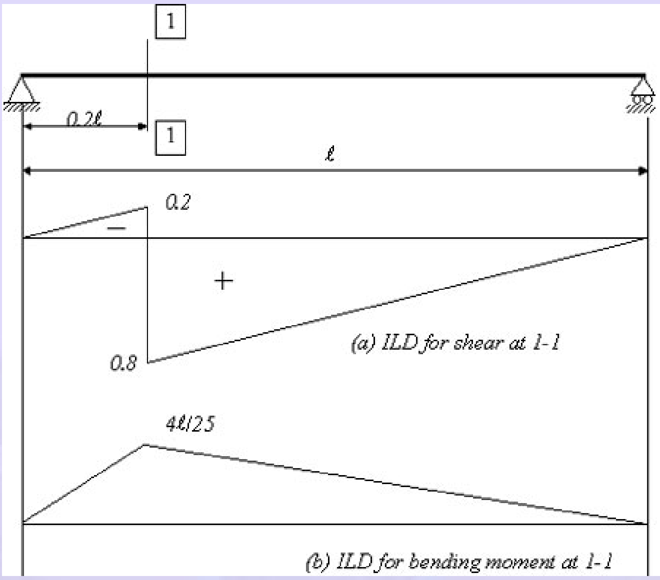


**Fig. 7.12 Influence lines for shear and bending moment**

Fig. 7.12(a) shows the influence line for shear at a section in a simply supported beam. It is assumed that positive shear occurs when the sum of the transverse forces to the left of a section is in the upward direction or when the sum of the forces to the right of the section is downward.

A unit force is placed at various locations and the shear force at sections 1-1 is obtained for each position of the unit load. These values give the ordinates of influence line with which the influence line diagram for shear force at sections 1-1 can be constructed. Note that the slope of the influence line for shear on the left of the section is equal to the slope of the influence line on the right of the section. This information is useful in drawing shear force influence line in all cases.

Influence line for bending moment at the same section 1-1 of the simple beam is shown in Fig. 7.12(b). For a section, when the sum of the moments of all the forces to the left is clockwise or when the sum to the right is counter-clockwise, the moment is taken as positive. The values of bending moment at sections 1-1 are obtained for various positions of unit load and influence line is plotted. The detailed calculation of ordinates of influence lines is illustrated for members of the truss girder in the following section.

### 7.5.2 Influence lines for truss girders

Influence lines for support reactions and member forces for truss may be constructed in the same manner as those for beams. They are useful to determine the maximum force that may act on the truss members. The truss shown in Fig.7.13 is considered for illustrating the construction of influence lines for trusses.

The member forces in $U_3U_4$, $U_3L_4$ and $L_3L_4$ are determined by passing a section X-X and considering the equilibrium of the free body diagram of one of the truss segments.

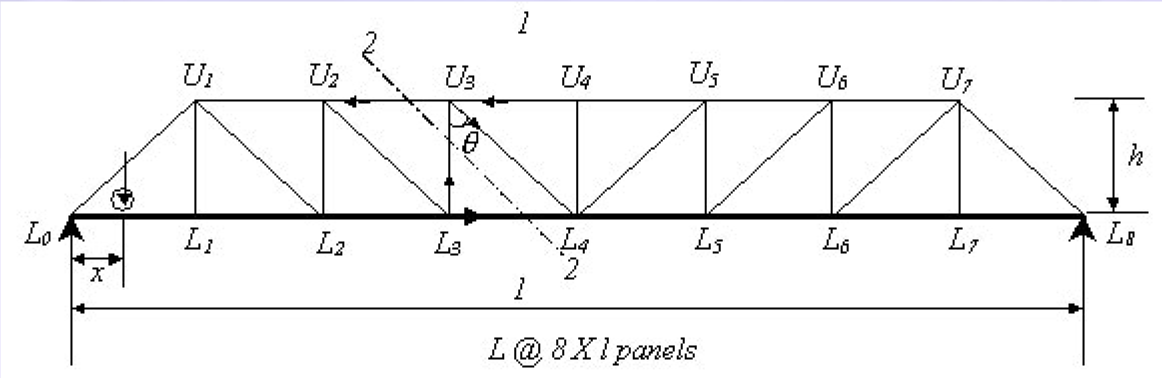


**Fig.7.13 A typical truss**

Consider a section 1-1 and assume unit-rolling load is at a distance x from $L_0$. Then, from equilibrium considerations reactions at $L_8$ and $L_0$ are determined. The reactions are:

$$\text{Reaction at } L_8 = \left(\frac{x}{L}\right)$$

$$\text{Reaction at } L_0 = \left(1 - \frac{x}{L}\right)$$

Consider the left-hand side of the section and take moments about $L_4$ by assuming appropriate directions for the forces in the members.

When unit load is in between $L_0$ and $L_4$:

$$\sum M_{L4} = 0$$

$$U_3 U_4 \text{ x } h - \left(\frac{x}{L}\right) \text{x } 4l = 0$$

$$U_3 U_4 = \frac{x}{h}\frac{4l}{L} = 0.5\frac{x}{h}$$

When unit load is in between $L_4$ and $L_8$:

Then, there will not be rolling unit load in the left-hand side section.

$$U_3 U_4 = \frac{4l}{h}\left(1 - \frac{x}{L}\right)$$

Note that the influence diagram gives force in the member $U_3U_4$ directly, due to the unit load.

### 7.5.2.2 Influence line diagram for member $U_3L_4$ (Inclined member) [Fig 7.14(b)]

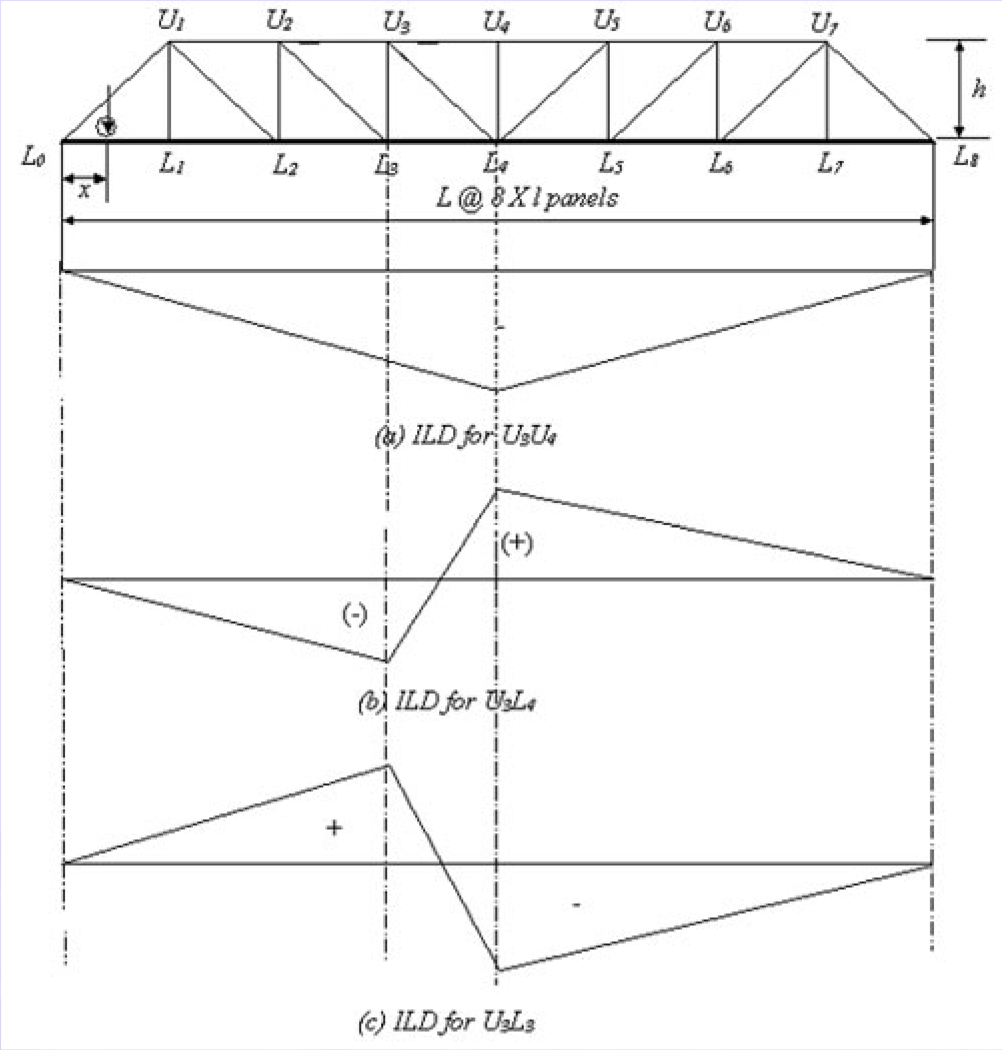


**Fig.7.14 Typical shapes of influence lines**

Again consider the left-hand side of the section 1-1, and use the equilibrium equation for vertical forces i.e

$\sum V = 0$ where, V represents the vertical force.

When unit load is in between $L_0$ and $L_3$:

$$\frac{x}{L} + U_3 L_4 \cos\theta = 0$$

$$\Rightarrow U_3 L_4 = \frac{-x}{L\cos\theta}$$

$$\text{Where, } \theta = \tan^{-1}\left(\frac{l}{h}\right)$$

When unit load is in between $L_4$ and $L_8$:

$$U_3 U_4 = \frac{1}{\cos\theta}\left(1 - \frac{x}{L}\right)$$

When unit load is in between $L_3$ and $L_4$:

Since the variation of force in member $U_3L_4$ is linear as the unit load moves from $L_3$ to $L_4$ joining the ordinates of influence line at $L_3$ and $L_4$ by a straight line gives the influence line diagram in that zone. Note that, $U_3L_4$ represents the force in that member.

### 7.5.2.3 Influence line diagram for $U_3L_3$ (Vertical member) [Fig. 7.14(c)]

Consider the left-hand side of the section 2-2 shown in Fig.7.13 for illustrating the construction of influence line for vertical member.

When unit load is in between $L_0$ and $L_3$:

By considering the equilibrium equation on the section left hand side of axis 2-2.

$$U_3 L_4 - \frac{x}{L} = 0$$

$$\Rightarrow U_3 L_4 = \frac{x}{L}$$

When unit load is in between $L_4$ and $L_8$:

$$U_3 L_4 = -\left(1 - \frac{x}{L}\right)$$

When unit load is in between $L_3$ and $L_4$:

Joining the ordinates of influence line at $L_3$ and $L_4$ by a straight line gives the influence line diagram between $L_3$ and $L_4$. $U_3L_3$ represents the force in that member.

Similarly influence line diagrams can be drawn for all other members. Typical shapes of influence line diagrams for the members discussed are shown in Fig.7.14. The design force in the member is obtained in the following manner. In this chapter, compressive forces are considered negative and tensile forces are positive.

**Case (1):** If the loading is Railway loading (UDL)

· Influence line diagram for force is drawn for that member

· The algebraic sum of areas of influence line under loaded length multiplied by magnitude of uniformly distributed load gives the design force.

**Case (2):** If the loading is Highway loading (Concentrated loading)

· Influence line diagram for force is drawn for that member

· The algebraic sum of the respective ordinates of influence line at the concentrated load location multiplied by concentrated loads gives design load of that member

· The series of concentrated loads are arranged in such a way that the maximum value of the desired member force is obtained.