

U Size reduction of Solids

Size reduction: The process of converting the object from one physical dimension of higher order to another dimension of smaller order.

- It is the operation carried out for reducing the size of bigger particles into smaller ones of desired size and shape with the help of certain external forces.
- Comminution is another term used for size reduction.

Objectives of size reduction

- To increase the surface area, because in most chemical reactions and some unit operations (drying, adsorption, leaching, etc.) involving solid particles, the reaction/transfer rate is directly proportional to the area of contact b/w the solid and the second phase
- To produce the solid particles of desired shape, size or size ranges, and specific surface.
- To separate unwanted particles effectively
- To dispose solid wastes easily
- To mix solid particles more intimately, and
- To improve the handling (storing and transportation) characteristics.

Size reduction methods.

→ Impact, compression, attrition and shear

Impact: The particle is subjected to a single violent force

- Instantaneous collision of one moving object against another.
- Both objects may be moving - ex: bat connecting with a fast moving ball
- One object may be motionless - ex: Rock being struck by a hammer below.

Gravity impact: The free falling material is momentarily stopped by the stationary object.

Ex: coal dropped onto a hard steel surface is an example of gravity impact.

→ It is most often used when it is necessary to separate two materials which have relatively different friability

Friable: Something that is crumbly or easily broken into a lot of little pieces.

→ The more friable is broken first, while the less friable material remains unbroken.

Dynamic impact: Ex: Materials dropped in front of a moving hammer

→ When crushed by dynamic impact, the material is unsupported and the force of impact accelerates movement of the reduced particles towards the breaker plate and/or other hammers.

Needed: i) for cubical particle, (ii) product must be well graded and must meet intermediate size specifications
iii) Materials are too hard and abrasive

Compression:-

- The particle is broken by two forces and the size reduction is done b/n two surfaces, with the work being done by one or both surfaces.
- Jaw crushers using this method of size reduction are suitable for reducing extremely hard and abrasive rock.
- As a mechanical reduction method, compression is chosen
 - (i) If the material is hard and tough,
 - (ii) If the material is abrasive
 - (iii) If the material is not sticky.
 - (iv) Where the finished product is to be relatively coarse in size, and
 - (v) When the material will break cubically.

Attrition:-

- It is a method of size reduction by rubbing or scrubbing the materials b/n two hard surfaces.
- Hammer mills operate with close clearances b/n the hammers and the screen bars, reduce the size of materials by attrition combined with shear and impact actions.
- Though it consumes more power, it is preferred for crushing the less abrasive materials such as limestone and coal.
- It is most useful when the material is friable or not too abrasive and a closed circuit system is not desirable to control the oversize.

shear:-

- It consists of a trimming or cleaving action rather than the rubbing action associated with attrition.
- It is usually combined with other size reduction actions eg. single roll crushers employ shear together with impact and compression.
- It is needed for friable material, primary crushing with a reduction ratio of 6 to 1, and production of a relatively coarse product.

principles of size reduction

properties of solids: For a particular size reduction operation

the choice of machine to be used mainly depends on

- i) the size and quantity of material to be handled.
- ii) the nature of the product required.

But properties of the feed material are most important.

Hardness: The resistance to scratching, and affects the power consumption

Toughness: Resistance of material to impact (reverse of friability)

Structure: Granular materials (coal or rock) can be easily crushed while fibrous materials need tearing action.

Friability: Tendency to fracture during normal handling.

Soapiness:- It is the measure of the co-efficient of friction μ' of

crystallinity: the surface of the material. If μ' is low, crushing is more difficult. The atoms in the crystal are arranged in a definite, repeating geometric pattern and there are certain planes in the crystal, called cleavage planes, along which breakage occurs.

Temperature sensitivity: The heat generated during size reduction can result in loss of heat-sensitive components from solids. Softening or melting may also be important - leading to clogging. In some cases, cryogenic crushing may be necessary using liquid nitrogen or dry ice.
 eg: in milling of spices or size reduction of meat.

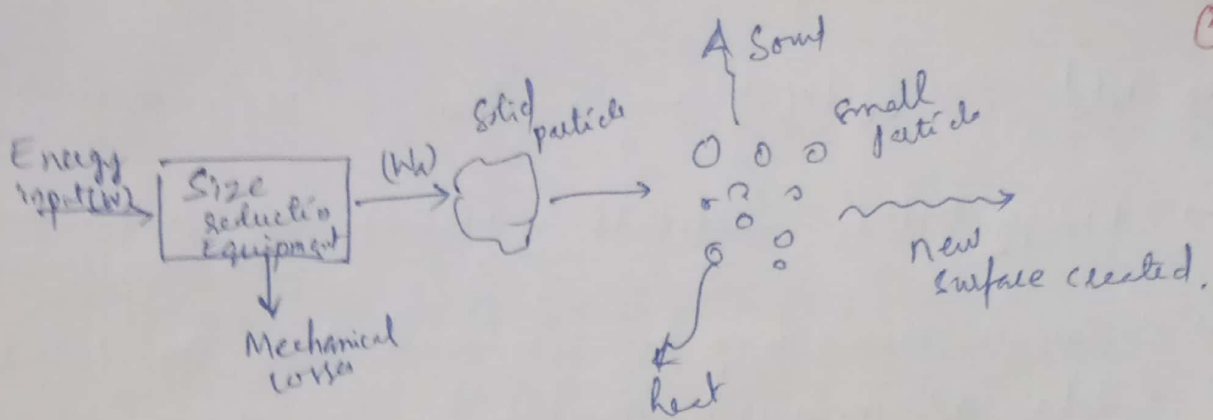
Factors affecting size reduction process

- (i) presence of moisture and sticky materials in equipments feed.
- (ii) presence of fines in the feed
- (iii) Segregation of feed particles in the crushing chamber
- (iv) Lack of feed control
- (v) Wrong motor size
- (vi) Insufficient crusher discharge area
- (vii) Insufficient capacity of the crusher's discharge conveyor
- (viii) Materials being extremely hard to crush
- (ix) Surface energy of solids
- (x) power consumption and
- (xi) selection of an appropriate crushing chamber.

Energy and power consumption in size reduction

- when external force/stress is applied for size reduction, the solid particles at first are twisted and strained.
- The work required to strain them is stored temporarily in the solids as the mechanical energy of stress.

- When additional force is applied to these already stressed particles, they are distorted beyond their ultimate strength and are suddenly broken into smaller particles, which ultimately generate new surfaces.
- The unit area of solid has a definite amount of surface energy and when its size is reduced, the surface area per unit mass, specific surface, increases.
- This creation of a new surface requires work, which is supplied by the release of energy of stress at the time of rupture.
- But it is important to note that only a small portion out of the total energy supplied to the equipment is utilized for the creation of a new surface and most of the energy is lost to overcome the friction (in the bearings and other moving parts of the machine)
- As heat (because by the principle of conservation of energy all energy of stress in excess of new surface energy created are converted to heat)
- And as sound.
- Thus, the energy efficiency is less and when most of the energy is lost, the cost of power becomes a major constraint.



Crushing efficiency

- Comminution may be a process of conversion of energy from one form to another.
- The energy is utilised in the form of potential (surface) energy, heat and sound.
- But only the potential energy is used.
- Therefore, the crushing efficiency can be defined as The ratio of the surface energy created to the energy absorbed (kinetic energy) by the solid.
- In another way, the ratio of the energy absorbed by the solid to form heat and the energy input to the machine.
- The crushing efficiency range: 0.001 to 1%.
- The quantities needed for calculation of efficiency:
 - i) Total energy input,
 - ii) energy lost during size reduction
 - iii) Total new surface created
 - iv) Specific surface energy.

- Total energy input - mechanical means or electrical instruments
- Energy lost is difficult to measure but we can measure energy consumption
- Surface area may be determined from size distribution data (or) measured directly by flow through a powder bed or by adsorption of gas molecules on the powder surface
- The specific surface energy of liquids can be measured with precision as it is numerically the same as the surface tension
 - ↳ But for solids, indirect methods based on mathematical utilisation of physicochemical quantities only are available whose accuracy largely depends on the assumptions made in measuring it.

Determination of power consumption:-

- Cost of power is a major expense in any size-reduction operation. and it is determined by using crushing efficiency.

$$\eta_c = \frac{\text{Surface energy created by crushing}}{\text{Total energy absorbed by the solid}}$$

W_a → Total energy absorbed by a unit mass of solid, J/kg

E_s → Surface energy per unit area, J/m² and

A_{sft}, A_{ssp} → Areas per unit mass of feed and product respectively (or, the specific surfaces), m²/kg.

Then the surface energy created by crushing will be $E_s (A_{ssp} - A_{sft})$

→ The crushing efficiency, η_c , becomes

$$\eta_c = \frac{E_s (A_{ssp} - A_{sft})}{W_a} \Rightarrow W_a = \frac{E_s (A_{ssp} - A_{sft})}{\eta_c}$$

→ Mechanical efficiency: the ratio of energy absorbed as the energy fed to the machine.

$$\eta_m = \frac{W_a}{W} \rightarrow \text{energy fed.}$$

Total energy input

$$W = \frac{W_a}{\eta_m}$$

$$W = \frac{E_s (A_{ssp} - A_{sft})}{\eta_m \eta_c}$$

→ If \dot{m} is the flow rate of solids to the machine then the power required, 'P', by the machine is the product of total energy input and the mass flow rate.

$$P = W \times \dot{m}$$

$$P = \frac{E_s (A_{ssp} - A_{sft})}{\eta_m \eta_c} \dot{m}$$

→ The expressions for the specific surfaces of feed and product

$$A_{sft} = \frac{6}{\phi_f \bar{D}_{vsf} \rho_{pf}} \quad \text{and} \quad A_{ssp} = \frac{6}{\phi_p \bar{D}_{vsp} \rho_{pp}}$$

- ϕ_f, ϕ_p → Sphericity of the feed and the product materials
- $\bar{D}_{vsf}, \bar{D}_{vsp}$ → Sauter mean diameter of feed and product, m
- ρ_{pf}, ρ_{pp} → Density of feed and product, kg/m³

→ For the homogeneous materials,
$$s_{pf} = s_{pf} = s_p \text{ (say)}$$

→ Substituting A_{spf} , A_{sp} in P.

$$P = \frac{6 E_s \dot{m}}{h_m \eta_c s_p} \left[\frac{1}{\phi_p \bar{D}_{vsf}} - \frac{1}{\phi_f \bar{D}_{vsf}} \right]$$

→ This relation tells us that the power requirement for crushing will be more for particles having higher surface energy and also for the higher flow rate.

→ All the particles (each having certain surface area) in an unit mass of solid particles have a definite amount of surface energy and when their size is reduced, their surface area as well as the surface energy per unit mass increases.

→ And, when this occurs, the power requirement becomes more and more for reducing fine particles to still finer ones than for breaking down large pieces of rock.

Laws of comminution

→ It is almost impossible to find out the accurate amount of energy requirement in order to effect size reduction of a given material, mainly because.

(i) There is a wide variations in the size and shape of particles both in the feed and product.

and (ii) Some energy is wasted as heat and sound, which can't be determined exactly.

→ But, a number of empirical laws have been proposed to relate the size reduction with the energy input to the machine.

→ They are Rittinger's law (1867), Kick's law (1885) and Bond's law (1952)

Rittinger's Law:- The work required for size reduction is proportional to the new surface area created.

$$W_R = \frac{P}{\dot{m}} = KE_s (A_{ssp} - A_{sft}) = 6KE_s \left[\frac{1}{\phi_p \bar{D}_{rsp}} - \frac{1}{\phi_p \bar{D}_{sft}} \right]$$

$$K = \frac{1}{n_c}$$

Rewritten as
$$W_R = \frac{6KE_s}{\phi_p} \left[\frac{1}{\bar{D}_{rsp}} - \frac{1}{\bar{D}_{sft}} \right] = K_R \left[\frac{1}{\bar{D}_{rsp}} - \frac{1}{\bar{D}_{sft}} \right]$$

$K_R = \frac{6KE_s}{\phi_p}$, is known as Rittinger's constant.

- Sometimes, the expression KE_s is used as Rittinger's const.
- But, $\frac{6KE_s}{\phi \rho_p}$ and KE_s are used as Rittinger's constant.
- However, the expression $K_R = \frac{6KE_s}{\phi \rho_p}$ is more valid as it involves both sphericity and density terms.
- The inverse of Rittinger's constant is known as Rittinger's number.
- Rittinger's law is applicable mainly to that part of the process, where ~~the increase~~ new surface is being created and holds most accurately for fine grinding where the increase in surface per unit mass of material is predominant.
- Also, this law is applied in cases where the energy input per unit mass of material is not too high.
- This law is applicable for feed size of less than 0.05 mm.

Kick's Law:-

→ The work required for crushing a given mass of material is constant for a given reduction ratio irrespective of the initial size.

→ The reduction ratio is "The ratio of initial particle size to final particle size."

$$W_k = \frac{P}{m} = K_k \ln \left(\frac{\bar{D}_{vsf}}{\bar{D}_{vsp}} \right)$$

K_k = Kick's Constant.

Ex:

→ If a given quantity of material is being crushed from 100 mm to 20 mm, or from 30 mm to 6 mm then

in both the cases the energy requirement will be same as the reduction ratio ($\frac{100}{20} = \frac{30}{6} = 5$).

→ This law is more accurate than Rittinger's law for coarse crushing where the surface area produced per unit mass is considerably less.

→ This law is applicable for feed size of greater than 50 mm.

Bond's Law:-

→ F. C. Bond suggested an intermediate law.

"The work required to form particles of size D_{pp} from a very large particle size is proportional to the square root of the surface to volume ratio (S_p/V_p) of the product."

→ This law is applicable for feed size b/n 0.05 to $\frac{50}{mm}$

$$\boxed{\frac{S_p}{V_p} = \frac{6}{\phi D_p}}$$

Mathematically

$$W_B = \frac{P}{m} = K \left[\left(\sqrt{\frac{S_p}{V_p}} \right) \right] = K \sqrt{\frac{6}{\phi D_{pp}}} = K \sqrt{\frac{6}{\phi}} \cdot \frac{1}{\sqrt{D_{pp}}} \\ = K_b \frac{1}{\sqrt{D_{pp}}}$$

$$K_b = K \sqrt{\frac{6}{\phi}} = \text{Bond's constant.}$$

→ more precisely

$$W_B = \frac{P}{m} = K_b \left[\frac{1}{\sqrt{D_{pp}}} - \frac{1}{\sqrt{D_{pf}}} \right]$$

→ When feed size becomes is very large, the term $\frac{1}{\sqrt{D_{pf}}}$ becomes negligible and the expression remains same

→ The Bond's constant (K_b), is dependent on the type of machine used and on the material to be crushed.

Work index:- The gross energy requirement in kilowatt hour per short-ton of feed (kwh/ton of feed) to reduce a very large particle to such a size that 80% of the product will pass through a 100- μ m or 0.1 mm screen.

$$\rightarrow W_i = k_b \frac{1}{\sqrt{D_{pp}}} \Rightarrow k_b = W_i \sqrt{D_{pp}}$$

P is in kW, m is in tonnes/hour

(i) D_{pp} is in μ m then $k_b = 10 W_i$ and

(ii) D_{pp} is in mm then $k_b = \sqrt{0.1} W_i = 0.3162 W_i$

\rightarrow Thus if 80% of feed particles pass through a D_{pf} mm screen and 80% product particle pass through a D_{pp} mm screen then

$$W_b = \frac{P}{m} = 0.3162 W_i \left[\frac{1}{\sqrt{D_{pp}}} - \frac{1}{\sqrt{D_{pf}}} \right]$$

Generalised Law:

\rightarrow All the above three laws can be derived from a generalised differential equation relating to work required for crushing and the particle size.

$$d(W) = d\left(\frac{P}{m}\right) = -K \frac{d(D_{vs})}{(D_{vs})^n}$$

putting $n=2, 1,$ and 1.5 in above eqn at integerity b/n suitable limits, we will get Rittinger's, Kick's and Bond's law respectively.

problem:- particles of the average feed size of 50×10^{-4} m are crushed to an average product size of 10×10^{-4} m at the rate of 20 tonnes/hr. At this rate, the crusher consumes 40 kW of power of which 5 kW are required for running the mill empty.

Calculate the power consumption if 12 tonnes/h of this product is further crushed to 5×10^{-4} m size in the same mill?

→ Assume that Rittinger's law is applicable.

Soln:-

Rittinger's law is $\frac{P}{\dot{m}} = K_R \left[\frac{1}{\bar{D}_{rsp}} - \frac{1}{\bar{D}_{rstf}} \right]$

$$\bar{D}_{rstf} = 50 \times 10^{-4} \text{ m}, \quad \bar{D}_{rsp} = 10 \times 10^{-4} \text{ m} \quad \text{and} \quad \dot{m} = 20 \text{ tonnes/hr}$$

→ As 5 kW of power is consumed for running the mill empty out of 40 kW of power fed to the mill, the actual power consumption is $P = 40 - 5 = 35 \text{ kW}$

$$\frac{35}{20} = K_R \left[\frac{1}{10 \times 10^{-4}} - \frac{1}{50 \times 10^{-4}} \right] \Rightarrow 1.75 = K_R (1000 - 200)$$

$$\Rightarrow 1.75 = 800 K_R \Rightarrow K_R = 2.1875 \times 10^{-3} \frac{\text{kWh}}{\text{tonne}}$$

→ This value of K_R is constant for the machine.

Now for $\bar{D}_{rstf} = 10 \times 10^{-4}$ m, $\bar{D}_{rsp} = 5 \times 10^{-4}$ m and $\dot{m} = 12$ tonnes/h

we have:

$$\frac{P}{12} = 2.1875 \times 10^{-3} \left[\frac{1}{5 \times 10^{-4}} - \frac{1}{10 \times 10^{-4}} \right]$$

$$P = 2.1875 \times 10^{-3} \times 12 (2000 - 1000)$$

$$P = 26.25 \text{ kW}$$

Problem:

(9)

A sample of material is crushed in a Blake jaw crusher such that the average size of the particles is reduced from 50 mm to 10 mm with energy consumption of 13 kW/(t/ls).

Determine the consumption of energy to crush the same material of 75-mm average size to an average size of 25 mm using Rittinger's law and Kick's law.

Soln: $\bar{D}_{vsf} = 50 \text{ mm}$, $\bar{D}_{vsp} = 10 \text{ mm}$

Energy consumption, $\frac{P}{\dot{m}} = 13.0 \text{ kW/(t/ls)} = 3.61 \frac{\text{kWh}}{\text{tonne}}$

Case I Rittinger's law:- $\frac{P}{\dot{m}} = K_R \left(\frac{1}{\bar{D}_{vsp}} - \frac{1}{\bar{D}_{vsf}} \right)$

Thus, $3.61 = K_R \left[\frac{1}{10} - \frac{1}{50} \right] = 0.08 K_R \Rightarrow K_R = 45.125 \frac{\text{kWh}}{\text{tonne}}$

→ When the same machine is used. 75 mm to 25 mm

$\frac{P}{\dot{m}} = 45.125 \left(\frac{1}{25} - \frac{1}{75} \right) = 1.218 \frac{\text{kWh}}{\text{tonne}}$

Case II Kick's law $\frac{P}{\dot{m}} = K_K \times \ln \left[\frac{\bar{D}_{vsf}}{\bar{D}_{vsp}} \right]$

Thus, $3.61 = K_K \times \ln \left(\frac{50}{10} \right) = 1.609 K_K$

$\Rightarrow K_K = 2.24 \frac{\text{kWh}}{\text{tonne}}$

→ Now 75 mm to 25 mm

$\frac{P}{\dot{m}} = 2.24 \times \ln \left(\frac{75}{25} \right) = 2.46 \frac{\text{kWh}}{\text{tonne}}$

Case III

Bond's law $\frac{P}{\dot{m}} = K_B \left(\frac{1}{\sqrt{D_{p1}}} - \frac{1}{\sqrt{D_{p2}}} \right)$

prob: 270 kW of power is required to crush 150 tonnes/h of a material.

If 80% the feed passes through a 50-mm screen and 90% the product passes through a 3-mm screen, calculate the work index of the material.

soln:

$$\text{Bond's law in } \frac{P}{\dot{m}} = 0.3162 W_i \left[\frac{1}{\sqrt{D_{pf}}} - \frac{1}{\sqrt{D_{pp}}} \right]$$

(i) Data: $\dot{m} = 150 \text{ tonnes/h}$, $P = 270 \text{ kW}$, $D_{pf} = 50 \text{ mm}$ and $D_{pp} = 3 \text{ mm}$

$$W_i = \frac{P}{0.3162 \times \dot{m} \times \left[\frac{1}{\sqrt{D_{pp}}} - \frac{1}{\sqrt{D_{pf}}} \right]} = \frac{270}{0.3162 \times 150 \times \left(\frac{1}{\sqrt{3}} - \frac{1}{\sqrt{50}} \right)}$$
$$= 13.06 \text{ kWh/tonne.}$$

(ii) For the same feed at the same feed rate, if $D_{pp} = 1.5 \text{ mm}$ the power required will be.

$$P = 0.3162 \times \dot{m} \times W_i \left[\frac{1}{\sqrt{D_{pp}}} - \frac{1}{\sqrt{D_{pf}}} \right]$$
$$= 0.3162 \times 150 \times 13.06 \left[\frac{1}{\sqrt{1.5}} - \frac{1}{\sqrt{50}} \right] = 418.16 \text{ kW.}$$

principals types of size-reduction machines

(10)

- 1) Crushers (coarse and fine) — compression
 - (a) Jaw crusher
 - (b) Gyratory crusher
 - (c) Crushing rolls
- 2) Grinders (Intermediate and fine) impact and attrition
 - (a) Hammer mills
 - (b) Rolling-compression mills
 - (i) Bowl mills
 - (ii) Rolling mills
 - (c) Attrition mills
 - (d) Revolving mills
 - (i) Rod mills
 - (ii) Ball mills ; pebble mill
 - (iii) Tube mills
- 3) Ultrafine grinders attrition
 - (a) Hammer mill with internal classification
 - (b) Fluid energy mill
 - (c) Agitated mills
- 4) Cutting machines
 - (a) Knife cutter, dices, slitters.

→ Factors for selecting size reduction equipment.

- i) proportion of the feed to be handled.
- ii) Nature of the product required
- iii) Quality of the material to be handled
- iv) Size of the material to be handled
- v) Speed of the size reduction equipment

- primary crusher is the one which crushes very large lumps to yield a product 150 to 250 mm in size.
- A secondary crusher is the one which takes the product from a crusher and reduces it to particles of about 6 mm size.
- Grinders are the machines which reduce crushed feed to powder. A
- An intermediate grinder yield a product that might pass a 40 mesh screen.
- A fine grinder gives a product most of which would pass a 200 mesh screen.
- Ultrafine grinders are the machines which accept feed particles having a size less than 6 mm and yield a product of size 1 to 50 μ m.
- Cutters are size reduction machines which give particles of definite size and shape, usually 2 to 10 mm in length.

Crushers

- slow speed, coarse reduction of large quantities of solids
- handle large lumps of hard materials
- mining industries.

Jaw crusher:-

- 1) Blake jaw crusher → Dodge jaw crusher

↙

The movable jaw is pivoted at the ~~bottom~~ ^{top}, thus giving greatest/minimum movement at the bottom.

↘

movable jaw is pivoted at the bottom thus giving greater movement at top.

↳ less used due to its tendency to choke because of the minimum movement of the jaw at bottom.

Difference b/n BJC at DJC:

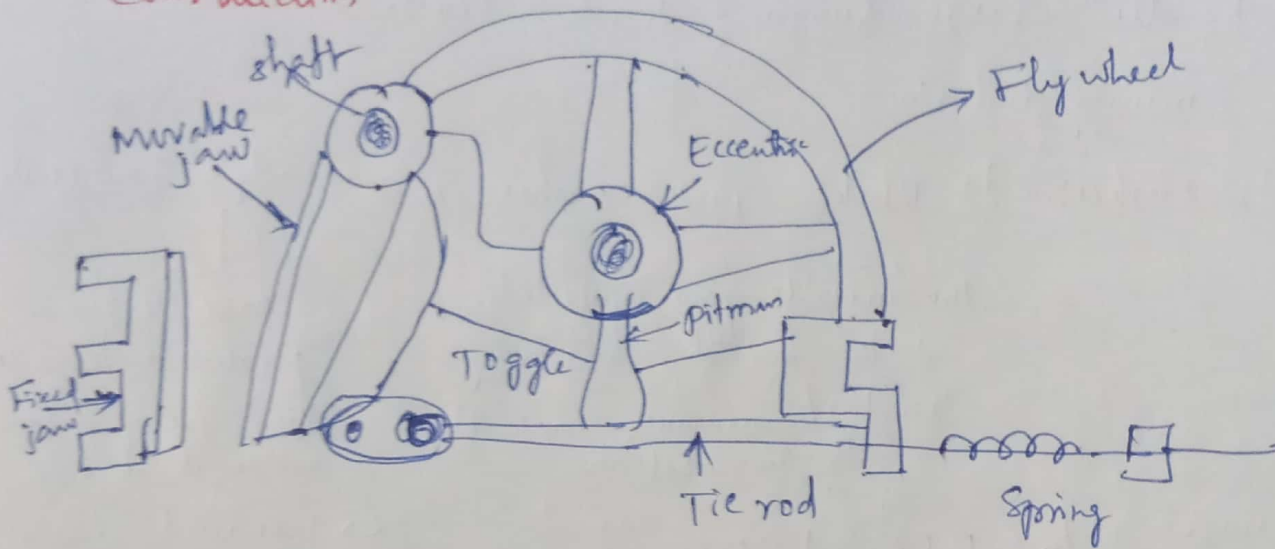
- i) BJC high production rate
- ii) large reduction ratio is not possible
- iii) comparatively made in large size
- iv) Does not give uniform product
- v) commonly/widely used

Blake Jaw crusher

principle;

Compression, it reduces size by compressive force cubical products, with minimum fines

Construction;



Blake jaw crusher

- The jaws are set to form a 'V' open at the top
- The swinging or movable jaw which reciprocates in a horizontal plane usually makes an angle of 20° to 30° with the fixed jaw (which is nearly vertical)
- The jaws are usually made of manganese steel or some other material that will withstand abrasion
- The faces of the crushing jaws are usually corrugated for concentrating the pressure on relatively small areas.

- eccentric cause the pitman to oscillate in a vertical direction, and this vertical movement is communicated horizontally (reciprocating motion) to the movable jaw by the toggles.
- The speed of the operation should not be high or otherwise a large quantity of fines is produced as the material cannot escape quickly and gets repeatedly crushed.
- Since the crushing action is intermittent, the loading on the machine is uneven and due to this the crusher incorporates a heavy flywheel.
- Since the maximum movement of the jaw is at the bottom (discharge), there will be little tendency for the crusher to choke.

protection of machine:-

- The machine is usually protected so that it is not damaged if accidental pieces of iron such as hammer heads, stray bolts, etc. enter into the crusher, by making one of the toggles in the driving mechanism relatively weak.
- That is, one particular toggle is made into two pieces which are held together with bolts that are purposely made the weakest part in the crusher.

so that, if stresses are set up, these bolts shear first.

→ Thus the failure is made at a point that can be easily and quickly repaired, instead of breaking some vital part of the machine.

Working

→ The material to be crushed is admitted between two jaws from the top.

→ The material caught between the upper part of the jaws is crushed to a smaller size during forward motion by compression.

→ The crushed material then drops/falls into the narrower space below during the backward motion and is re-crushed as the jaws close next time.

→ After sufficient reduction, the crushed material drops out the bottom of the machine.

→ The jaws usually open and close 250-400 times per minute.

Capacity of Jaw Crushers

The Theoretical capacity of a jaw crusher is

$$Q = \frac{\rho_p A W_j N_j (1 - \epsilon)}{60} \text{ kg/h}$$

ρ_p - density of materials

A - Area of swing, m^2

W_j - jaw width, m

N_j - Number of swings per minute, min^{-1}

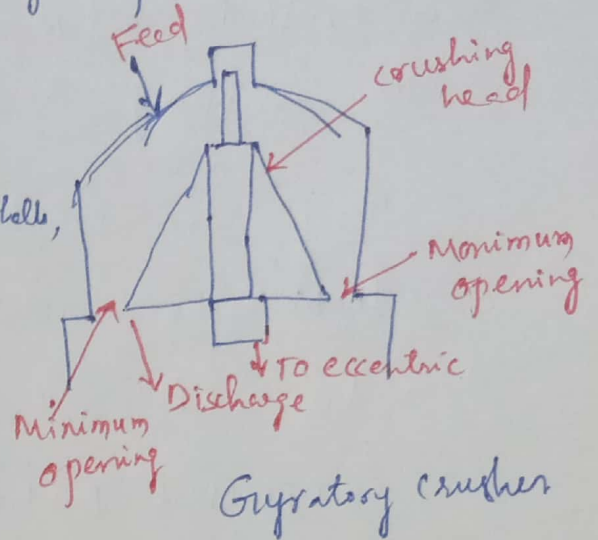
and ϵ - porosity of particles

Gyratory crusher

principle: It works on the principle of compression

Construction and Working:

→ It consists of two vertical conical shells, the outer shell having its apex in downward direction while the inner cone is positioned with its apex upward.



- The inner shell acts as the crushing head, which is in the form of truncated cone and is mounted on an oscillating shaft.
- The upper end of this cone is held in a flexible bearing while the lower end is connected to an eccentric
- The eccentricity causes the conical crushing head to oscillate between open side setting and closed side setting (OSS and CSS) discharge openings.
- Hence, the crushing action takes place around the whole of the cone and is continuous.
- The eccentricity also determines the capacity of gyratory crushers.

- The material to be crushed is fed from the top and is crushed between the stationary outer shell and the crushing head.
- They are crushed several times before being discharged from the bottom.
- An additional crushing effect occurs between the compressed particles, resulting in less wear of the crusher materials. This is known as interparticular crushing.
- ↳ The material to be crushed is charged from the top. The conical head gyrates inside the casing.
- At any point on the periphery of the casing, the bottom of the crushing head moves towards and then away from the stationary wall.
- The solids caught in the 'V' shaped space between the head and the casing are broken and rebroken until they drop out from the bottom of the machine.
- The speed of the crushing head usually lies b/w 125-425 gyrations per minute.

Features of the Gyratory Crushers;

(14)

1. Continuous in action, 2. fluctuations in stresses are smaller
3. Load on the motor is nearly uniform
4. power consumption per ton of material crushed is smaller
- and 5. requires less maintenance than jaw crusher.

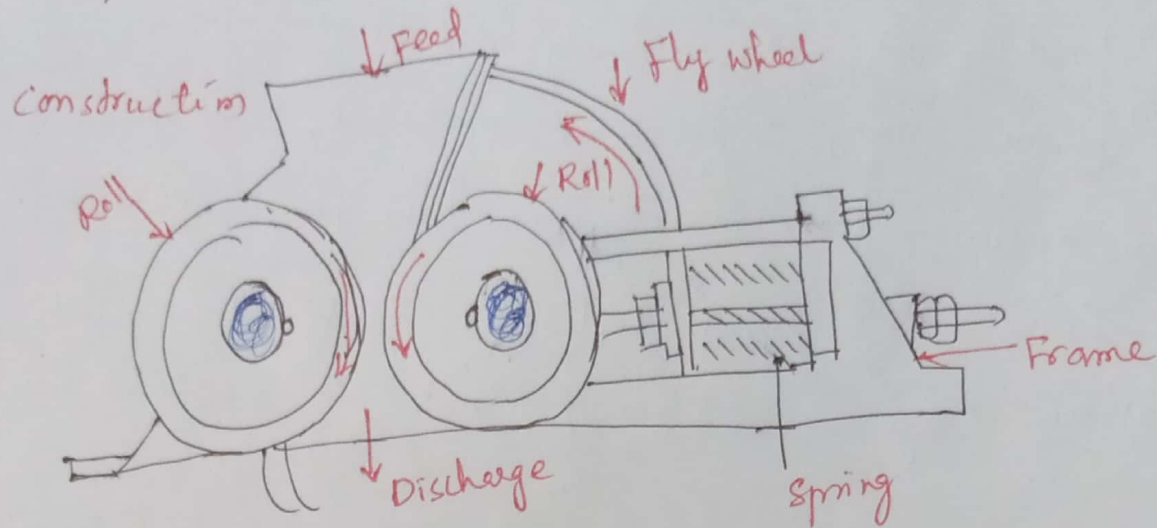
→ Since the capital cost is high, it is suitable only where large quantities of materials are to be handled.

Jaw crusher	Gyratory crusher
→ Reciprocating	Gyratory
→ Discharge is discontinuous	Discharge is continuous
→ primary crusher (larger size feed)	Secondary crusher (smaller size feed)
→ The load on the motor is not uniform	Load on the motor is uniform nearly
→ More maintenance is required	Less maintenance is required.
→ power consumption per ton of material crushed is more	power consumption is low
→ Low capital cost	high capital cost
→ It has smaller capacity when used to produce/ effect a small size reduction	Large capacity

Roll Crushers

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principle: Size reduction is achieved by compression



- Smooth roll crusher consists of two heavy metal rolls of the same dimension (diameter) placed side by side each other on the horizontal position.
- The rolls mounted on the shafts are rotated towards each other at the same speed.
- One of the shafts moves in the fixed bearings while other moves in the movable bearings.
- The clearance b/n the rolls can be adjusted according to the size of feed and the size of product required.
- One of the rolls is driven directly and the other by friction with the solids being crushed.
- The rolls have relatively narrow faces and are large in dia, therefore they can nip moderately large lumps.

- (15)
- The material fed to the machine is reduced in size by compression and discharged from the bottom.
 - The machine is protected by spring loading (i.e., by mounting the bearings of one of the roll shafts against coiled springs) against damage due to tramp and very hard material.
 - The speed of rolls varies from 50 to 300 rev/min.
 - Secondary crushers, feed 12-75 mm product 12 mm to about 20 mesh.

Working

- The material to be crushed is fed from the top.
- As the rolls rotate, the material gets caught between them and gets reduced in size by compression and discharged from the bottom.

Industrial Applications:-

- Used in situations in which fines are to be minimised
- They are employed for crushing of oil seeds, coal, phosphate rocks, abrasive materials, lime, limestone petroleum coke, and explosive materials in gun powder industries.

Selection of Crushing Rolls and Angle of Nip

- while selecting the rolls for a certain duty, it is necessary to know
- (i) the size of feed, (ii) the size of product and (iii) the amount of material to be handled.
- The coefficient of friction between the roll surface and the material to be crushed, incorporated with a relation b/w the size of the feed and the size of the product fixes the diameter of rolls and also determines whether a particle will be drawn into rolls and gets crushed or not.

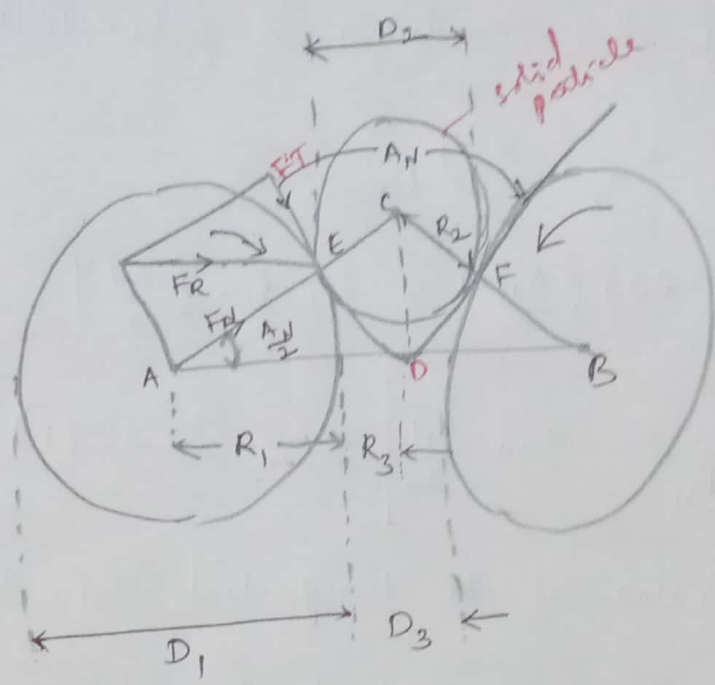
Assumptions:

- (i) The particle to be crushed is spherical
- (ii) The roll surfaces are smooth,
- and (iii) The gravity of the feed particle is negligible.

→ Consider a spherical particle of diameter D_2 that is to be crushed.

→ It is positioned between a pair of crushing rolls of diameter D_1 .

→ Let D_3 be the minimum spacing b/w the rolls which is also the maximum dimension of the crushed particles and A_N be the angle b/w two common tangents to the particle and each of the rolls.



Angle of nip:-

The angle formed by the tangents to the roll faces at a point of contact with a particle to be crushed.

Forces exerted by crushing rolls on a spherical particle.

- Let F_T and F_N be the tangential and normal forces acting on the particle respectively at a point of contact with the rolls,
- Let F_R be the resultant force of the above two forces.
- It may be noted that the particle will be nipped and gets crushed if the resultant force F_R is directed downwards
- Otherwise the particle will slide on the rolls or be thrown up and will not be crushed at all.
- In other words, if the vertical component of the tangential force, $F_{T_v} = F_T \cos\left(\frac{A_N}{2}\right)$ is greater than the vertical component of the normal force $F_{N_v} = F_N \sin\left[\frac{A_N}{2}\right]$ then only the particle will be nipped and crushed between the rolls.

→ Mathematically, this condition may be written as

$$F_{T_V} > F_{N_V}$$

$$\Rightarrow F_T \cos\left(\frac{A_N}{2}\right) > F_N \sin\left[\frac{A_N}{2}\right]$$

$$\Rightarrow \boxed{\frac{F_T}{F_N} > \tan \frac{A_N}{2}} \quad \text{--- (1)}$$

→ But the ratio of the tangential force to the normal force is the coefficient of the friction, μ

$$\therefore \mu > \tan \frac{A_N}{2}$$

→ When $\mu = \tan \frac{A_N}{2}$, under this limiting condition of crushing, the angle A_N is called the angle of nip.

→ For all the practical purposes, the value of the angle A_N is usually taken as 32° .

→ Now from the triangle ACD, we have

$$\cos\left[\frac{A_N}{2}\right] = \frac{AD}{AC}$$

$$\Rightarrow \boxed{\cos \frac{A_N}{2} = \frac{R_1 + R_3}{R_1 + R_2}} \quad \text{--- (2)}$$

where $R_3 =$ half the distance between two rolls.
 $= \frac{D_3}{2}$

→ Eqn (2) gives the relationship b/w the size of feed, radius of rolls and the gap b/w the rolls with the angle of nip.

→ Eqn (2) can also be written in terms of diameters as (7)

$$\cos \frac{A_N}{2} = \frac{D_1 + D_3}{D_1 + D_2}$$

→ For $A_N = 32^\circ$, we have from Eqn (2):

$$\cos \left(\frac{32^\circ}{2} \right) = \cos 16^\circ = 0.961 = \frac{R_1 + R_3}{R_1 + R_2}$$

→ The limiting value for the angle $\frac{A_N}{2}$ at which the resultant force F_R acts horizontally is called the angle of bite and under this condition, there will be little or no crushing at all.

Capacity of crushing rolls.

→ The theoretical capacity of a crushing roll 'Q' in kg/h is given by

$$Q = 60 \pi D_1 D_3 b N \rho$$

b — breadth of roll face, m

N — number of revolutions per minute, rpm

ρ — density of the material to be crushed, kg/m³

→ The volumetric capacity is affected by speed, nip, diameter, and breadth of roll face.

→ And, actually capacity is usually b/m 10-30% of the theoretical one.

problem: What should be the maximum product size obtained from a feed size of 60 cm, using a double-roll crusher having rolls of 140-cm diameter and 50-cm width face, if the co-efficient of friction is 0.28? Compare the result when the coefficient of friction is 0.32.

Soln:

Angle of nip is $A_N = 2 \tan^{-1} \mu$ and

$$\cos \frac{A_N}{2} = \frac{D_1 + D_3}{D_1 + D_2}$$

$$\mu = 0.28 ; D_1 = 140 \text{ cm} = 1.4 \text{ m} ; D_2 = 60 \text{ cm} = 0.6 \text{ m}$$

$$\text{Now, } A_N = 2 \tan^{-1} \mu = 2 \tan^{-1} (0.28) = 31.28^\circ$$

$$\text{Thus } \cos \frac{31.28^\circ}{2} = \frac{1.4 + D_3}{1.4 + 0.6} \Rightarrow D_3 = 0.526 \text{ m} = 52.6 \text{ cm}$$

→ 52.6 cm is the maximum size of product obtained from this crusher.

→ And when $\mu = 0.32$, the angle of nip, $A_N = 35.48^\circ$.

$$\text{Thus } \cos \frac{35.48^\circ}{2} = \frac{1.4 + D_3}{1.4 + 0.6}$$

→ $D_3 = 0.228 \text{ m} = 22.8 \text{ cm}$ is the minimum size of product obtained from this crusher.

Note: It is evident from the above results that with the increase of μ , smaller and smaller product size can be obtained using the same feed size and the same crusher.

problem:- A slaked lime manufacturer uses a roll crusher to crush limestone (specific gravity = 2.66). The crushing rolls have rolls of 150 cm as diameter by width of 50-cm. The clearance b/w rolls is 1 cm. The rolls run at 100 rpm. The angle of nip is 30° . Find out (i) the maximum feed size to the crusher, and (ii) the theoretical capacity of the crushing rolls.

soln:-

(i)

$$\cos \frac{\alpha_N}{2} = \frac{D_1 + D_3}{D_1 + D_2}; \quad \alpha_N = 30^\circ, \quad D_1 = 1.5 \text{ m}, \quad D_3 = 0.01 \text{ m}$$

$$\cos \frac{30^\circ}{2} = \frac{1.5 + 0.01}{1.5 + D_2} \Rightarrow D_2 = 0.0647 \text{ m} = 6.47 \text{ cm}$$

→ 6.47 cm is the maximum feed size to the crusher.

(ii) The theoretical capacity of crushing rolls is

$$Q = 60 \pi D_1 D_3 b N \rho \text{ kg/h}$$

$D_1 = 1.5 \text{ m}; \quad D_3 = 0.01 \text{ m}; \quad b = 0.5 \text{ m}; \quad N = 100 \text{ rpm}$
and specific gravity = 2.66

Thus density of limestone = $2.66 \times 10^3 \text{ kg/m}^3$

Theoretical Capacity

$$Q = 60 \times \pi \times 1.5 \times 0.01 \times 0.5 \times 100 \times 2.66 \times 10^3$$

$$\Rightarrow Q = 376048.64 \text{ kg/h}$$

$$Q = 376.049 \text{ tonne/h}$$

Grinders

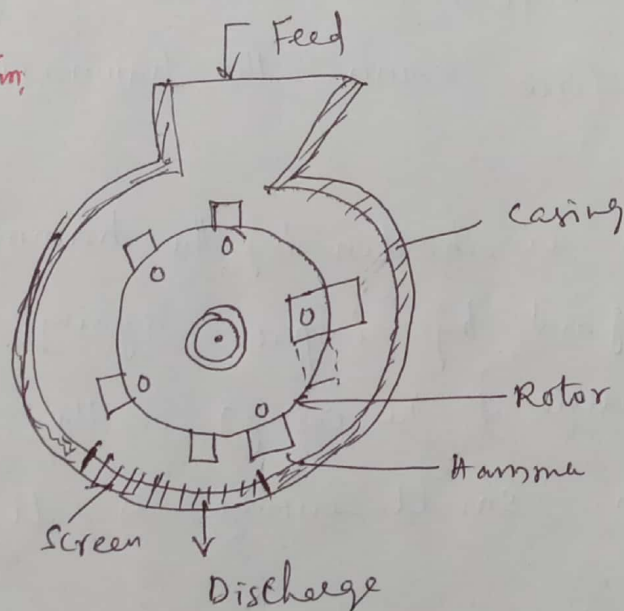
(19)

- Subdividing the solids to a finer product than crushing
- The size reduction machines employed for an intermediate duty.
- A grinder is often charged with the product from a crusher which it reduces to powder

Hammer mill

principle: Size reduction is achieved by impact and attrition

Construction:



- The hammer mill consists essentially of a high speed rotor turning inside a cylindrical casing.
- The rotor is mounted on a shaft which is usually horizontal. The swing hammers are pinned to a rotor disk.
- The hammers are rectangular bars of metal with plain or enlarged ends.
- The particles are broken by the set of swing hammers
- The product falls through a gate or screen which forms the lower portion of the casing.

- Several rotor disks each carrying four to eight swing hammers are often mounted on a single shaft.
- The rotor disk diameter ranges from 150 mm to 250 mm.
- As the hammers are hinged, the presence of any hard material does not cause damage to the equipment.
- The hammers can be readily replaced when they wear out.

Working:-

- The material to be crushed is fed from the top of casing. The shaft is rotated at a high speed and centrifugal force causes the hammers to swing out radially.
- The material is beaten by the hammers around inside of the casing and by impact against the beaker plates (located on inside of the casing) or the screen is crushed until it is small enough to fall through the screens.
- Hammer mills are employed to grind tough fibrous solids like bark or leather, steel turnings, hard rock, sticky clay, etc.

Tumbling mills

- A cylindrical shell rotating with its axis either horizontal or at a small angle to the horizontal and charged with a grinding medium to about half its volume constitutes a tumbling/grinding mill
- Used as fine grinding machines
- Categorized into ball, rod, tube and pebble mills
 - ↓ steel balls
 - ↓ rods
 - ↓ small balls
 - ↓ ceramic pebbles
- May be operated batch wise or continuously.

Characteristics of various tumbling mills

Characteristic	ball mill	Rod mill	Tube mill	pebble mill
principle of comminution	Impact	Rolling compression and attrition	Impact	Impact
Material of construct of grinding media	Steel	High Carbon Steel	steel	Ceramic pebbles made of flint or porcelain
Diameter of grinding media	12-125 mm	50 mm	-	-
Feed Size	Upto 50 mm	Upto 25 mm	upto 25 mm	upto 25 mm
product size	Fine	Uniform fine	Fine	Fine.
L/D ratio	1 to 1.5:1	1.5 to 3:1	2 to 4:1	1 to 2:1
Applications	Coal, Pigment, feldspar for pottery	particularly for sticky material and not suitable for tough material	same as ball mill but residue time is more	paint and pigment industries cosmetic industries

Ball mills

→ popular due to their low operating and maintenance costs regardless of whether the material displays Mohs hardness values of over 4 or is soft - such as limestone or bauxite

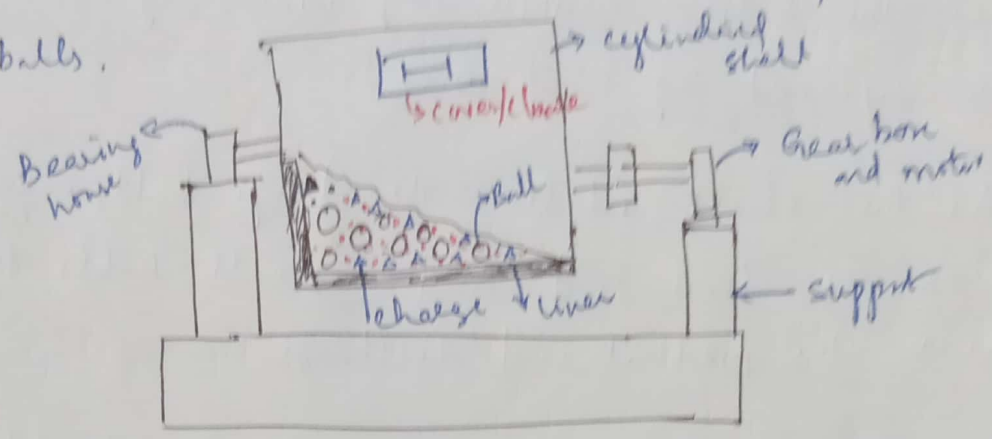
principle: Impact of balls, which fall from the top of the shell on to the feed particles near the bottom of the shell

Construction:

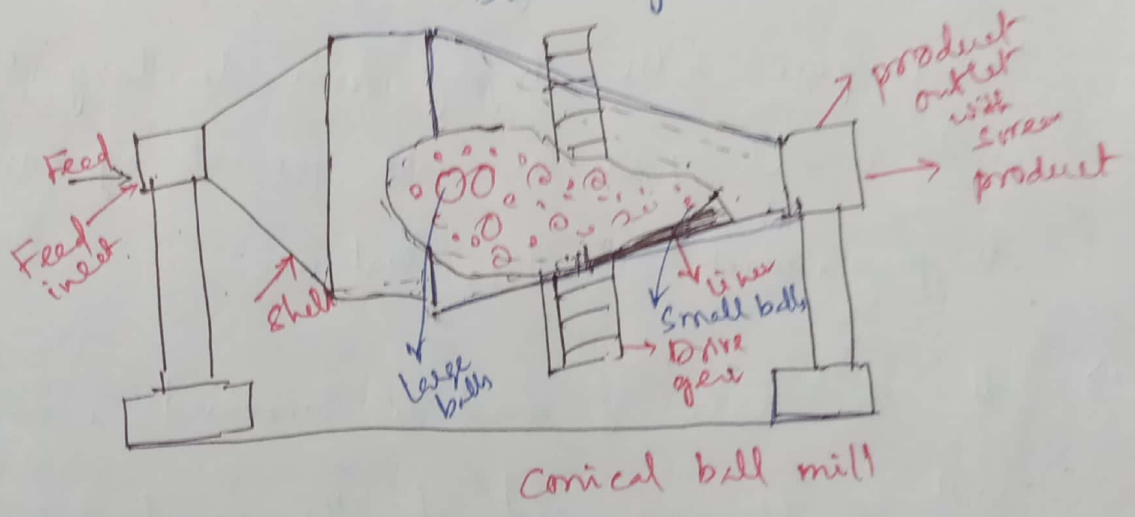
- A ball mill consists of a hollow cylindrical shell rotating about its axis.
- The axis of the shell may be either horizontal or at a small angle to the horizontal.
- It is partially filled with balls
- The inner surface of the cylindrical shell is usually lined with an abrasion-resistant material (manganese steel or rubber)
- The length of the mill is approximately equal to its diameter
- The balls occupy about 30-50% the volume of the mill.
- The diameter of the ball used lies b/w 12mm - 125mm
- The optimum diameter is approximately proportional to the square root of the size of the feed.
- shell speed 600 - 1000 rpm
- For large ball mill 3m dia and 4.25m length

→ It may be operated in a batch or continuous fashion, wet or dry.

→ In a continuously operated mill, the outlet is normally covered with a coarse screen to prevent the escape of the balls.



Batch operated Ball mill



Conical ball mill

Working:

→ In case of continuously operated ball mill, the material to be ground is fed from the left through a 60° cone and the product is discharged through a 30° cone to the right.

→ As the shell rotates, the balls are lifted up on the rising side of the shell and then they cascade down (or chip down on to the feed) from near the top of the shell.

- In doing so, the solid particles in b/m the balls are ground and reduced in size by impact.
- The mill contains balls of various ages and sizes since the balls continually wear by attrition and are replaced by new ones.
- As the shell rotates, the large balls segregate near the feed end and small balls segregate near the product end/discharge.
- The initial breaking of the particles (feed) is done by the largest balls dropping from the largest distance and small particles are ground by small balls dropping a much smaller distance.
- If the rate of feed is increased, a coarser product will be obtained and if the speed of the rotation is increased (less than critical speed), the fineness for a given capacity increases.

Applications:-

- The ball mill is used for grinding materials such as coal, pigments, and feldspar for pottery.
- Grinding can be carried out either wet or dry but the former is carried at low speeds.

Factors influencing the size of the product.

- a) Feed rate: Feed rate \uparrow size reduction \downarrow
(material is in the mill for a shorter time)
- b) properties of feed material: With a hard material, a smaller size reduction is achieved.
- c) Weight of balls: With a heavy charge of balls, we get a fine product.
- We can increase the weight of the charge by increasing the number of balls or by using a ball material of higher density.
- optimum conditions are obtained when the volume of the balls is equal to 50% that of the mill.
- So, the variation in the weight of balls is done by using materials of different densities.
- d) Speed of rotation of the mill: At low speeds, the balls simply roll over one another and little grinding is obtained, while at very high speeds, the balls are simply carried along the walls of the shell and little or no grinding takes place. So for an effective grinding the ball mill should be operated at a speed (optimum speed) equal to 50 to 75% of the critical speed.

e) Level of the material in the mill:- At low level of material in the mill results into a reduction in the power consumption. If the level of material is increased, the cushioning action increases and power is wasted by the production of undersize material in an excessive quantity.

Advantages of the ball mill

- i) low installation cost.
- ii) cost of power required is low
- iii) Suitable for materials of all degrees of hardness
- iv) Suitable for batch as well as continuous operation
- v) For grinding of certain explosive materials since it can be used with an inert atmosphere.
- vi) open and closed circuit grinding.
- vii) Cheap grinding medium

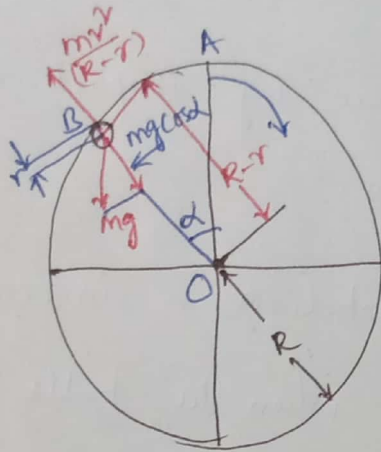
Theory

- The speed of rotation is a crucial factor for ball mills.
- At low speed speeds of rotation, the balls are lifted and simply roll back over feed materials
- Size reduction is caused by attrition and little crushing action takes place.

- Under this condition, the mill is said to be cascading. (4)
- At slightly higher speeds, the balls are carried up further inside the mill and fall back due to gravity on to the feed particles at the bottom.
- Grinding takes place by impact and the mill is said to be cataracting.
- If the speed of rotation is increased further and further a stage is reached when the balls are carried along with the walls of the mill due to higher centrifugal force and do not fall at all, and the mill is said to be centrifuging.
- The minimum speed at which centrifuging occurs is called critical speed of the mill.
- At the critical speed, the balls will be at the uppermost position of the mill and there will be no resultant force acting on the ball as the centrifugal force will be balanced by the weight of the ball.

Derivation of the critical speed of a ball mill

→ The speed at which the outermost balls break contact with the wall depends on the balance between centrifugal force and gravitational force.



Forces on ball in Ball mill

- Consider the ball at point 'B' on the periphery of the ball mill.
- Let 'R' be the radius of the mill and 'r' be the radius of ball,
- 'R-r' represents the distance b/w centre of the ball and the axis of the mill.
- Let 'α' be the angle b/w OB and vertical through the point O. (OA)
- The forces acting on the ball are:
- The force of gravity, mg (m - mass of the ball)
 - The centrifugal force, $\frac{mv^2}{R-r}$ (v - peripheral speed)

- The component of gravity opposing the centrifugal force (centripetal component) is $mg \cos \alpha$.
- As long as the centrifugal force exceeds the centripetal component of the force of gravity, the particle will not lose contact with the wall.
- As the angle ' α ' decreases, the centripetal force increases.
- Unless the speed crosses the critical value, a stage is reached where the above opposing forces are equal and the ball is ready to fall away from the wall.
- The angle at which the said phenomenon occurs is found out by equating the two opposing forces.

Thus,

$$mg \cos \alpha = \frac{mv^2}{(R-r)}$$

$$\cos \alpha = \frac{v^2}{(R-r)g}$$

- The relationship b/w the peripheral speed and the speed of rotation is

$$v = 2\pi N (R-r)$$

$$\rightarrow \cos \alpha = \frac{4\pi^2 N^2 (R-r)}{g}$$

- at critical speed $\alpha = 0$, $\cos \alpha = 1$ and $N = N_c$

$$\cos \alpha = 1 = \frac{4\pi^2 N_c^2 (R-r)}{g} \Rightarrow N_c = \frac{1}{2\pi} \sqrt{\frac{g}{R-r}} \quad \text{critical speed.}$$

Comparison of crushing and Grinding operation

S.No	Crushing	Grinding
1	compression	impact and attrition
2	large pieces to small lumps	crushed feed to powders
3	mostly open circuit	always closed circuit
4	Dry feed.	dry and wet feed
5	reduction ratio exceeds 6 to 8	the reduction ratio is high as 100
6	one shift operation as residence time is less and throughput is large	residence time is larger and throughput is smaller
7	primary and secondary	grinders and ultrafine grinders
8	Heavy duty, low speed	light duty, high speed
9	feed size 1500 - 40 mm product size 50 - 5 mm	Feed size 5 - 2 mm product size 0.1 mm (about 200 mesh)
10	Energy consumption per unit mass of product is low due to coarse particle production	high due to fine particle production

in addition to the hammers the rotor shaft carries two fans, which draw air through the mill inward toward the drive shaft and then discharge into ducts leading to collectors for the product. On the rotor disks are short radial vanes for separating over-size particles from those of acceptable size. Acceptably fine particles are carried past the radial vanes; particles that are too large are thrown back for further reduction in the grinding chamber. The maximum particle size of the product is varied by changing the rotor speed or the size and number of the separator vanes. Mills of this kind reduce 1 or 2 tons/h to an average particle size of 1 to 20 μm , with an energy requirement of about 40 kWh/t (50 hp · h/ton).

Fluid energy mills

In these mills the particles are suspended in a high-velocity gas stream. In some designs the gas flows in a circular or elliptical path; in others there are jets that oppose one another or vigorously agitate a fluidized bed. Some reduction occurs when the particles strike or rub against the walls of the confining chamber, but most of the reduction is believed to be caused by interparticle attrition. Internal classification keeps the larger particles in the mill until they are reduced to the desired size.

The suspending gas is usually compressed air or superheated steam, admitted at a pressure of 7 atm (100 lb_f/in.²) through energizing nozzles. In the mill shown in Fig. 28.10 the grinding chamber is an oval loop of pipe 25 to 200 mm (1 to 8 in.) in diameter and 1.2 to 2.4 m (4 to 8 ft) high. Feed enters near the bottom of the loop through a venturi injector. Classification of the ground particles takes place at the

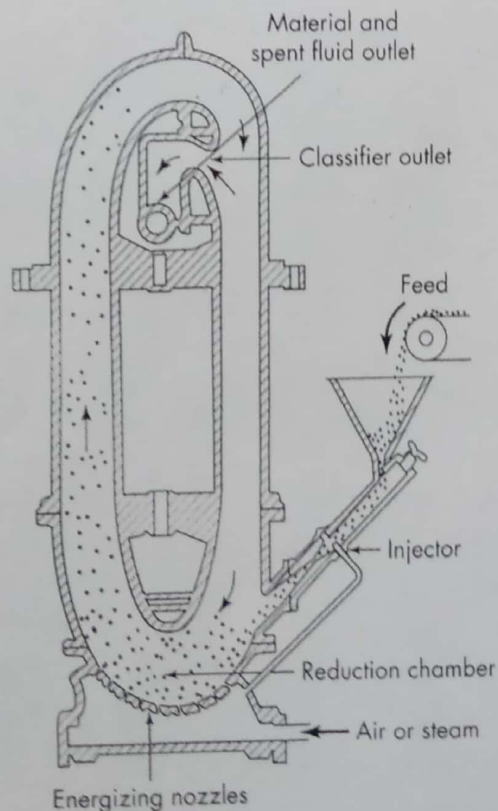


FIGURE 28.10

Fluid-energy mill. (By permission, Fluid Energy Processing and Equipment Co.)

Ultrafine Grinders

→ Mills which reduce solids to fine particles averaging 1 to 20 μm in size are called as ultrafine grinders.

Fluid Energy Mill

- Grinding takes place by attrition.
- Size reduction results from attrition b/n rapidly moving particles of the material being ground.
- A source of compressed air or gas or high pressure superheated steam that enters the grinding chamber through nozzles in the periphery at high speed provides energy to the particles to achieve high velocities.
- In fluid energy mill there are no moving parts and grinding media.
- It consists of a flat horizontal cylindrical chamber provided with tangentially arranged jet nozzles in the inner wall.
- The energy for milling (grinding) is supplied by compressed air or nitrogen gas.

- The compressed air/gas issuing through the nozzles forms a very high velocity tangential circle within the grinding chamber
- The material to be ground is fed into the same tangential circle through a venturi feeder.
- The material on the circle gets rapidly accelerated, causing it to impact against itself, hence breaking the particles to the low micron range.
- The particles that are larger in size are held towards the outer periphery of the chamber by centrifugal force.
- While the particles smaller in size travel in a spiral movement towards the central outlet from where they exit into a cyclone below for bottom discharge.
- Handles 150 microns to 2 micron.
(powders from pulveriser)
- Food products, antibiotics, dyes, cosmetics, ... etc.

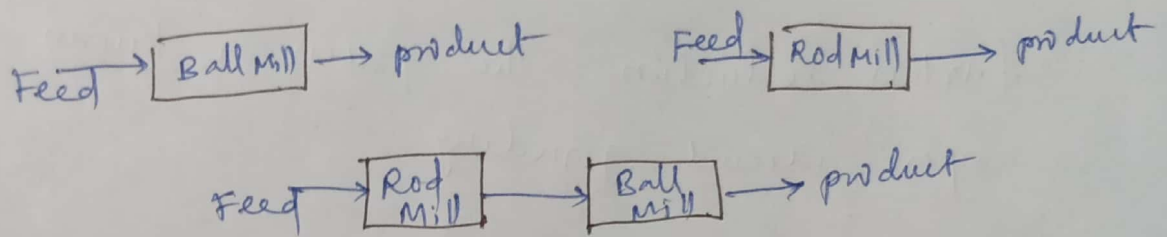
Open circuit and Closed Circuit Grinding (7)

- In many machines, the feed material is reduced to satisfactory size by passing it once through the machine.
- If the material is passed only once through the machine (crushing or grinding), and no attempt is made to return the oversize material to it for further reduction, the process is known as open circuit grinding.
- If the partially ground material from the machine is sent to a size separation unit, from where the undersize is withdrawn as the product and the oversize material is returned to the machine for regrind, the process is known as closed circuit grinding.
- In case of coarse particles, the size separation unit is a screen or grizzly while it is some form classifier in case of fine powders.
- Closed circuit grinding though useful for any crusher, it is commonly employed to machines yielding a fine product.

Open circuit Grinding

- It consists of one or more grinding mills arranged in series or parallel without classification equipment.
- This method discharges a final ground as it comes from a mill and there is no return of coarse discharge back to the mill.

→ Ex:



↳ Conditions that favour OCG:-

- (i) Small reduction ratios
- (ii) Coarse reduction of particles.

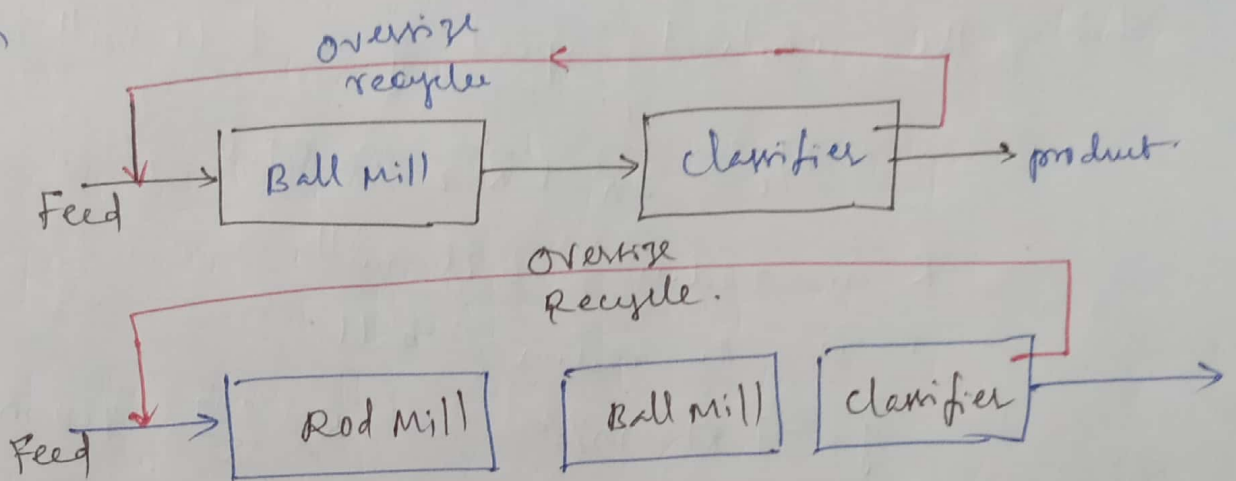
↳ Advantages

- (i) Simplicity of operation
- (ii) Minimum Equipment requirements

Closed Circuit Grinding

- It consists of one or more grinding mills with classification equipment
- the mills discharge ground product to classifier which returns the coarse product from it to the mill for further grinding.

Ex: (i)



- Advantages: (i) Higher capacity, (ii) lower power consumption per ton of product
- (iii) Suitable for reduction to fine and ultrafine sizes
- (iv) Avoids coarse material in final ground product
- (v) Eliminate overgrinding by removing fines early.

→ conditions that favour C.C.G.: (i) larger reduction ratios (ii) fine reduction of particles.

Equipment	Industry
Jaw crusher	Cement
Ball mill	Paint
Ultrafine grinder	Cosmetic & pharma
cutters	Leather Tanning
Hammer mill	Food

problems: 1. Calculate the operating speed of the mill from

the data given below:

Diameter of ball mill = 800 mm,

Diameter of ball = 60 mm

If (i) operating speed is 55% less than the critical speed.

(ii) critical speed is 40% more than the operating speed.

Soln: The critical speed of a ball mill is given by

$$N_c = \frac{1}{2\pi} \sqrt{\frac{g}{R-r}} \text{ in r.p.s}$$

R is radius of ball mill

r is the radius of ball.

$$g = 9.81 \text{ m/s}^2$$

$$R = \frac{800}{2} = 400 \text{ mm} = 0.4 \text{ m}$$

$$r = \frac{60}{2} = 30 \text{ mm} = 0.03 \text{ m}$$

$$N_c = \frac{1}{2\pi} \sqrt{\frac{9.81}{0.40 - 0.03}} = \underline{0.82 \text{ r.p.s}}$$

(I) operating speed is 55% less than the critical speed

$$55\% \text{ of the critical speed} = 0.55 \times 0.82 = 0.45 \text{ r.p.s}$$

$$\text{operating speed} = 0.82 - 0.45 = 0.37 \text{ r.p.s (22 rpm)}$$

(or)

$$\text{operating speed} = (1 - 0.55) \times \text{critical speed}$$
$$= (1 - 0.55) \times 0.82 = 0.37 \text{ r.p.s.}$$

(II) Critical speed is 40% more than the operating speed

$$\text{critical speed} = 1.40 (\text{operating speed})$$

$$\text{operating speed} = \frac{\text{critical speed}}{1.40}$$

$$= \frac{0.82}{1.40}$$

$$= 0.586 \text{ r.p.s (35 rpm)}$$

problem 2 What rotational speed, in rpm, would you recommend for a ball mill 1200 mm in diameter charged with 75 mm balls? (9)

soln:

$$N_c = \frac{1}{2\pi} \sqrt{\frac{g}{R-r}} \text{ in r.p.s}$$

$g = 9.81 \text{ m/s}^2$; R - radius of the ball mill

$$\text{Diameter} = 1200 \text{ mm} \Rightarrow R = \frac{1200}{2} = 600 \text{ mm} = 0.60 \text{ m}$$

$$\text{Diameter of the ball} = 75 \text{ mm} \Rightarrow r = \frac{75}{2} = 37.5 \text{ mm} = 0.0375 \text{ m}$$

$$N_c = \frac{1}{2\pi} \sqrt{\frac{9.81}{0.60 - 0.0375}} = 0.665 \text{ r.p.s}$$

$$= 39.90 \text{ rpm} = \underline{40 \text{ rpm}}$$

operating speed of the ball mill is 50 to 75% of the critical speed

$$\text{operating speed} = 50 \text{ to } 75\% \text{ of } 40 \text{ r.p.m.}$$

$$= \underline{20 \text{ to } 30 \text{ r.p.m.}}$$

The rotational speed that can be recommended is
b/m 20 - 30 r.p.m.

problem 3

Calculate the operating speed of the ball mill $R = 250 \text{ mm}$, $r = 20 \text{ mm}$, and operating speed is 50% of the critical speed of the mill.

soln:

$$N_c = \frac{1}{2\pi} \sqrt{\frac{g}{R-r}} = \frac{1}{2\pi} \sqrt{\frac{9.81}{0.25 - 0.02}} = 1.04 \text{ rps} = \underline{62 \text{ rpm}}$$

$$(i) \text{ operating speed of ball mill} = 0.5 N_c$$

$$= 0.5 \times 62$$

$$= \underline{31 \text{ rpm}}$$

problem 4: In a ball mill of 2000 mm diam, 100-mm diameter steel balls are being used for grinding, presently for the material being ground, the mill is running at 15 rpm. At what speed will the mill have to run if the 100 mm balls are replaced with 50 mm balls, all other conditions remaining same?

soln:

$$\cos \theta = \frac{4\pi^2 N^2 (R-r)}{g}$$

$$D = 2000 \text{ mm} ; d = 100 \text{ mm} ; N = 15 \text{ rpm}$$

$$R = 1000 \text{ mm} = 1 \text{ m} ; r = 50 \text{ mm} = 0.05 \text{ m} ; N = 0.25 \text{ rps}$$

$$\cos \theta = \frac{4 \times \pi^2 \times (0.25)^2 \times (1 - 0.05)}{9.81} = 0.238$$

Now, the 100-mm steel balls are replaced with 50-mm balls,

→ Hence, $r = \frac{50}{2} = 25 \text{ mm} = 0.025 \text{ m}$.

$$N = \sqrt{\frac{g \cos \theta}{4\pi^2 (R-r)}}$$

$$\Rightarrow N = \sqrt{\frac{9.81 \times 0.238}{4\pi^2 (1 - 0.025)}} = \sqrt{0.0606}$$

⇒ $N = 0.2461 \text{ rps} = 14.77 \text{ rpm}$ is the speed of the ball mill when the balls are replaced with 50 mm balls.